Applied Statistics

Central Limit Theorem






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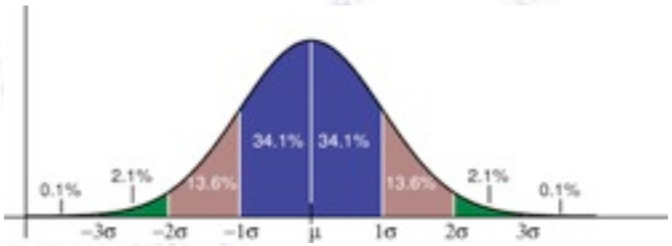

"Statistics is merely a quantization of common sense"

Adding random numbers

If each of you chose a random number from your own favorite distribution*, and we added all these numbers, repeating this many times…

What would you expect?

* OK - to be nice to me, you agree to have similar RMSs in these distributions!

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Gaussian!!!

…by the central limit theorem!

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Adding random numbers

If each of you chose a random number from your own favorite distribution*, and we added all these numbers, repeating this many times…

Gaussian!!!

the central limit theorem!

<u>Central Limit Theorem:</u>

The sum of N *independent* continuous random variables x_i with means μ_i and variances σ_i^2 becomes a Gaussian random variable with mean $\mu = \Sigma_i \mu_i$ and variance $\sigma^2 = \Sigma_i \sigma_i^2$ in the limit that N approaches infinity.

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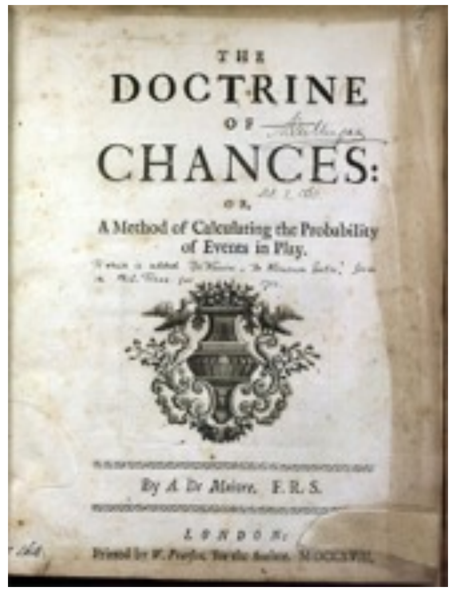
The Central Limit Theorem holds under fairly general conditions, which means that the Gaussian distribution takes a central role in statistics...

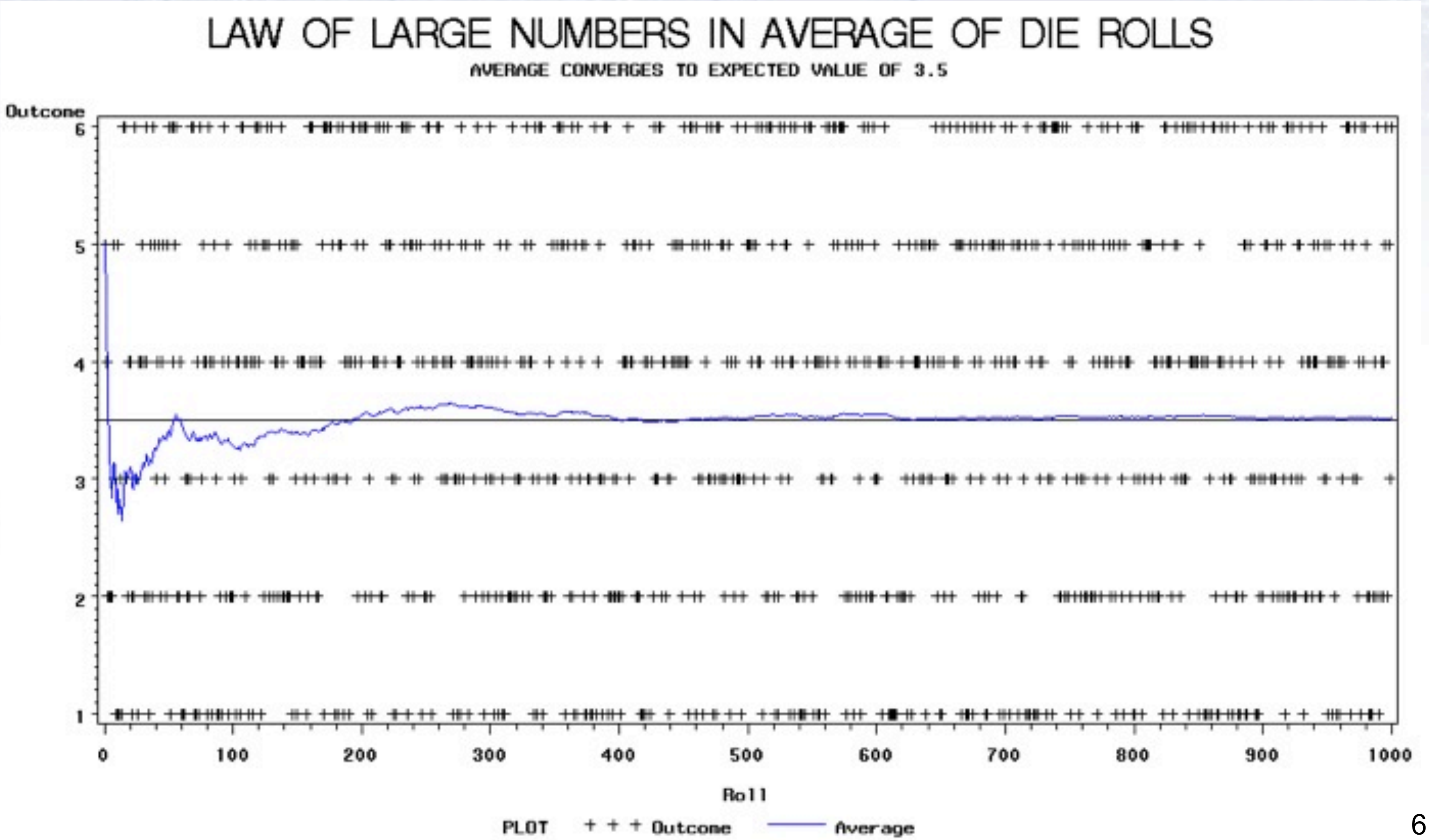
The Gaussian is "the unit" of distributions!



