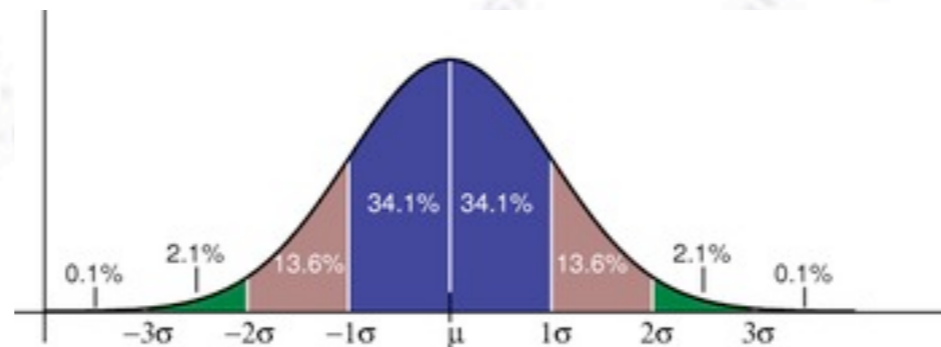


# Applied Statistics

## Confidence intervals and Limits



Troels C. Petersen (NBI)



*"Statistics is merely a quantization of common sense"*

The background is a technical map, likely a magnetic field map, showing contour lines and magnetic field lines. A prominent feature is a contour line labeled 'MAGNETIC' with a value of '10°15'W'. Other contour lines are labeled with values like 170, 180, 190, 200, 210, 220, 230, 240, 250, 260, 270, 280, 290, 300, 310, 320, 330, 340, 350, 360, 370, 380, 390, 400, 410, 420, 430, 440, 450, 460, 470, 480, 490, 500, 510, 520, 530, 540, 550, 560, 570, 580, 590, 600, 610, 620, 630, 640, 650, 660, 670, 680, 690, 700, 710, 720, 730, 740, 750, 760, 770, 780, 790, 800, 810, 820, 830, 840, 850, 860, 870, 880, 890, 900, 910, 920, 930, 940, 950, 960, 970, 980, 990, 1000. The map also shows a grid of latitude and longitude lines. The text 'THE BITTER END YACHT CLUB' is visible in the upper right corner.

# Confidence intervals



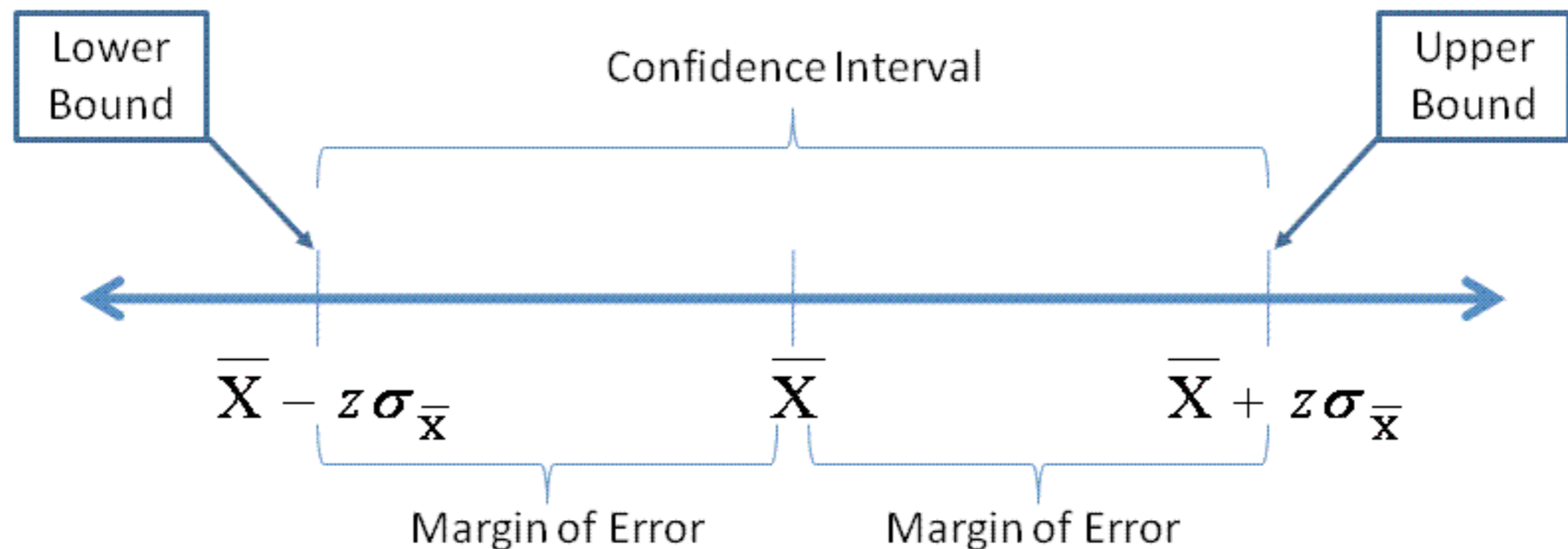
# Confidence intervals

*“Confidence intervals consist of a range of values (interval) that act as good estimates of the unknown population parameter.”*

It is thus a way of giving a range where the true parameter value probably is.

