# Applied Statistics

## Mean and Width






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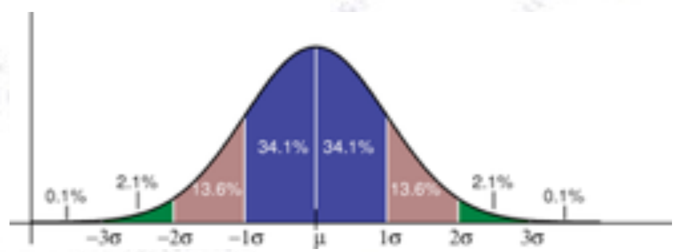


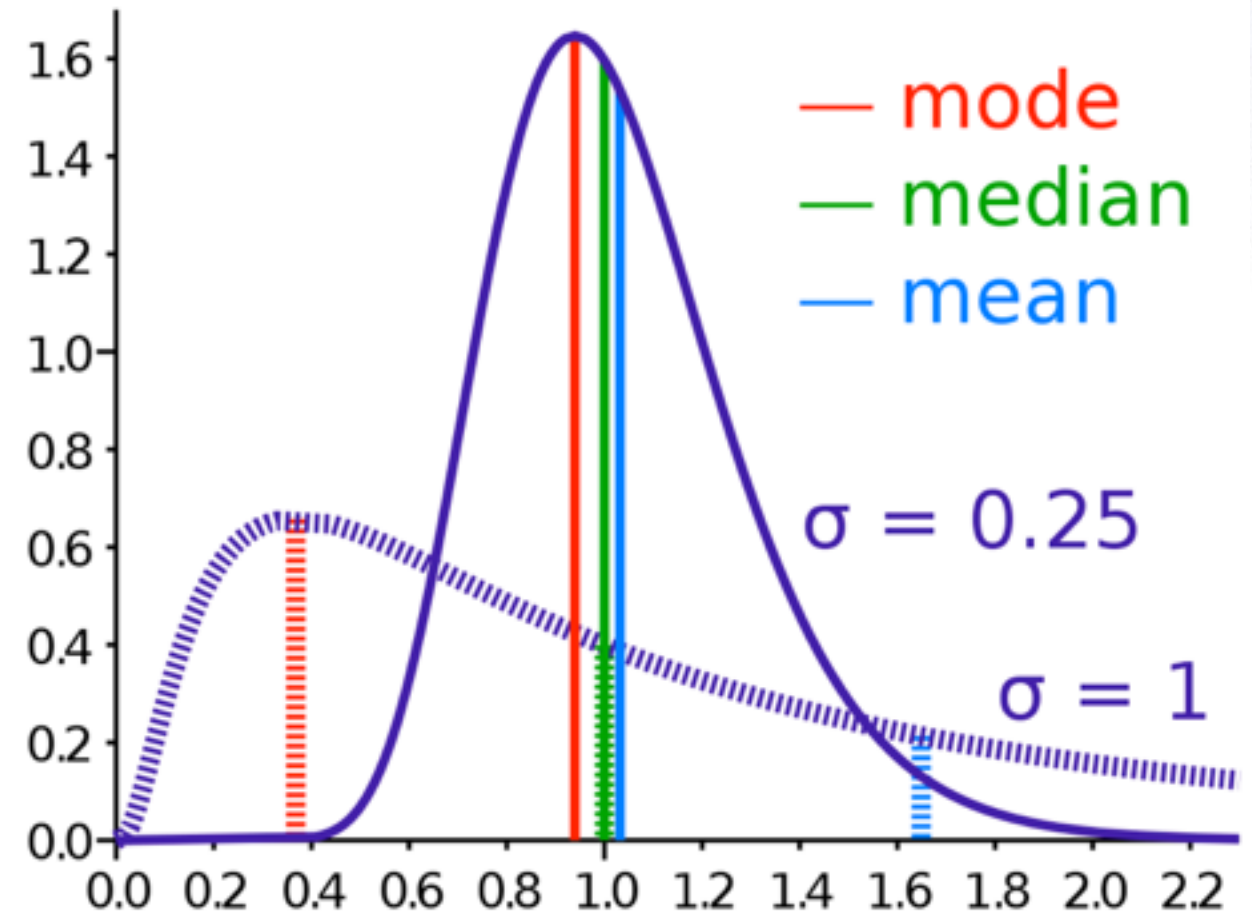

*"Statistics is merely a quantisation of common sense"*

# Defining the mean

There are several ways of defining "a typical" value from a dataset:

a) Arithmetic mean   b) Mode (most probably)   c) Median (half below, half above)

d) Geometric mean   e) Harmonic mean   f) Truncated mean (robustness)