Since measurements are often affected by many small effects, uncertainties tend to be Gaussian (until otherwise proven!).

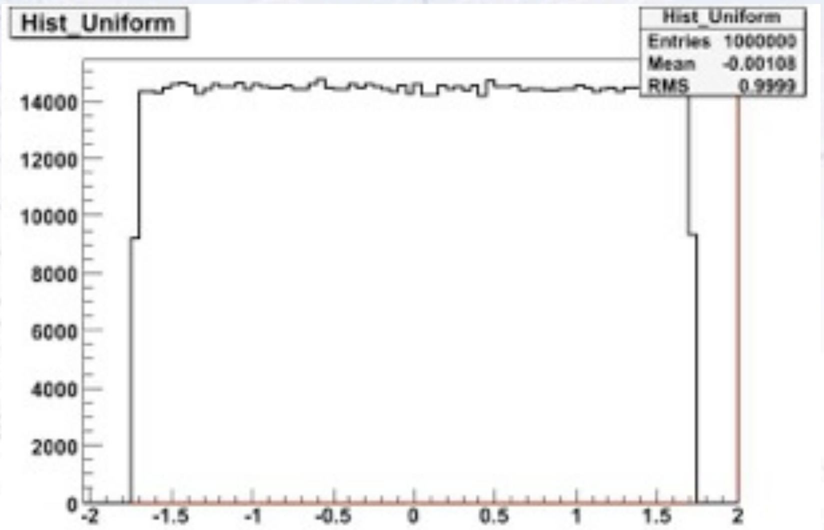
Statistical rules often require Gaussian uncertainties, and so the central limit theorem is your new good friend..

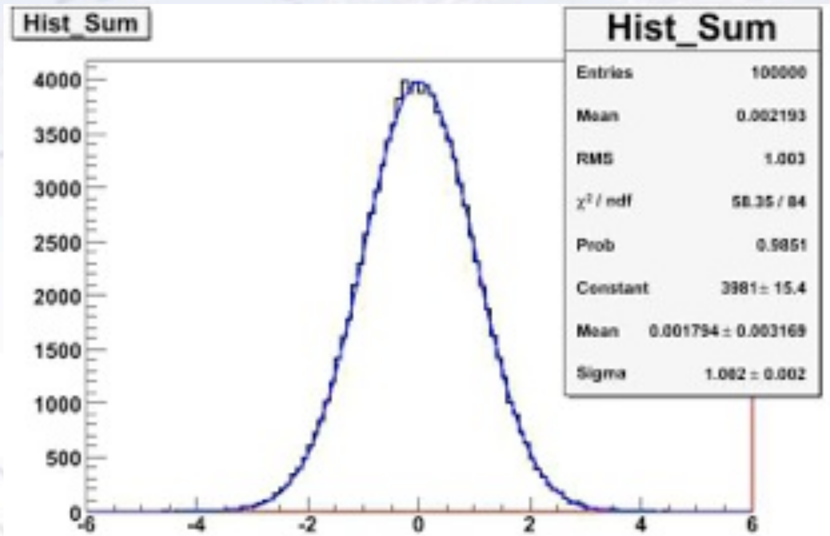
Law of large numbers



Example of Central Limit Theorem

Take the sum of 100 uniform numbers! Repeat 100000 times to see what distribution the sum has…