A very simple confidence interval for a Gaussian distribution can be constructed as:  
(z denotes the number of sigmas wanted)

$$\bar{x} \pm z \frac{s}{\sqrt{n}}$$



# Confidence intervals

Confidence intervals are constructed with a certain **confidence level C**, which is roughly speaking the fraction of times (for many experiments) to have the true parameter fall inside the interval:

$$Prob(x_- \leq x \leq x_+) = \int_{x_-}^{x_+} P(x) dx = C$$

Typically,  $C = 95\%$  (thus almost  $2\sigma$ ), but  $90\%$  and  $99\%$  are also used occasionally.

There is a choice as follows:

1. Require symmetric interval ( $x_+$  and  $x_-$  are equidistant from  $\mu$ ).
2. Require the shortest interval ( $x_+ - x_-$  is a minimum).
3. Require a central interval (integral from  $x_-$  to  $\mu$  is the same as from  $\mu$  to  $x_+$ ).

For the Gaussian, the three are equivalent!

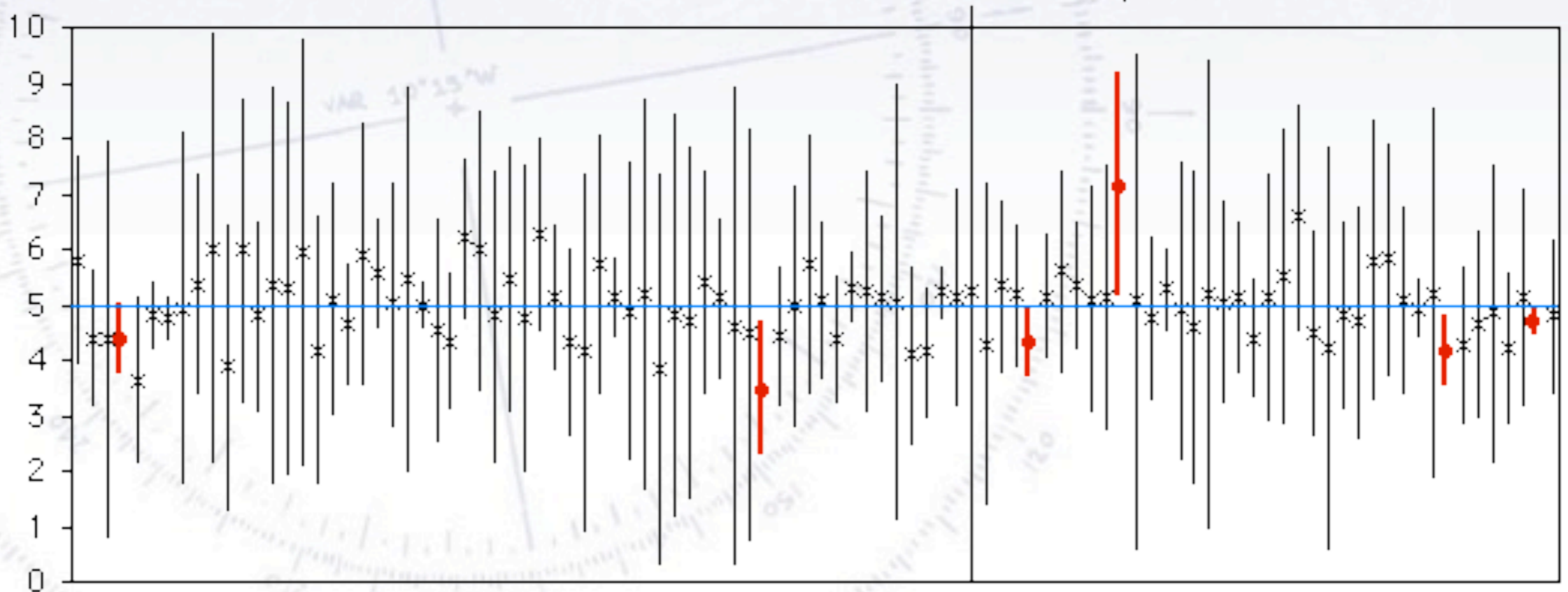
Otherwise, 3) is usually used.



# Confidence intervals

The confidence interval does not ALWAYS include the true value - only C fraction.

