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1

1.1

Generally usefull stuff:

$$f(x) = x \cdot \exp(-x) \quad (1)$$
$$f'(x) = (x-1) \cdot \exp(-x) \quad (2)$$
$$f''(x) = (x-2) \cdot \exp(-x) \quad (3)$$
$$F(x) = -\exp(-x) \cdot (x+1) \quad (4)$$

Mean m

$$m = \int_0^\infty f_m(x)\, dx, \text{ with } f_m(x) = x \cdot f(x) \quad (5)$$
$$F_m(x) = -\exp(-x) * (x^2 + 2x + 2) \quad (6)$$
$$m = \lim_{a\to\infty} F_m(a) - F_m(0) \quad (7)$$
$$= \lim_{a\to\infty} 2 - \exp(-x) * (x^2 + 2x + 2) = 2 \quad (8)$$

Mode e

Three conditions have to be fulfilled for any value e to be a maximum:

$$f'(e) = (e-1) \cdot \exp(-e) = 0 \quad (9)$$
$$f''(e) = (e-2) \cdot \exp(-e) < 0 \quad (10)$$
$$e > 0 \quad (11)$$
$$\Rightarrow e \in \{1\} \quad (12)$$

Since there is only one maximum, it is at the same time the mode.

Median c

$$0.5 = \int_0^c f(x)\, dx = F(c) - F(0) \quad (13)$$
$$\Rightarrow 0.5 = (c+1) \cdot \exp(-c) \quad (14)$$
$$c > 0 \quad (15)$$
$$c \approx 1.67835 \quad (16)$$

The right hand side of 14 is a transcendental function of c. See numerical code for solution.

RMS r

$$r^2 = \int_0^\infty f_r(x)\,dx, \text{ with } f_r(x) = (x-m)^2 \cdot f(x) \tag{17}$$

$$F_r(x) = -\exp(-x) * (x^2 - x^2 + 2x + 2) \tag{18}$$

$$r^2 = \lim_{a\to\infty} F_r(a) - F_r(0) = F_r(0) = 2 \tag{19}$$

$$\Rightarrow r = \sqrt{2} \tag{20}$$

1.2

Assuming a binomial process with probability of success (hitting the city) $p = 0.02$ in each of the $n = 100$ trials. The probability $P_{100,0.02,0}$ of zero successes $k = 0$ is:

$$P_{100,0.02,0} = \frac{n!}{k! \cdot (n-k)!} \cdot p^k \cdot (1-p)^{n-k} \approx 0.13262 \tag{21}$$

The number of trials necessary to have no sucesses with a probability of less than $1 - 0.95$ is bounded by the following inequality:

$$1 - 0.95 > P_{n,0.02,0} = \frac{n!}{k! \cdot (n-k)!} \cdot p^k \cdot (1-p)^{n-k} \tag{22}$$

$$\Rightarrow 0.05 > (1-p)^n = 0.98^n \tag{23}$$

$$\Rightarrow n > 148.284 \tag{24}$$

At least $n = 149$ trials are necessary to have at least one hit with a confidence of 0.95.

1.3

The drop in number of deaths d and injuries i are:

$$1 - \frac{d_{2012}}{d_{2011}} = 1 - \frac{167}{220} \approx 0.241 = 24.1\% \tag{25}$$

$$1 - \frac{i_{2012}}{i_{2011}} = 1 - \frac{3611}{4039} \approx 0.106 = 10.6\% \tag{26}$$

The variance on two incident numbers n_1, n_2 is (assuming a Poisson process) the same as the numbers themselves. The variance on the difference is the sum of the individual differences. It follows that the error on the difference is $\sigma_n = \sqrt{|n_1 + n2|}$. The ratio of differences and errors for d and i are:

$$\frac{|d_1 - d_2|}{\sigma_d} \approx 2.7 \tag{27}$$

$$\frac{|i_1 - i_2|}{\sigma_i} \approx 4.9 \tag{28}$$

This means both drops are likely to reflect a change in rates (with more than 0.99 confidence).

2

2.1

$$\bar{v} = (97 \pm 4)ms^{-1} \tag{29}$$
$$E_{kin} = (1300 \pm 200)J \tag{30}$$
$$E_{kin,corr} = (1280 \pm 120)J \tag{31}$$

The last measurement deviates about 2σ from the mean. This is not a surprising occurence in a series of 7 measurements.