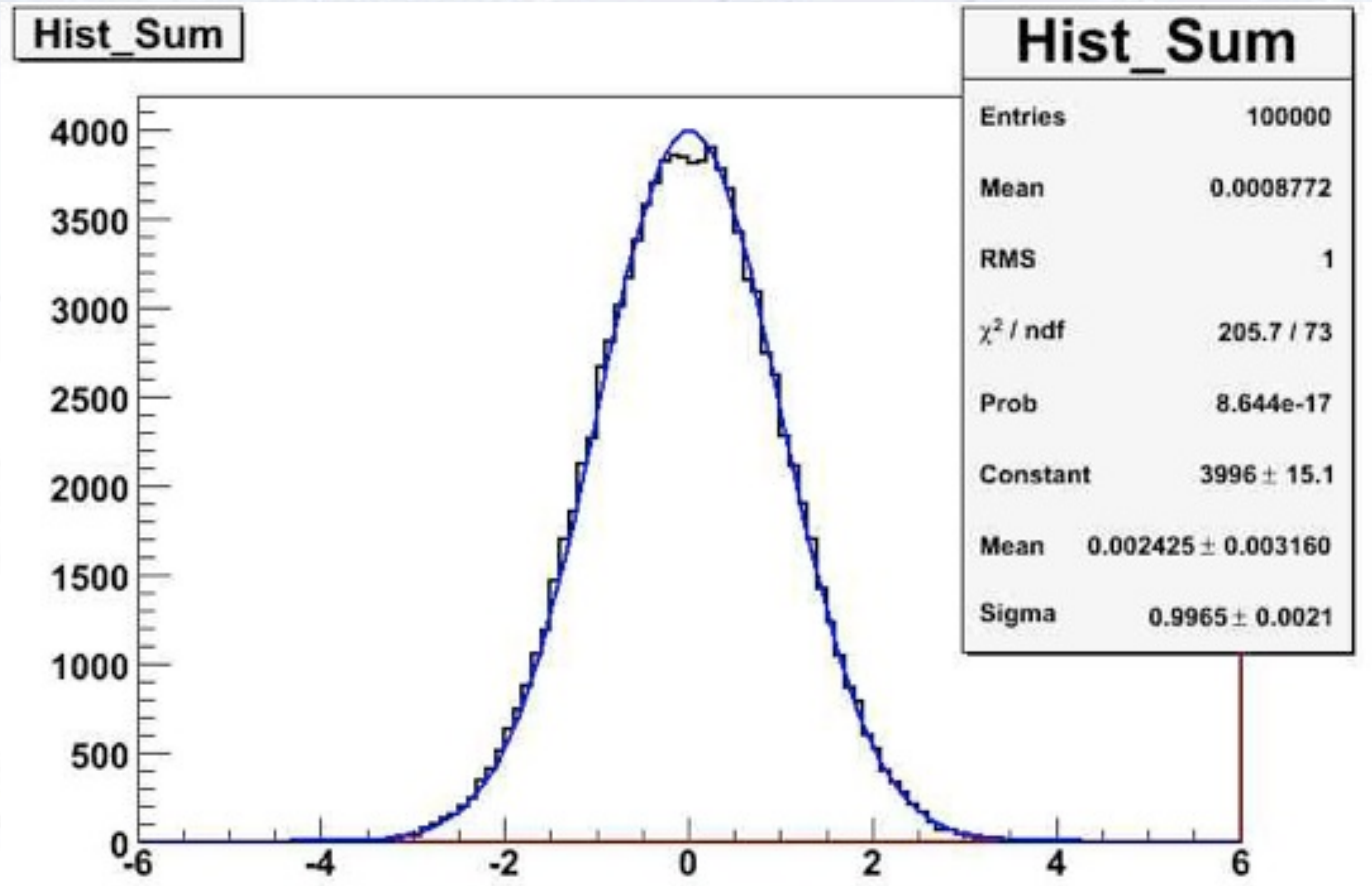


The result is a bell shaped curve, a so-called **normal** or **Gaussian** distribution.

It turns out, that this is very general!!!