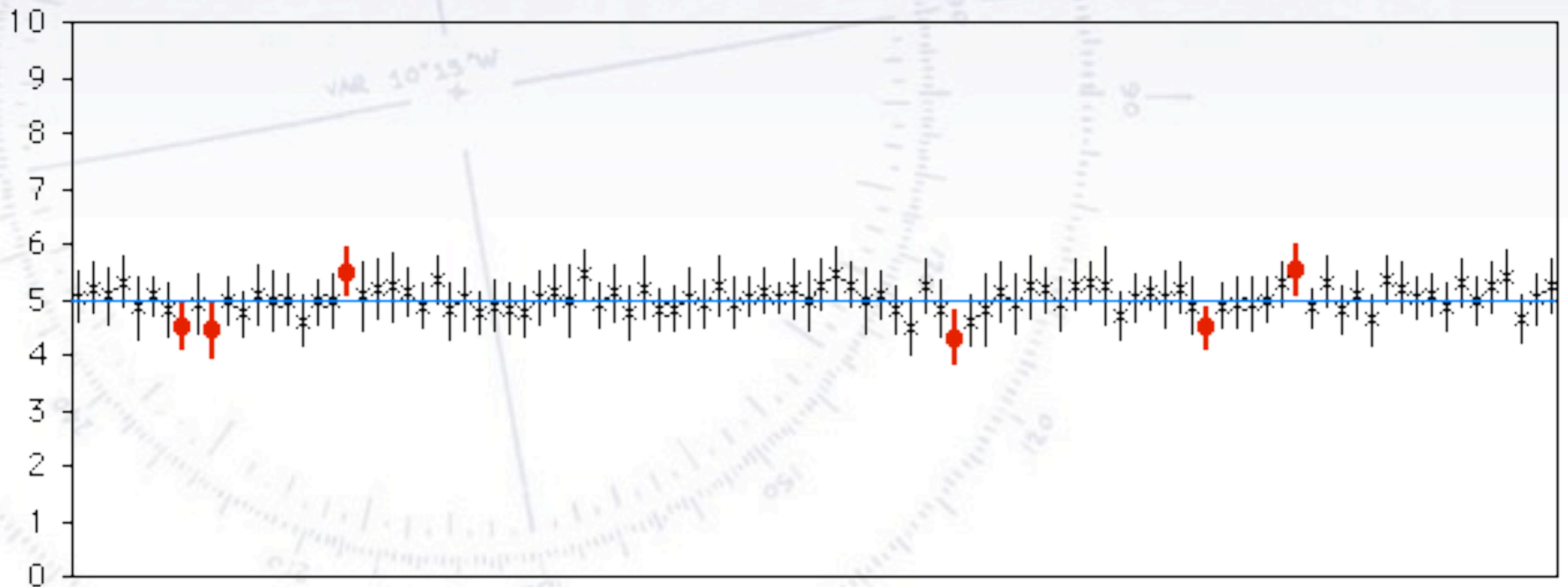
mean and 95% confidence intervals for 100 samples,  $N=3$



# Confidence intervals

The confidence interval does not ALWAYS include the true value - only C fraction.

mean and 95% confidence intervals for 100 samples, N=20



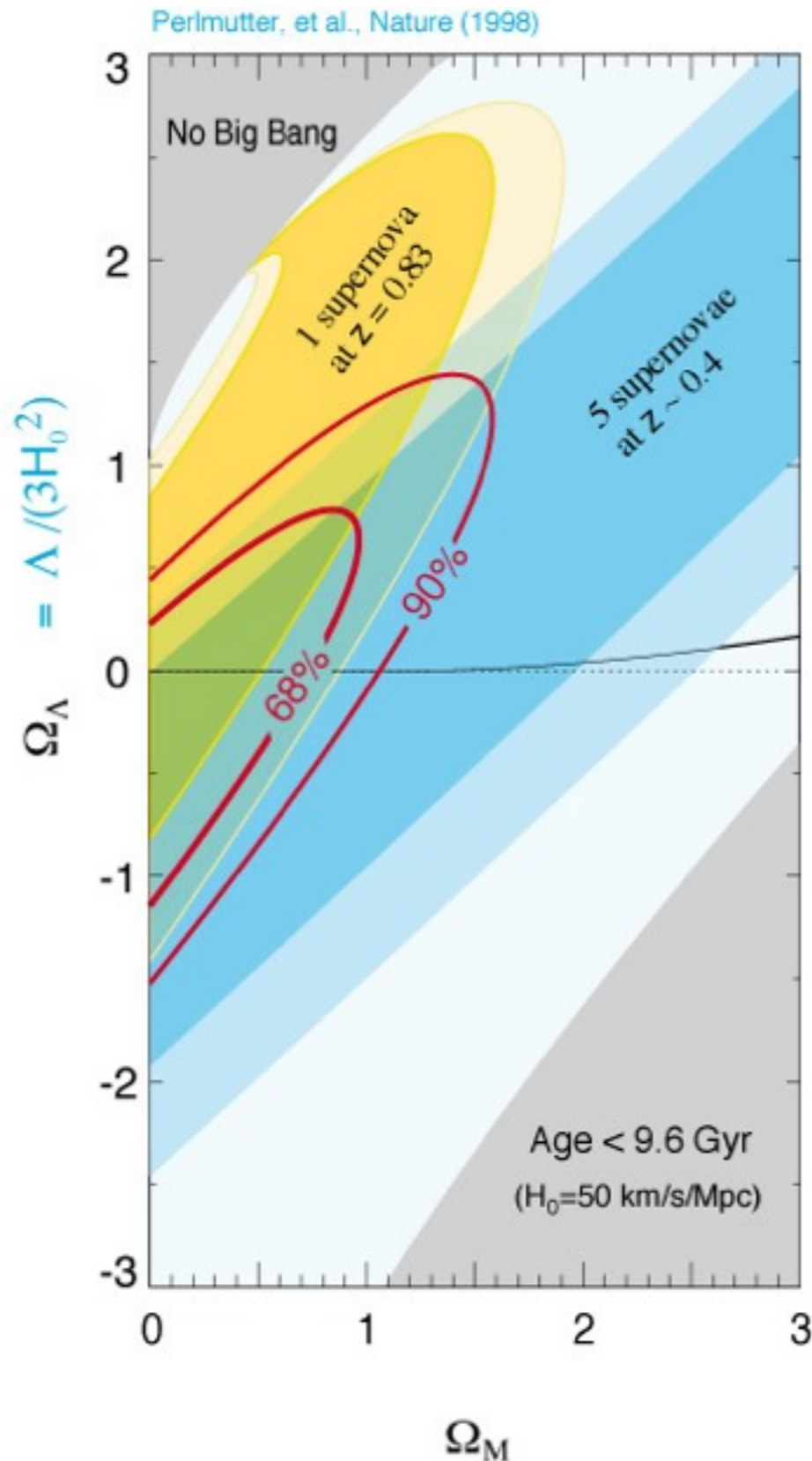
...and higher statistics does not help you!