2.2

Note that for $\theta = 1.54 \pm 0.02$ error propagation using the derivative is not suitable for the tan, because the derivative changes to quickly.

$$\text{For } \theta = 0.54 \pm 0.02 : \tag{32}$$
$$\sin(\theta) = 0.51 \pm 0.02 \tag{33}$$
$$\cos(\theta) = 0.857 \pm 0.010 \tag{34}$$
$$\tan(\theta) = 0.60 \pm 0.03 \tag{35}$$
$$\text{For } \theta = 1.54 \pm 0.02 : \tag{36}$$
$$\sin(\theta) = 0.9995 \pm 0.00006 \tag{37}$$
$$\cos(\theta) = 0.03 \pm 0.02 \tag{38}$$
$$\tan(\theta) = 32 \pm^{61}_{12} \tag{39}$$

2.3

$$n_2 = 1.50 \pm 0.02 \tag{40}$$

2.4

$$\Delta N_{N_0} = \Delta N_0 \exp(-t/\tau) \tag{41}$$
$$\Delta N_\tau = \Delta\tau \cdot N_0 \cdot t \exp(-t/\tau)/\tau^2 \tag{42}$$
$$\Delta N_{N_0} = \Delta N_\tau \tag{43}$$
$$\Rightarrow \Delta N_0 \exp(-t/\tau) = \Delta\tau \cdot N_0 \cdot t \exp(-t/\tau)/\tau^2 \tag{44}$$
$$\Rightarrow t/\tau = \frac{\tau \Delta N_0}{N_0 \Delta\tau} = 1 \tag{45}$$

3

$$1 = \int_{-1}^{2} a\, x^2\, dx = 3a \tag{46}$$

$$\Rightarrow a = \frac{1}{3} \tag{47}$$

$$\text{mean: } m = \int_{-1}^{2} a\, x^3\, dx = \frac{15a}{4} = \frac{5}{4} \tag{48}$$

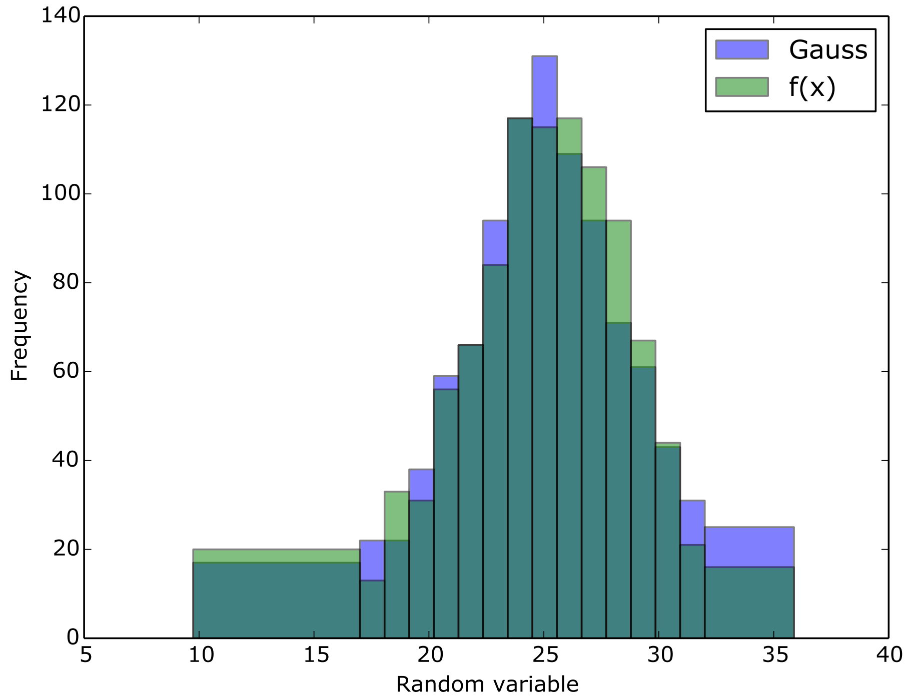
$$\text{square of width: } w^2 = \int_{-1}^{2} (x-m)^2 a\, x^2\, dx = m^2 - \frac{5}{2}m + \frac{11}{5} = \frac{51}{80} \tag{49}$$

$$\tag{50}$$

To generate random numbers according to this distribution, one can calculate the cumulative distribution $c(x) = \frac{1}{9}(x^3 + 1)$ and invert it:

$$x(c) = \begin{cases} (9c-1)^{\frac{1}{3}} & \text{if } x > \frac{1}{9} \\ -(1-9c)^{\frac{1}{3}} & \text{otherwise} \end{cases} \tag{51}$$

A uniformly distributed random variable c on the interval $[0, 1]$ can now be transformed into the desired result using $x(c)$.