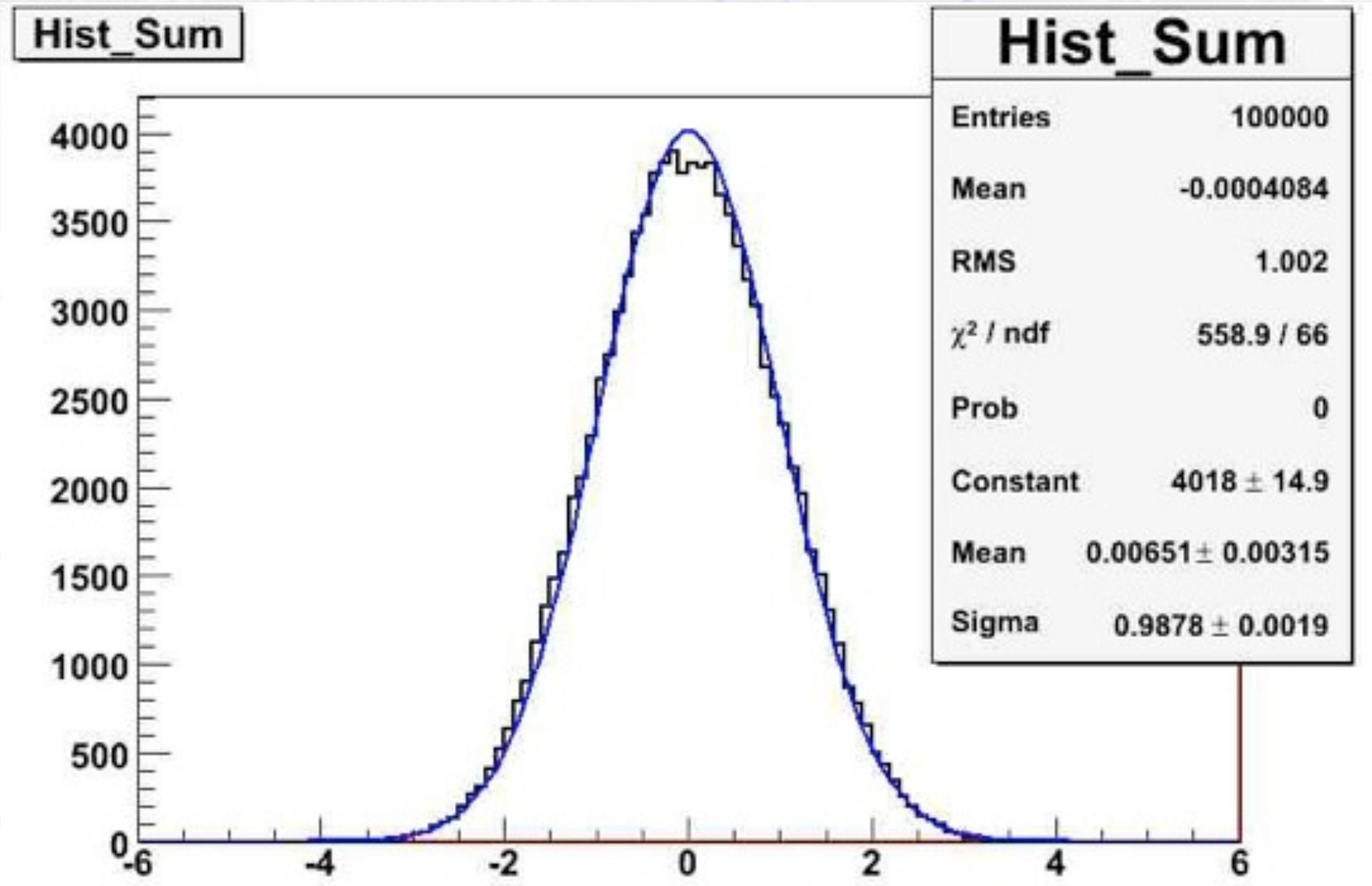
Example of Central Limit Theorem

Now take the sum of just **10** uniform numbers!