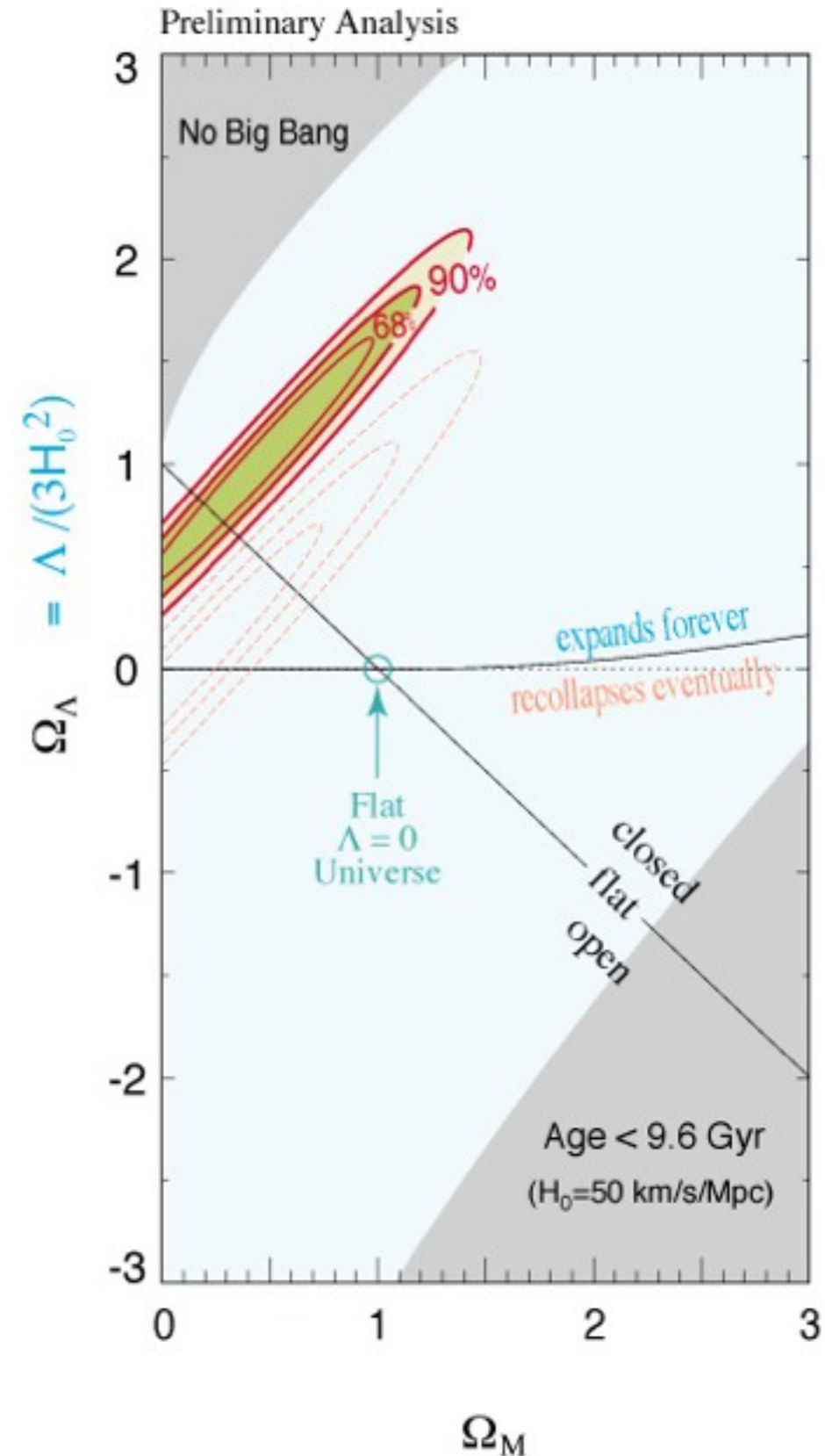
# Example

from cosmology

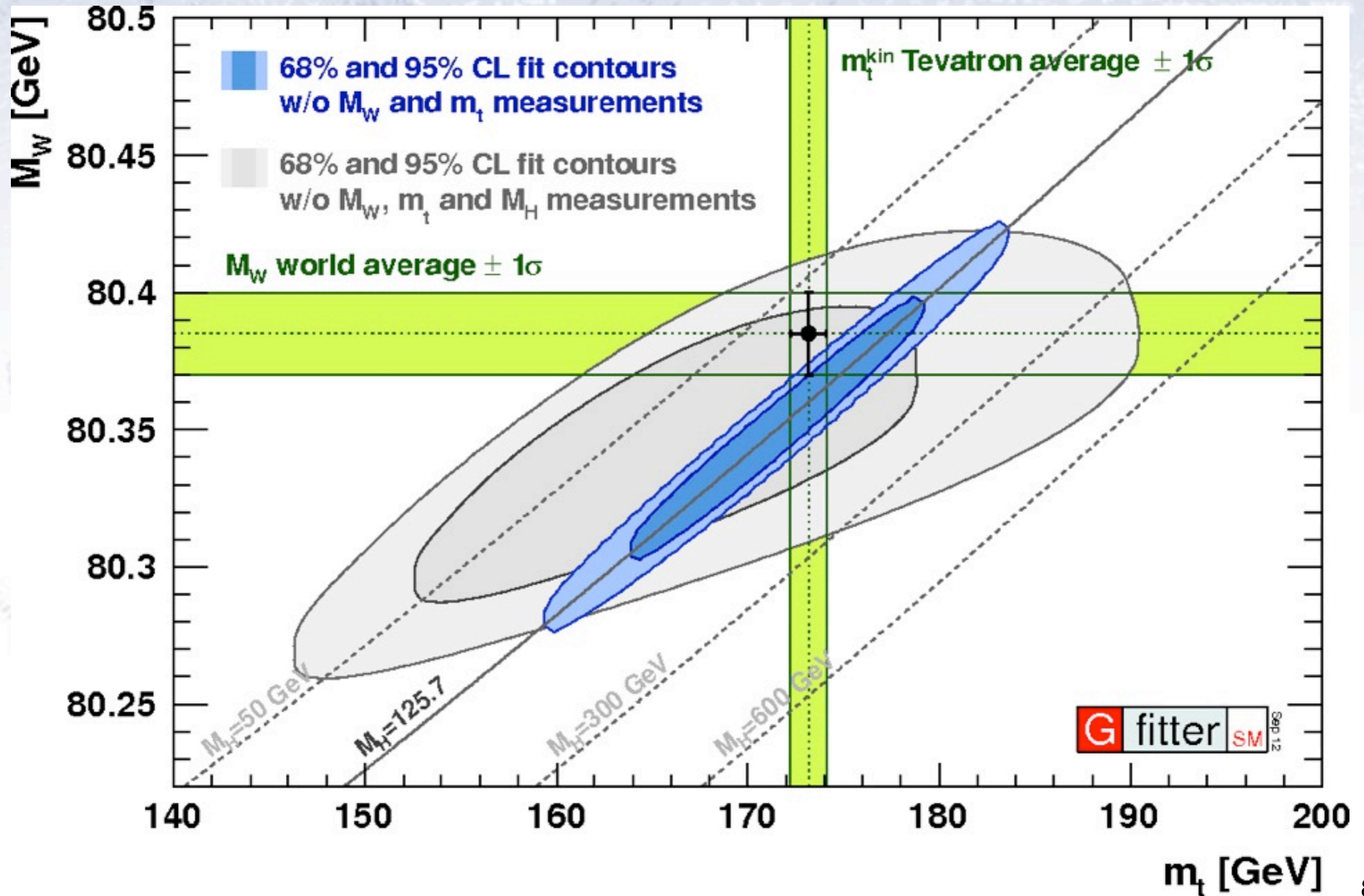
Results:  $\Omega$  vs  $\Lambda$   
from 6 supernovae



