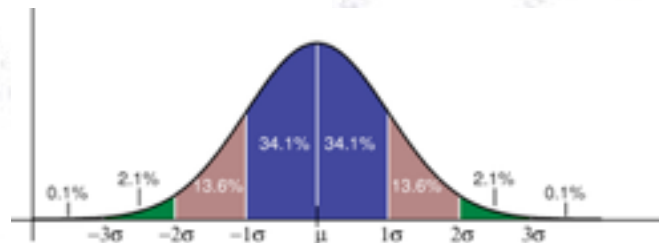


Applied Statistics

Correlations



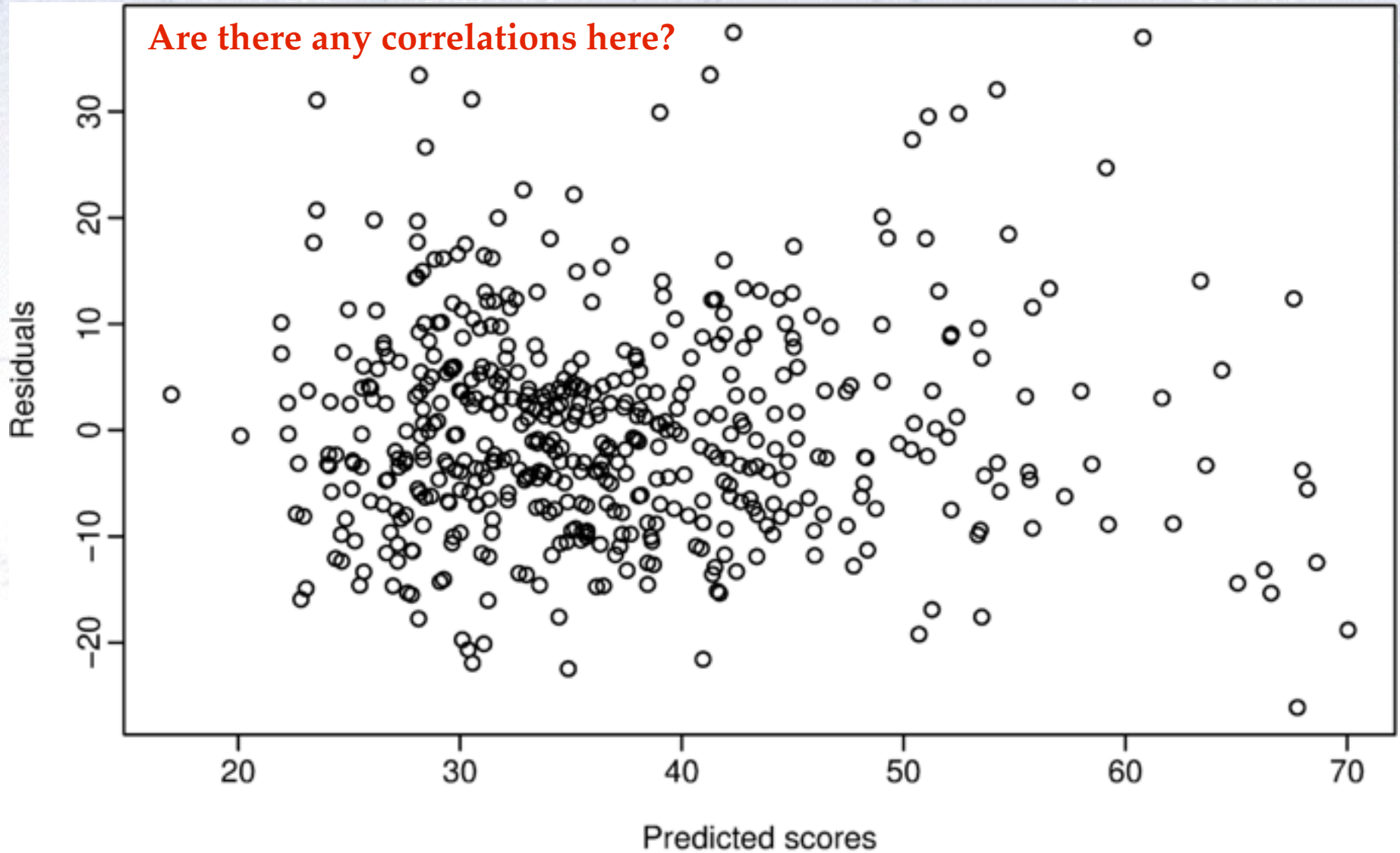
Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"