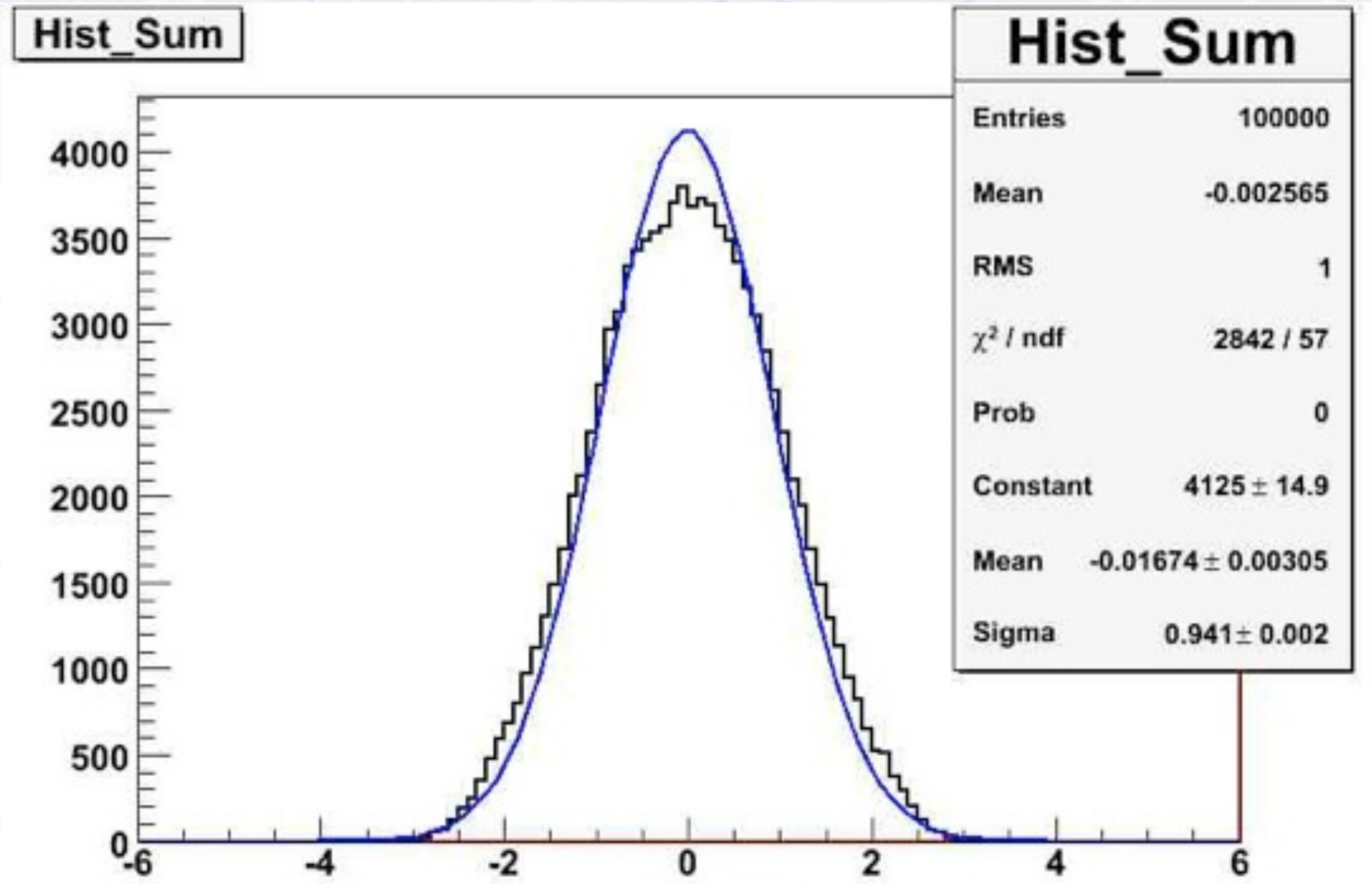
Example of Central Limit Theorem

Now take the sum of just **5** uniform numbers!