# Mean and Width

It turns out, that the best estimator for the **mean** is (as you all know):

$$\hat{\mu} = \frac{1}{N} \sum_i x_i = \bar{x}$$

For the **width** of the distribution (a.k.a. **standard deviation** or **RMS**) it is:

$$\hat{\sigma} = \sqrt{\frac{1}{N} \sum_i (x_i - \mu)^2}$$

Note the "hat", which means "estimator". It is sometimes dropped...

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$$\hat{s} = \sqrt{\frac{1}{N-1} \sum_i (x_i - \bar{x})^2}$$

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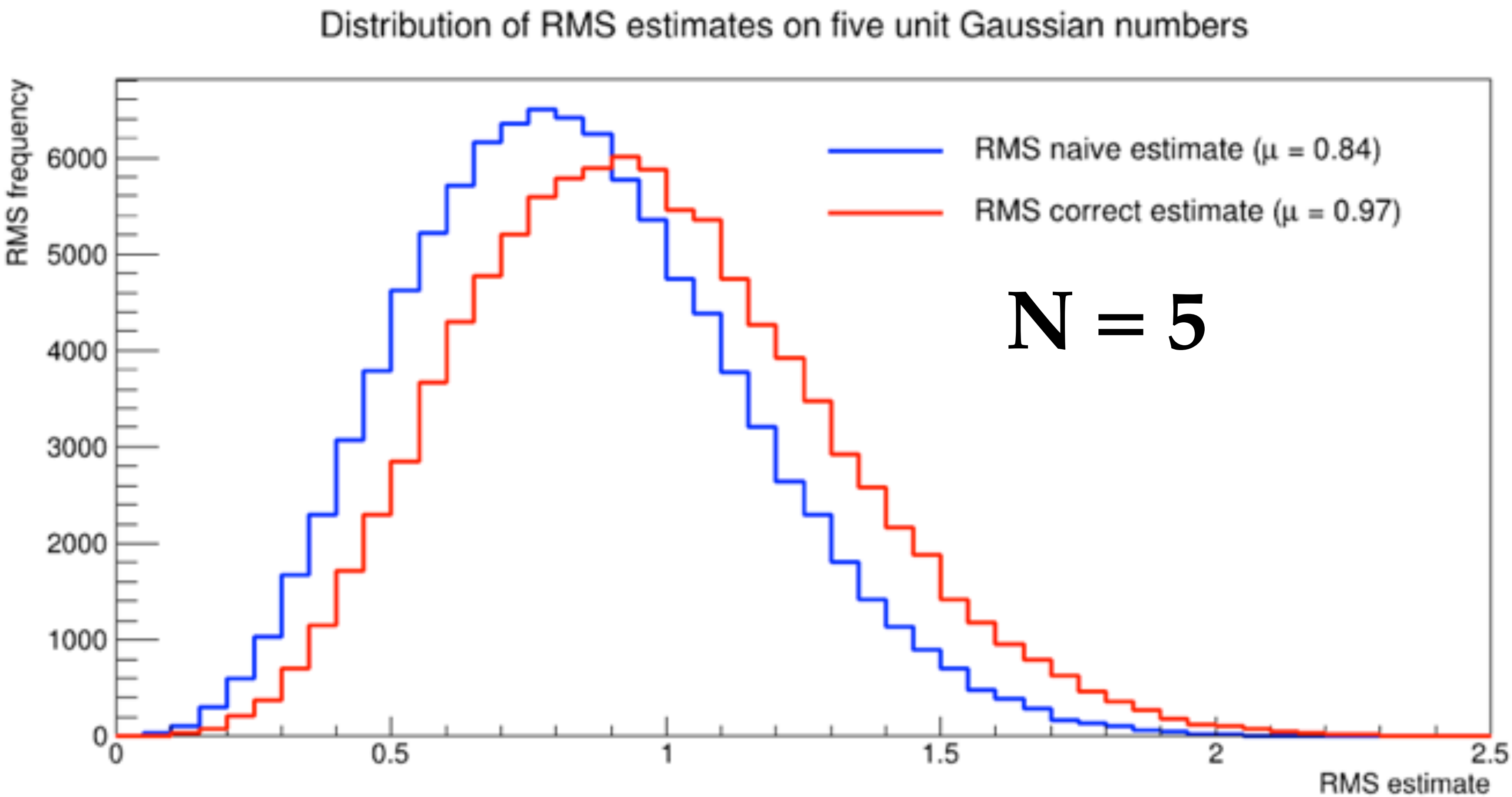
# How incorrect is the naive RMS?