Correlation

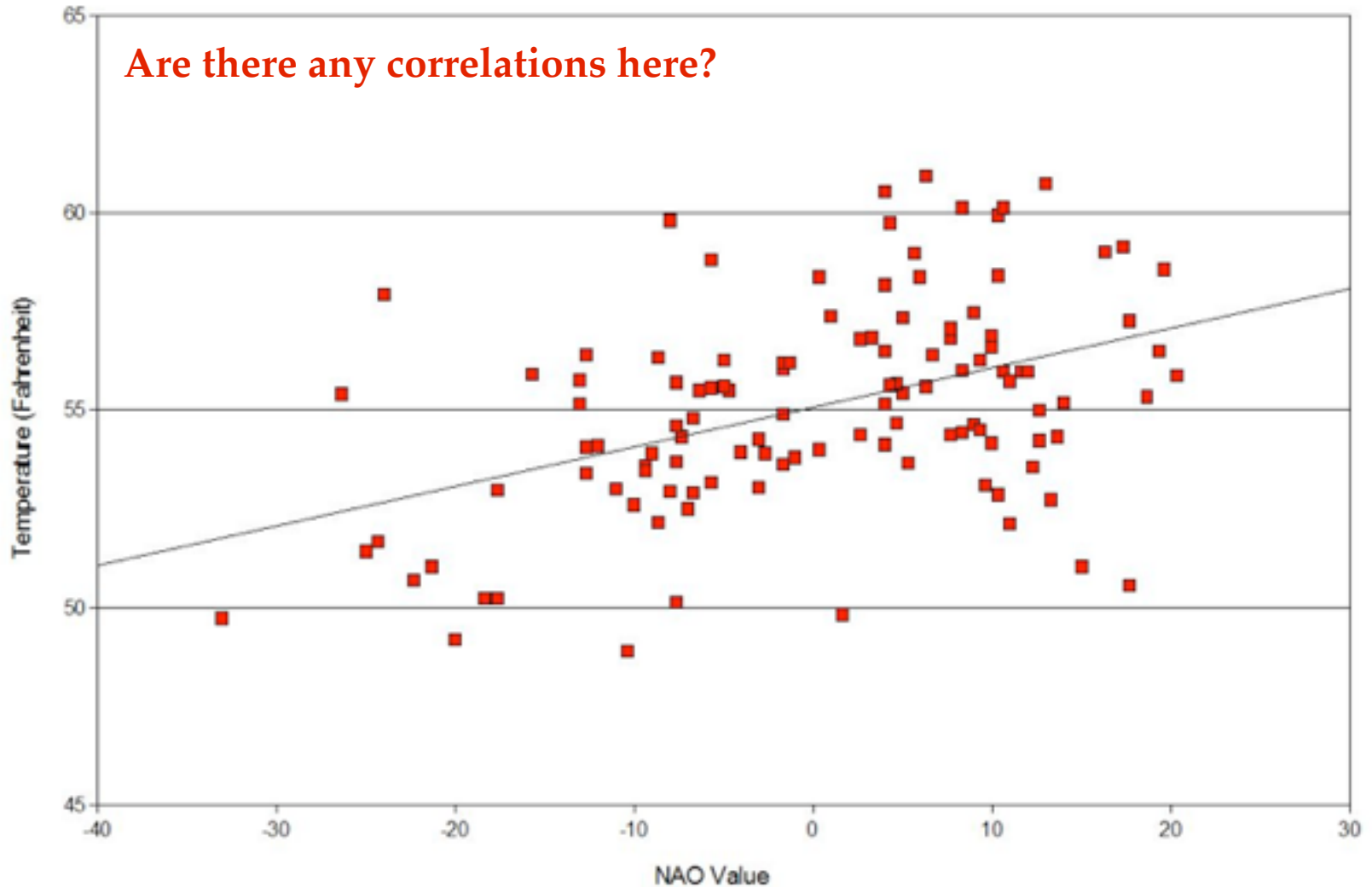
Are there any correlations here?