Results:  $\Omega$  vs  $\Lambda$   
from 40 supernovae

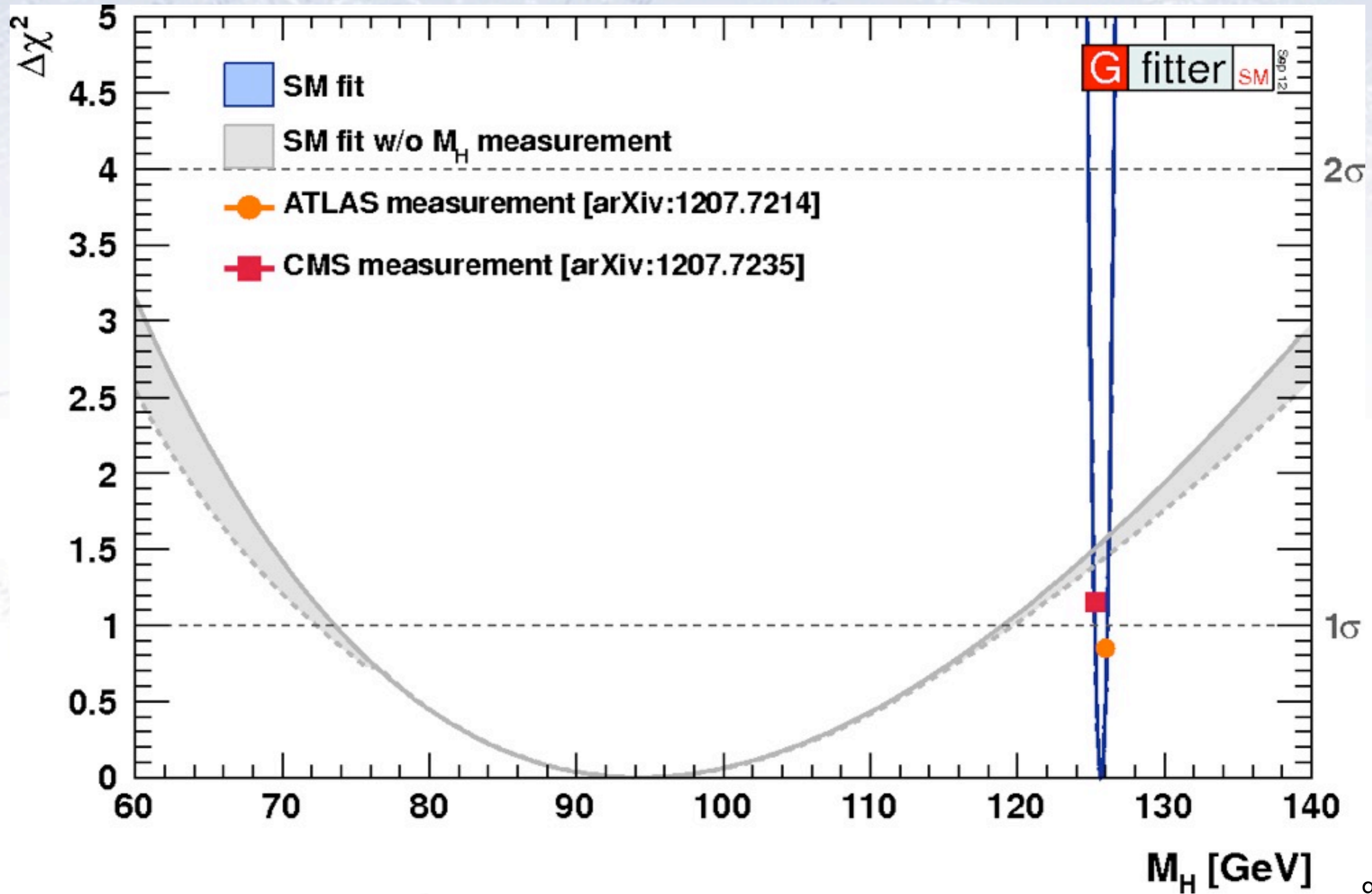


# Example from particle physics





# Example from particle physics



The background is a faded map showing magnetic isotherms (lines of equal magnetic intensity) and a magnetic declination line. The declination line is labeled 'VAR 10°15' W'. The isotherms are labeled with values such as 170, 180, 190, 200, 210, 220, 230, 240, 250, 260, 270, 280, 290, 300, 310, 320, 330, 340, 350, 360, 370, 380, 390, 400, 410, 420, 430, 440, 450, 460, 470, 480, 490, 500, 510, 520, 530, 540, 550, 560, 570, 580, 590, 600, 610, 620, 630, 640, 650, 660, 670, 680, 690, 700, 710, 720, 730, 740, 750, 760, 770, 780, 790, 800, 810, 820, 830, 840, 850, 860, 870, 880, 890, 900, 910, 920, 930, 940, 950, 960, 970, 980, 990, 1000. The map also shows a coastline and the text 'THE BITTER END YACHT CLUB'.