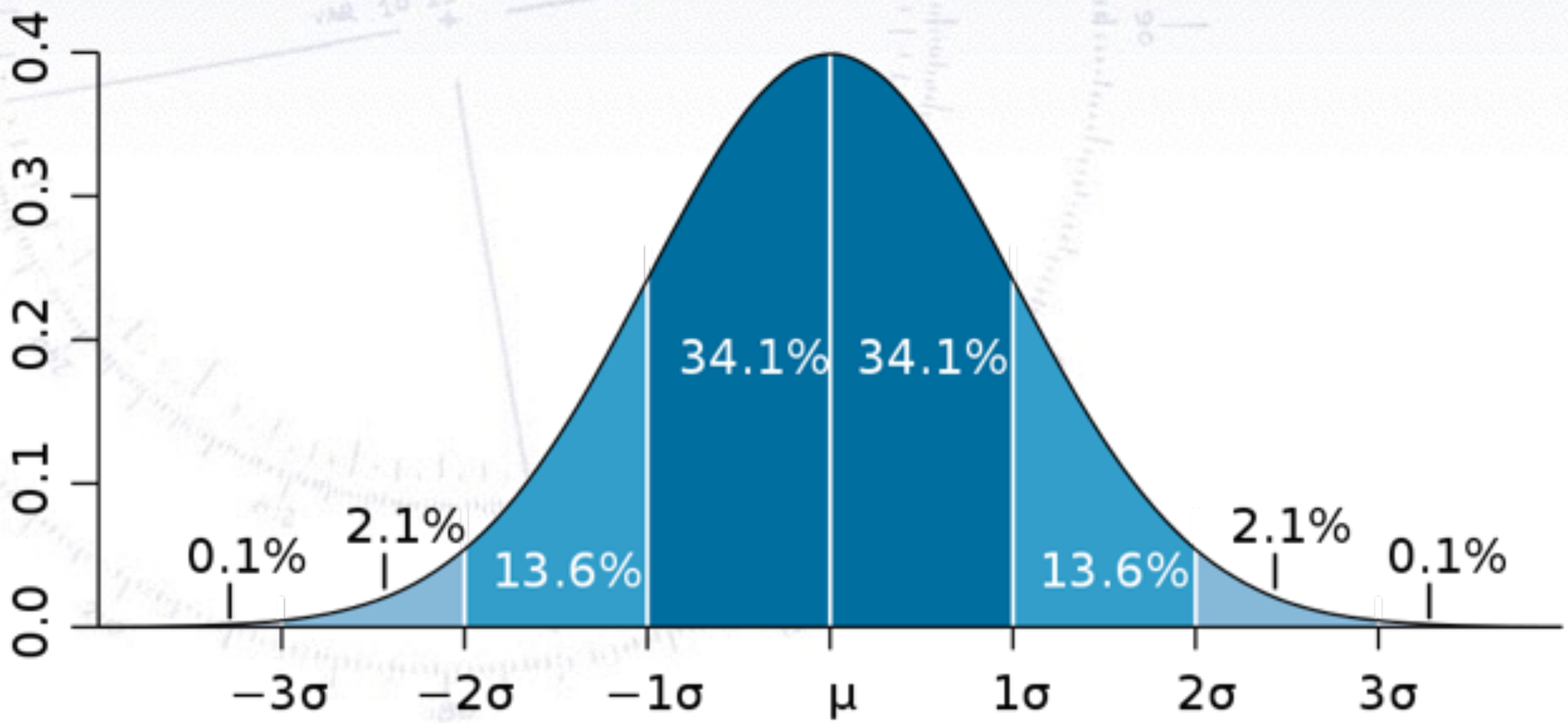
Such questions can most easily be answered by a small simulation:




So, the "naive" RMS underestimates the uncertainty a bit...