Correlation

North Atlantic Oscillation (NAO) Effects

Upper Texas Coast Temperature



Correlation

Recall the definition of the Variance, V :

$$V = \sigma^2 = \frac{1}{N} \sum_i^n (x_i - \mu)^2 = E[(x - \mu)^2] = E[x^2] - \mu^2$$

Likewise, one defines the **Covariance**, V_{xy} :

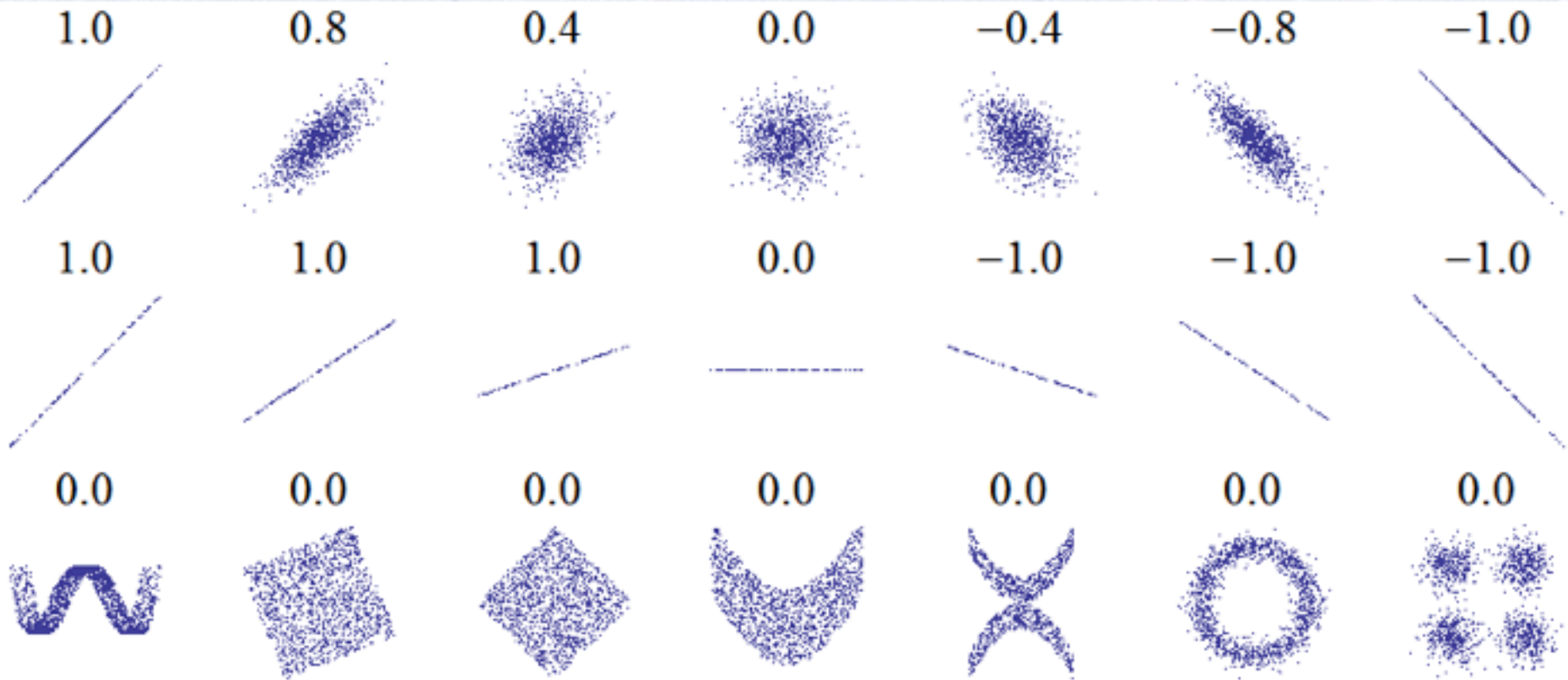
$$V_{xy} = \frac{1}{N} \sum_i^n (x_i - \mu_x)(y_i - \mu_y) = E[(x_i - \mu_x)(y_i - \mu_y)]$$

“Normalising” by the widths, gives the (linear) correlation:

$$\rho_{xy} = \frac{V_{xy}}{\sigma_x \sigma_y} \quad -1 < \rho_{xy} < 1$$
$$\sigma(\rho) \simeq \sqrt{\frac{1}{n}(1 - \rho^2)^2 + O(n^{-2})}$$

Correlation

Correlations in 2D are in the Gaussian case the “degree of ovalness”!