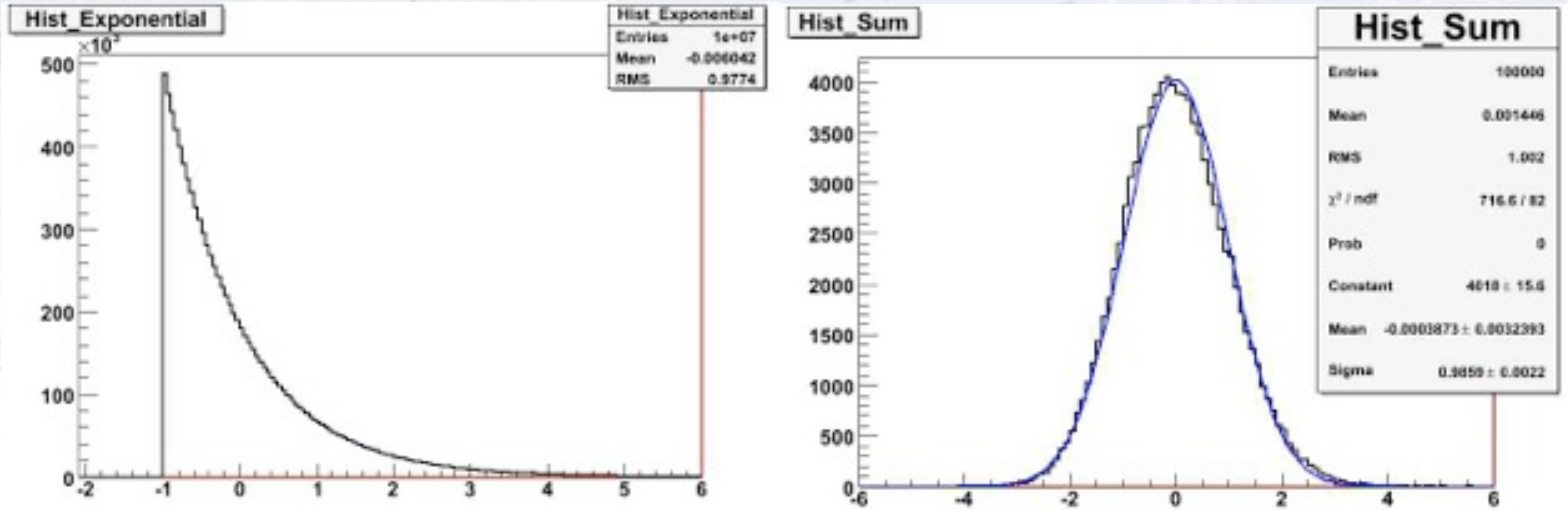
Example of Central Limit Theorem

Now take the sum of just **3** uniform numbers!



Example of Central Limit Theorem

This time we will try with a much more "**nasty**" function. Take the sum of 100 *exponential* numbers! Repeat 100000 times to see the sum's distribution…



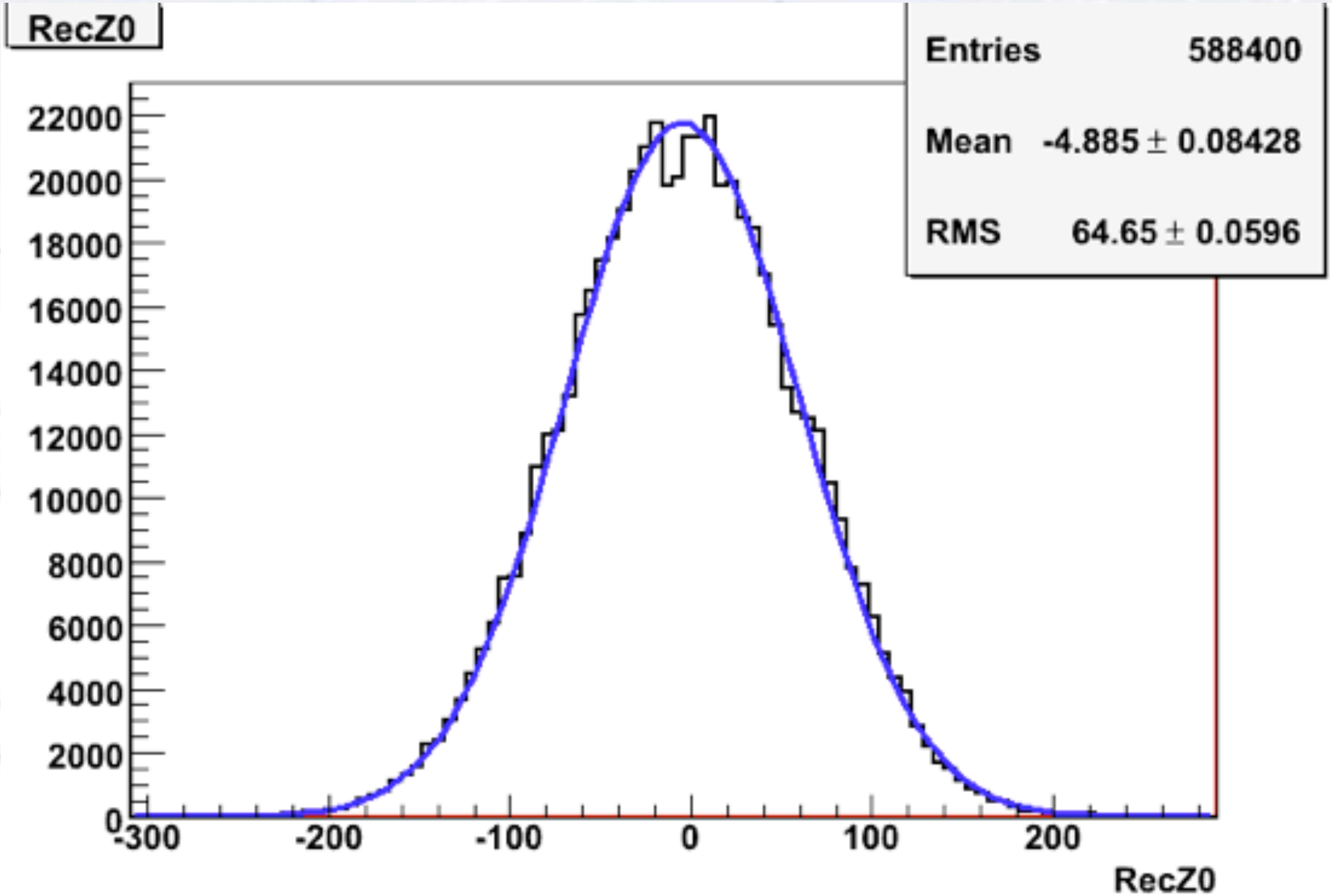
Even with such a non-Gaussian skewed distribution, the sum quickly becomes