# Confidence limits



# Confidence limits

Imagine that you do an experiment to search for an unknown but predicted phenomenon (aether, planet Vulcan, dark energy, Higgs particle, etc.), and that find **nothing!**

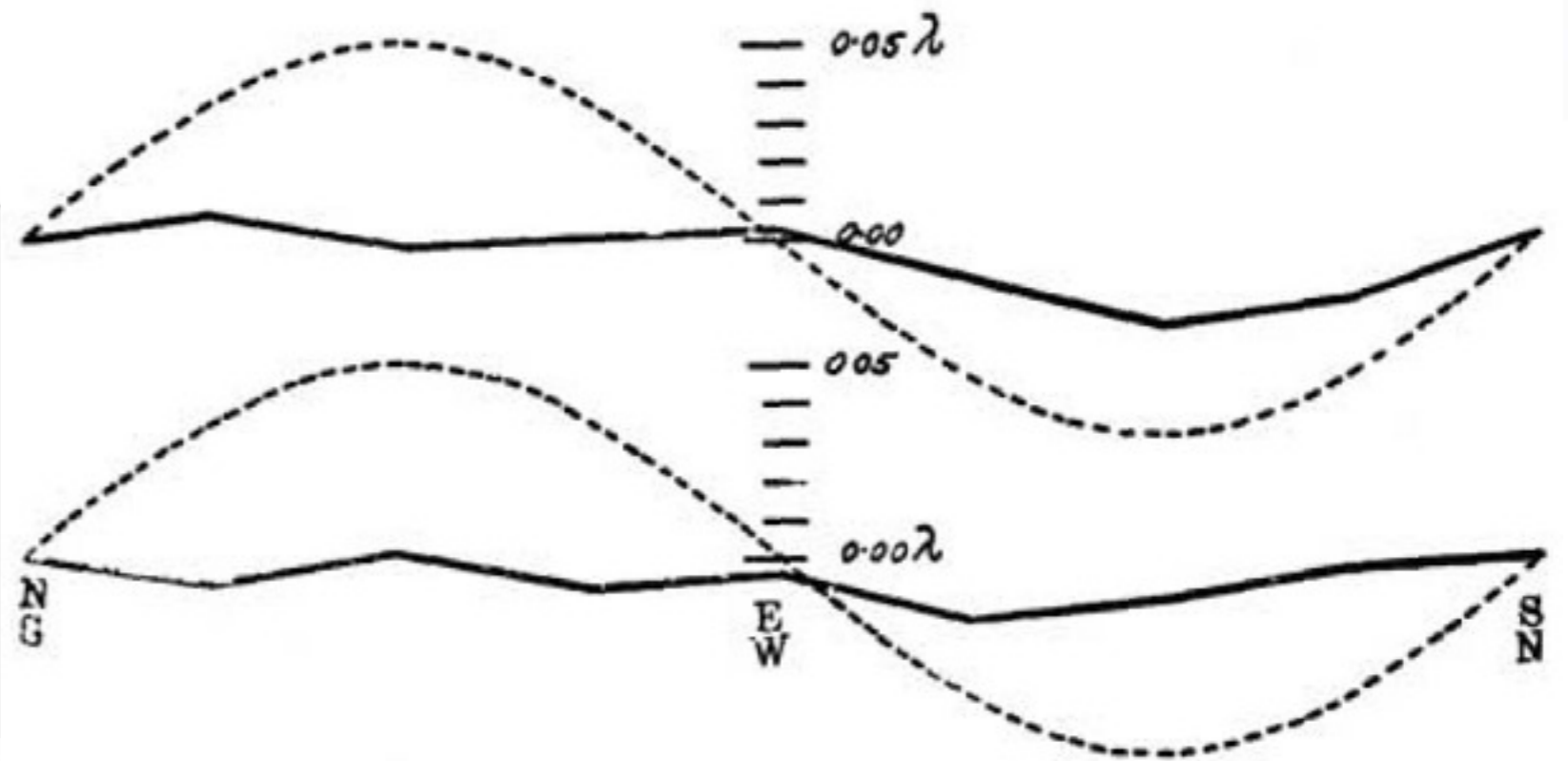
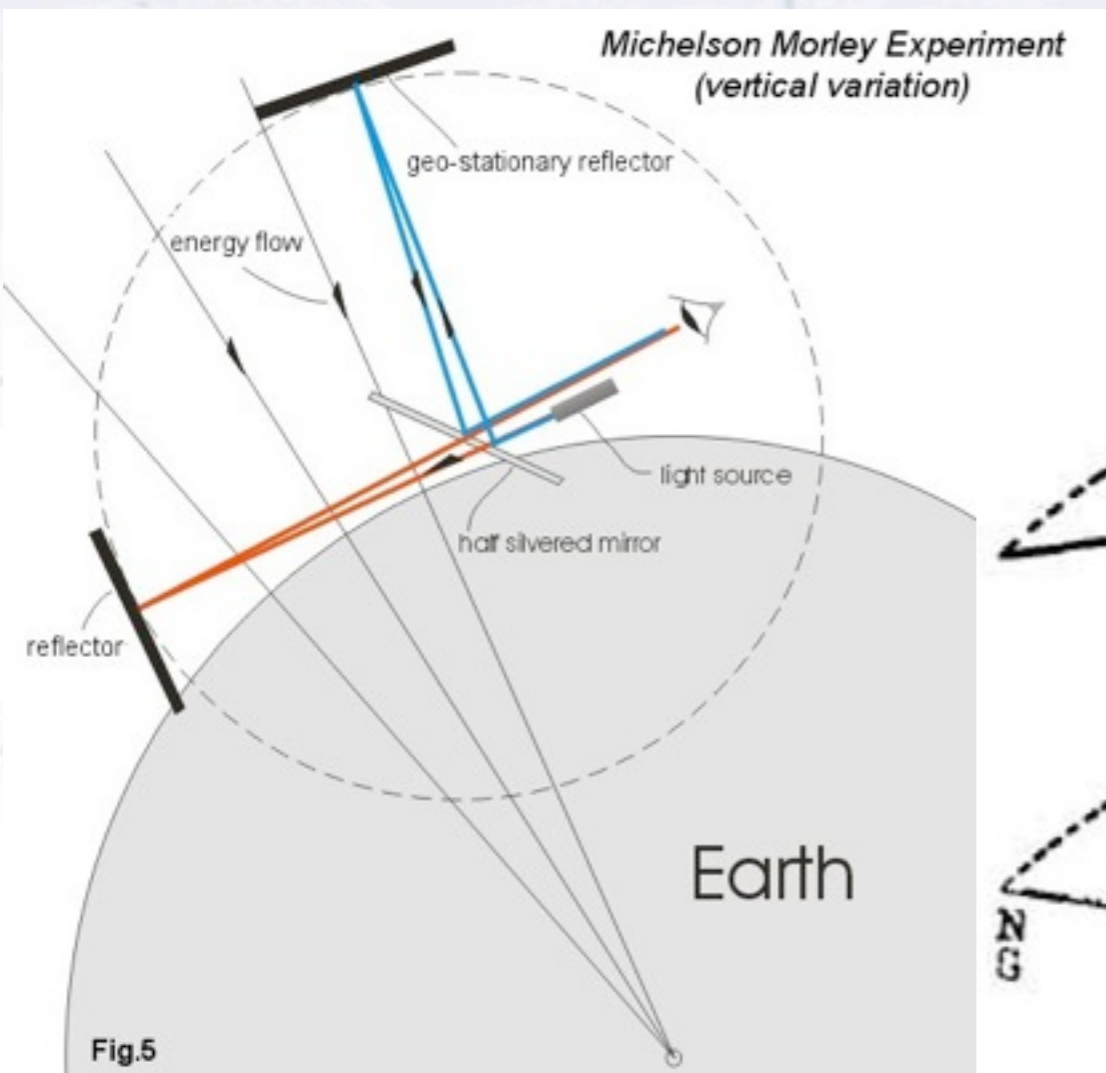
Reporting this result, you wish to state *what you would have discovered, if it had been there*, i.e. something along the lines:

*“If the aether had affected the speed of light by  $X\%$ , we would have seen the effect with 95% confidence”.*

This is a **confidence limit** (much like a one-sided confidence interval).

# Confidence intervals

In the case of Michelson-Morley, a limit could be set on the “degree of dragging” of the aether (though they didn’t do this, as statistics was still in its infancy!).





# Confidence limits - Poisson

Poisson statistics is a neat special case, perhaps best explained by numbers:

Table 9.3 Poisson lower and upper limits for  $n_{\text{obs}}$  observed events.

$n_{\text{obs}}$	lower limit $a$			upper limit $b$		
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.01$	$\beta = 0.1$	$\beta = 0.05$	$\beta = 0.01$
0	-	-	-	2.30	3.00	4.61
1	0.105	0.051	0.010	3.89	4.74	6.64
2	0.532	0.355	0.149	5.32	6.30	8.41
3	1.10	0.818	0.436	6.68	7.75	10.04
4	1.74	1.37	0.823	7.99	9.15	11.60
5	2.43	1.97	1.28	9.27	10.51	13.11
6	3.15	2.61	1.79	10.53	11.84	14.57
7	3.89	3.29	2.33	11.77	13.15	16.00
8	4.66	3.98	2.91	12.99	14.43	17.40
9	5.43	4.70	3.51	14.21	15.71	18.78
10	6.22	5.43	4.13	15.41	16.96	20.14

Example: If you in a day observe 0 red cars on Blegdamsvej, you can with 95% confidence say that there are fewer than 3.00 pr. day.