Note how ALL of the bottom distributions have $\rho = 0$, despite obvious correlations!

Correlation

The correlation matrix V_{xy} explicitly looks as:

$$V_{xy} = \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 & \cdots & \sigma_{1N}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2N}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_N^2 & \sigma_{N2}^2 & \cdots & \sigma_{NN}^2 \end{bmatrix}$$

Very specifically, the calculations behind are:

$$V = \begin{bmatrix} \mathbb{E}[(X_1 - \mu_1)(X_1 - \mu_1)] & \mathbb{E}[(X_1 - \mu_1)(X_2 - \mu_2)] & \cdots & \mathbb{E}[(X_1 - \mu_1)(X_n - \mu_n)] \\ \mathbb{E}[(X_2 - \mu_2)(X_1 - \mu_1)] & \mathbb{E}[(X_2 - \mu_2)(X_2 - \mu_2)] & \cdots & \mathbb{E}[(X_2 - \mu_2)(X_n - \mu_n)] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}[(X_n - \mu_n)(X_1 - \mu_1)] & \mathbb{E}[(X_n - \mu_n)(X_2 - \mu_2)] & \cdots & \mathbb{E}[(X_n - \mu_n)(X_n - \mu_n)] \end{bmatrix}.$$

Correlation and Information

Correlations influence results in complex ways!

They need to be taken into account, for example in **Error Propagation!**

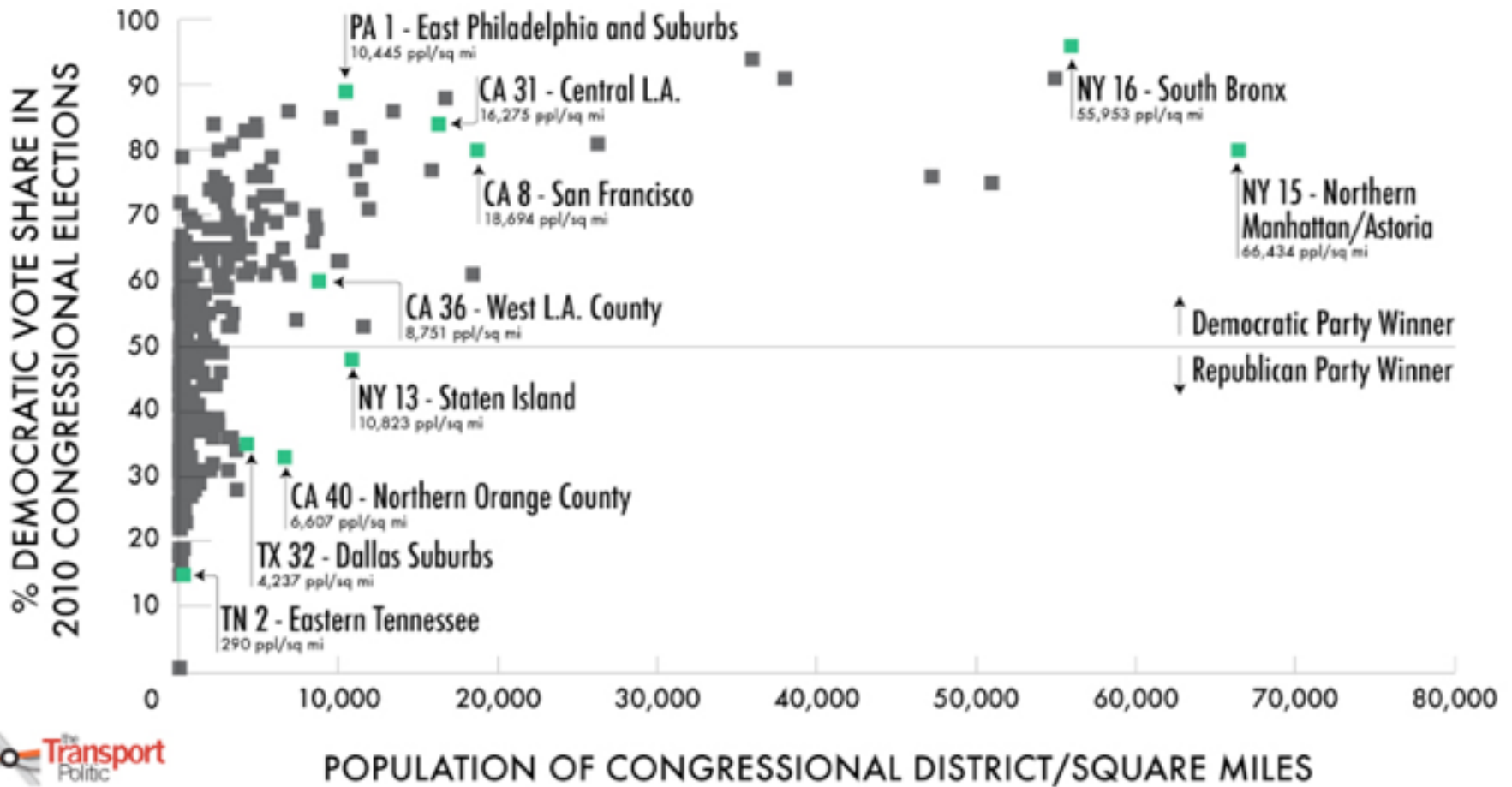
Correlations may contain a significant amount of information.