Gaussian!!!

It turns out, that this fact saves us from much trouble: Makes statistics "easy"!

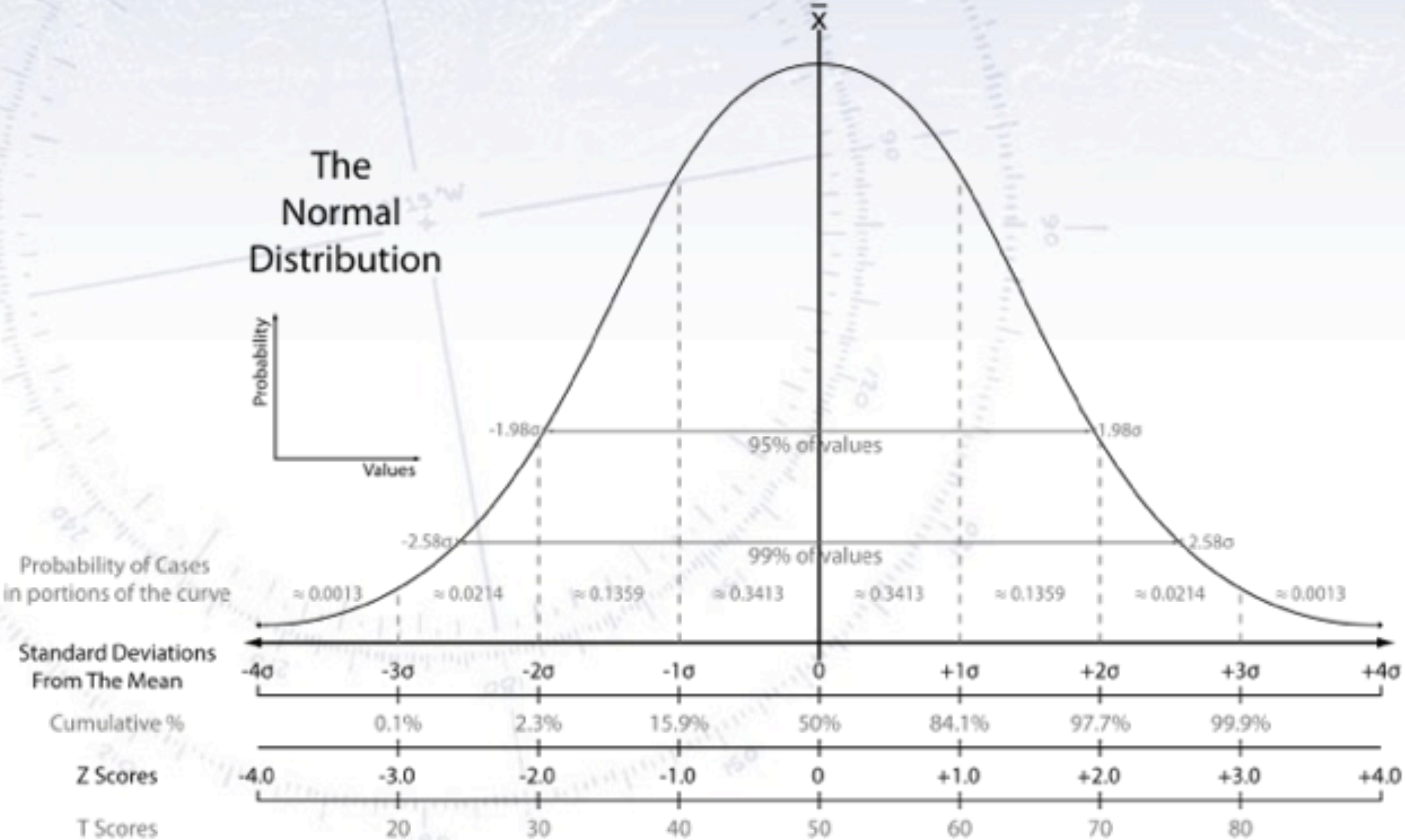
Example of Central Limit Theorem

Looking at z-coordinate of tracks at vertex from proton collisions in CERNs LHC accelerator by the ATLAS detector, this is what you get:



The Gaussian distribution

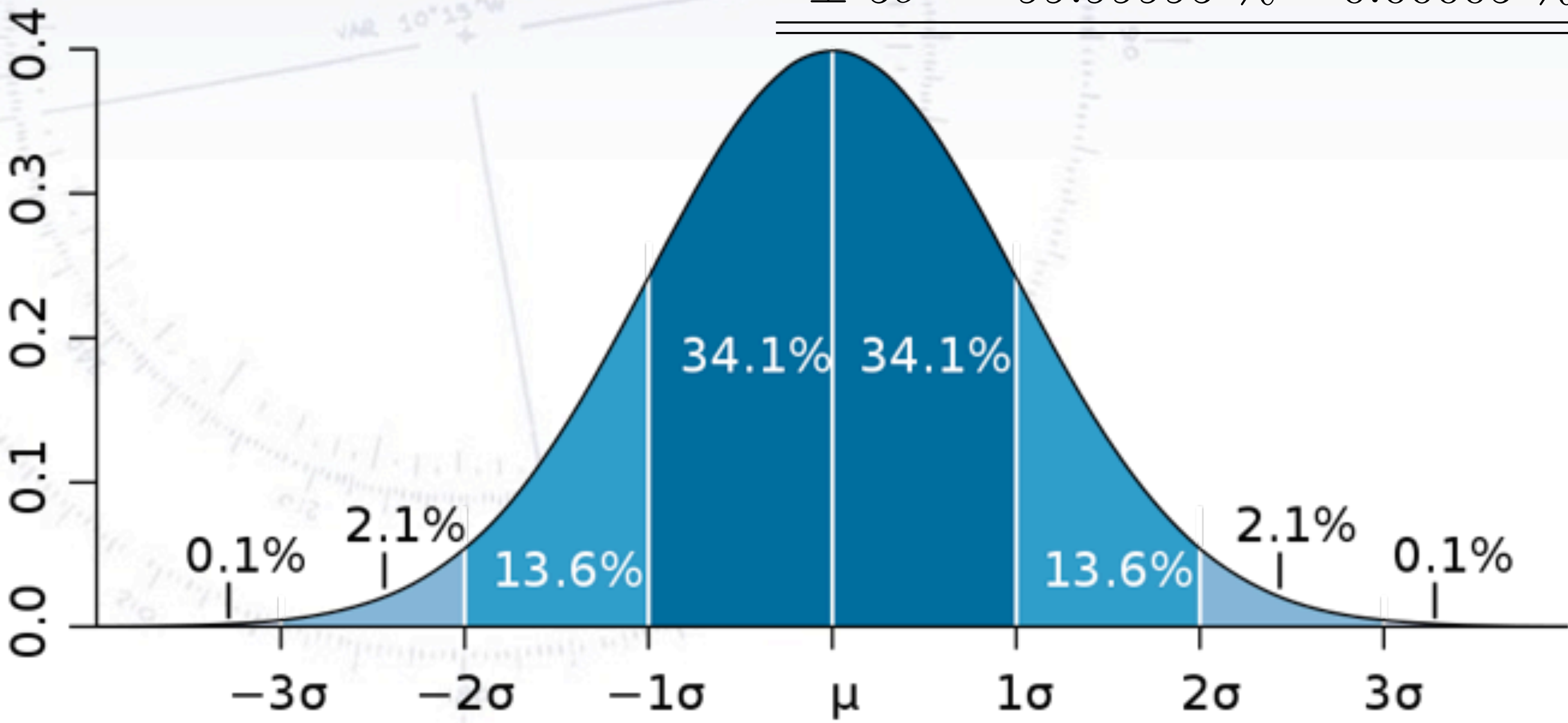
Almost every field of science have their own terms for features of the Gaussian, also known as the “normal” distribution.



The Gaussian distribution

It is useful to know just a few of the most common Gaussian integrals:

Range	Inside	Outside
$\pm\ 1\sigma$	**68** %	32 %
$\pm\ 2\sigma$	**95** %	5 %
$\pm\ 3\sigma$	**99.7** %	0.3 %
$\pm\ 5\sigma$	99.99995 %	0.00005 %



Summary

The Central Limit Theorem

...is your good friend because it…

ensures that uncertainties tend to be Gaussian!