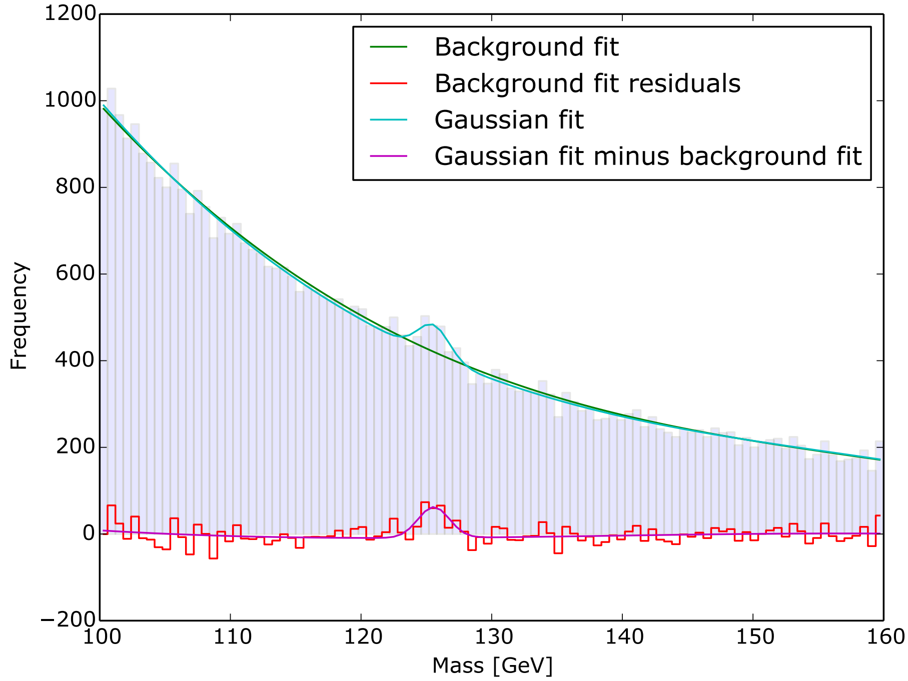
The histogram above shows a sample of a thousand values from the the sum of 20 distributions $f(x)$ and a gaussian with the same width and mean. The p-value for the hypothesis of independence of a Chi-square contingency test of the two samples is 0.41 and the respective means are 25.02 ± 0.12 for the gaussian and 25.04 ± 0.11 for the convolution of $f(x)$. Neither the difference and uncertainties of the means nor the Chi-square test indicate that the hypothesis of equality of the two underlying pdfs can be rejected with a confidence of at least 0.95.

4

4.1

The Wald-Wolfowitz test on the residuals of the background fit returns 103 and an expected number of runs of 100 ± 7. This does not indicate, that the fit is insufficient (runs and expectation are consistent).

The significance of the largest gaussian (shown in the graph below) is: 3.3σ



4.1

The third result was removed, because its distance to the mean corresponds to $\approx 20\sigma$. The true value is $2.2\mu s$, the unweighted mean of the results is $(1.99 \pm 0.06)\mu s$, the weighted mean is $(1.91 \pm 0.03)\mu s$. Both are completely inconsistent with the true value (by $\approx 10\sigma$). Using the weighted mean and uncertainty as best combined measurement, yields $\chi^2 \approx 21.2$ and $p \approx 0.006$.

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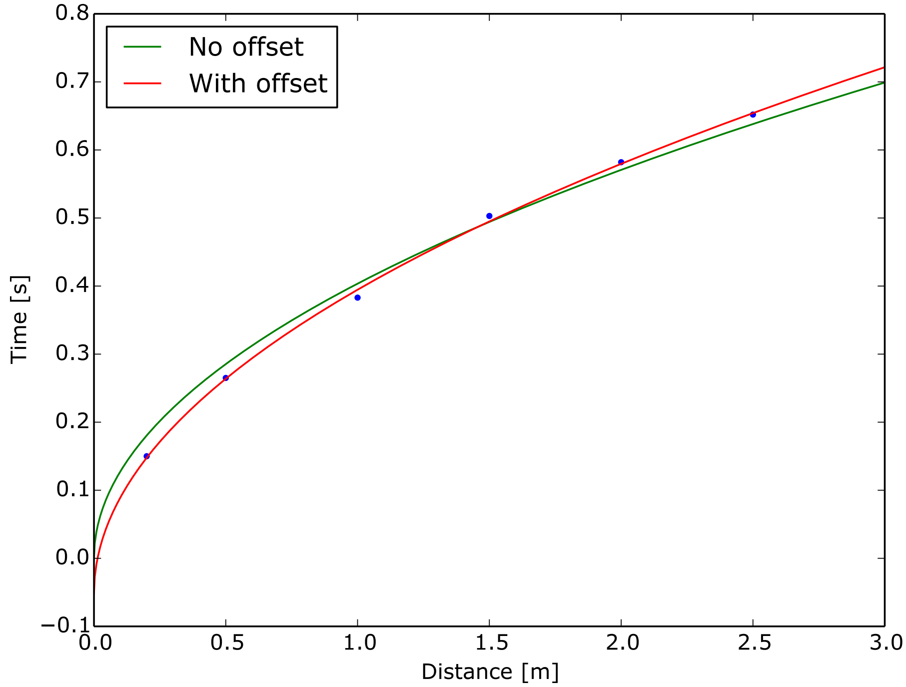

The fit quality of the first hypothesis is very bad ($\chi^2 \approx 21.6$, $p \approx 0.00061$). Adding an offset time as a parameter improves the fit enough to be reasonable ($\chi^2 \approx 2.16$, $p \approx 0.71$. Weighing the two probabilities for obtaining a χ^2 this bad or worse against each other makes me ≈ 0.99914 certain that the hypothesis of an exact release time $t_0 = 0$ should be rejected.