We will consider this more when we play with multivariate analysis.

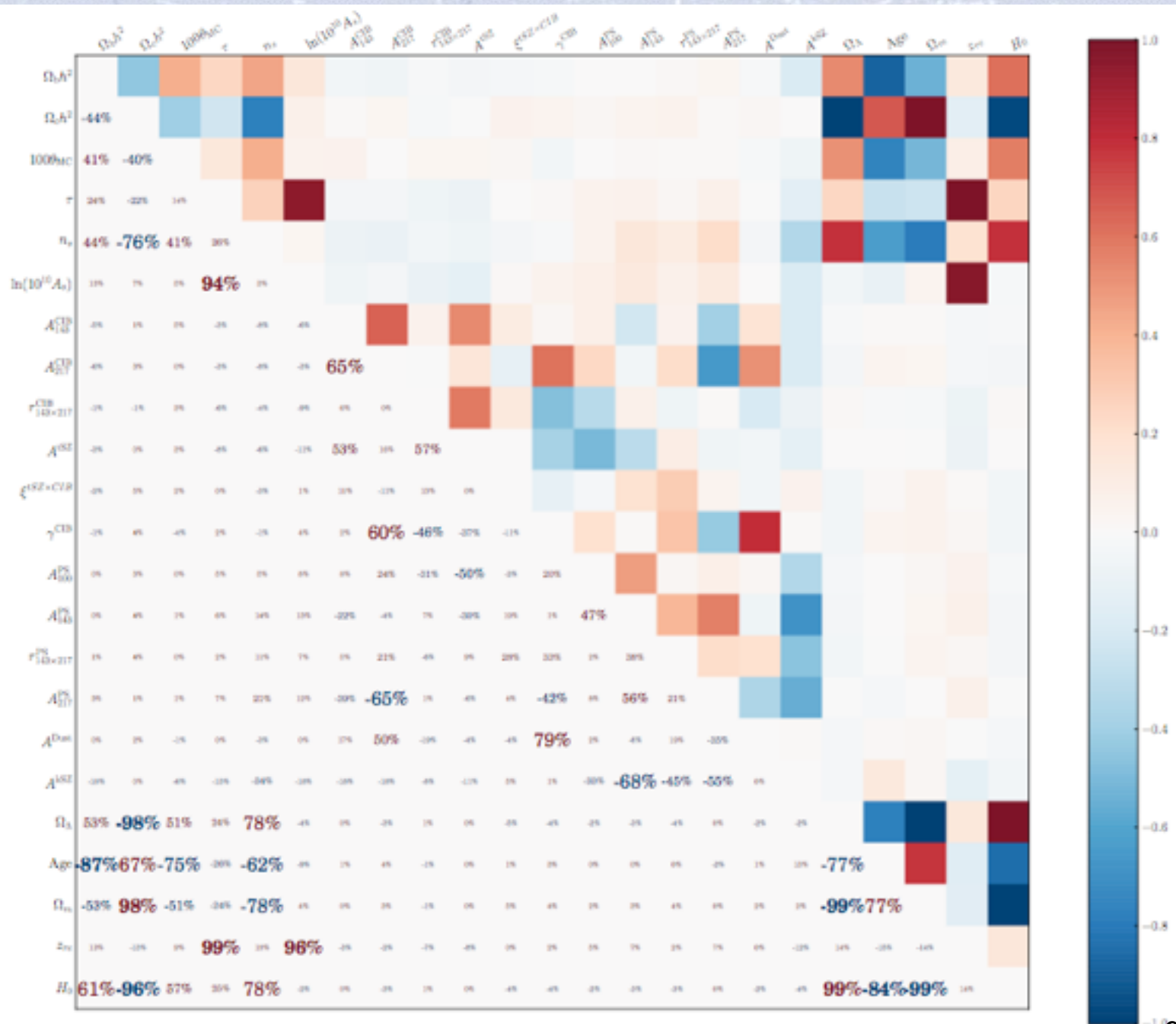


Correlation example

RELATING DENSITY AND VOTING PATTERNS IN U.S. CONGRESSIONAL DISTRICTS



Planck example



Correlation Vs. Causation

“Com hoc ergo propter hoc”

(with this, therefore because of this)

Fig 1
IS FACEBOOK DRIVING
THE GREEK DEBT CRISIS?

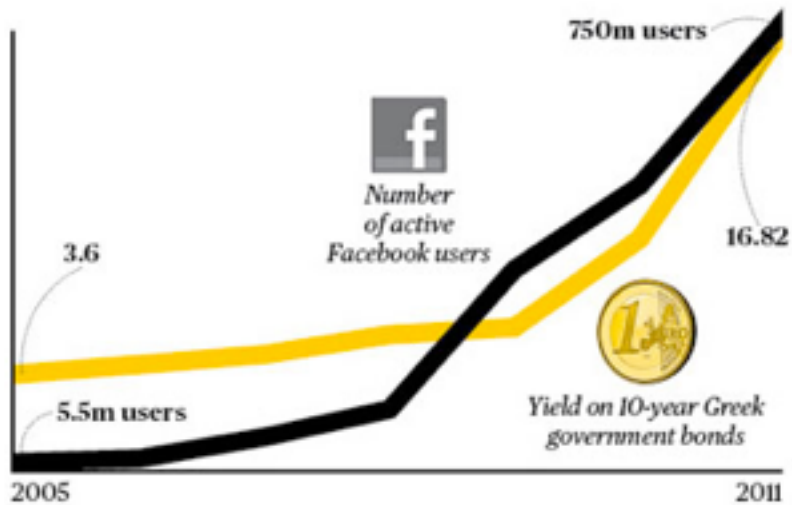
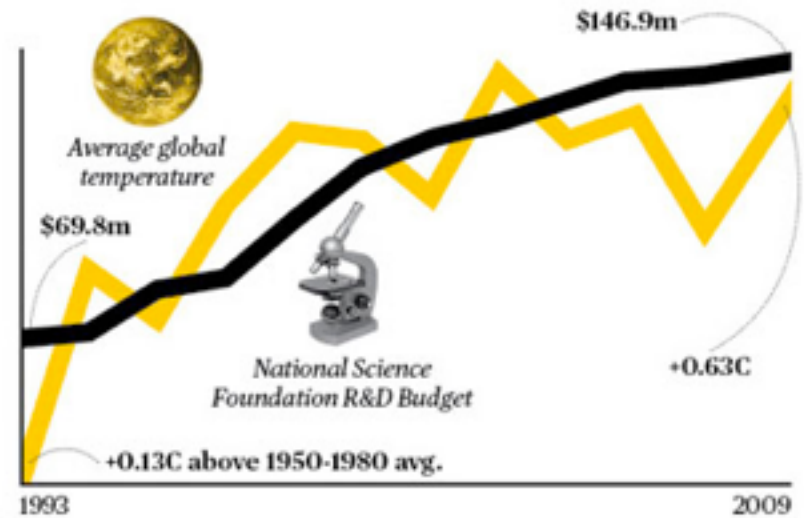


Fig 2
IS GLOBAL WARMING A HOAX
PROPAGATED BY SCIENTISTS?



Correlation Vs. Causation

“Com hoc ergo propter hoc”

(with this, therefore because of this)