# Relation between RMS and Gaussian width...

When a distribution is Gaussian, the RMS corresponds to the Gaussian width σ:



# Mean and Width

What is the **uncertainty on the mean?** And how quickly does it improve with more data?

$$\hat{\sigma}_\mu = \hat{\sigma}/\sqrt{N}$$

<u>Example:</u>

**Cavendish Experiment**

(measurement of Earth's density)

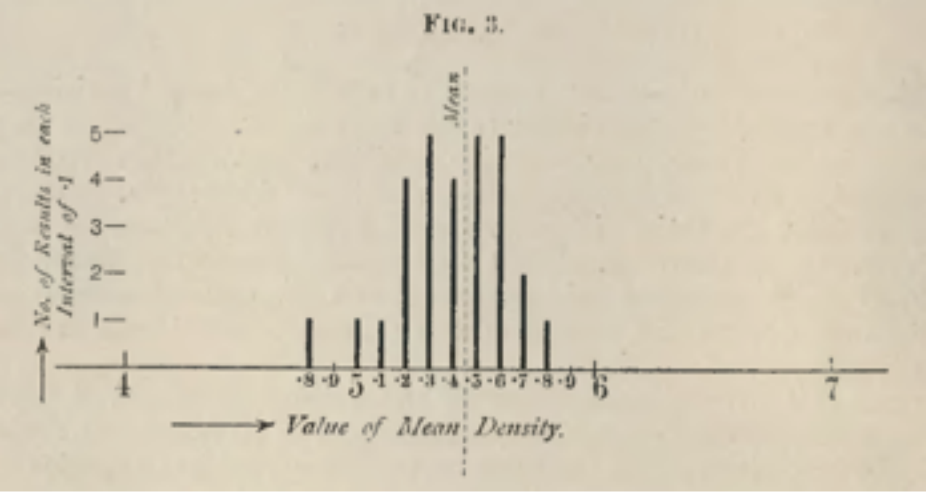
N = 29

mu = 5.42

sigma = 0.333

sigma(mu) = 0.06

**Earth density = 5.42 ± 0.06**



# Mean and Width

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Please, commit to memory now!

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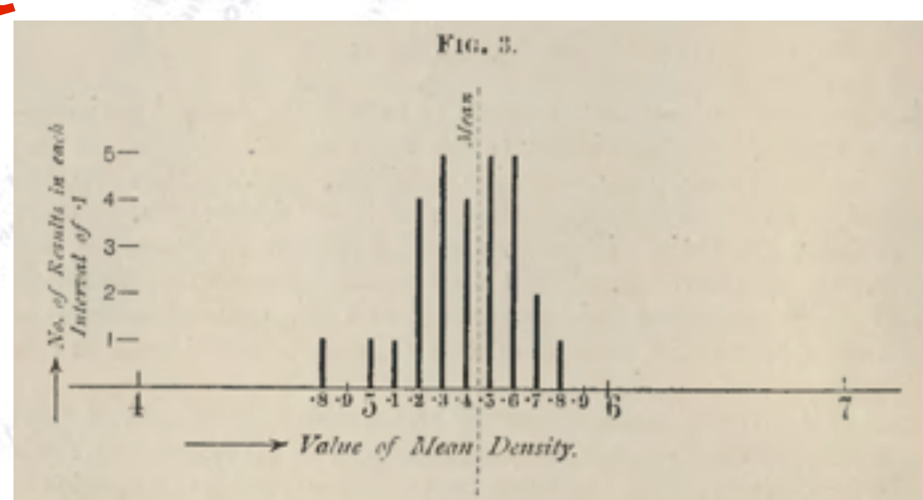


# Weighted Mean

What if we are given data, which has different uncertainties?
How to average these, and what is the uncertainty on the average?

$$\hat{\mu} = \frac{\sum x_i/\sigma_i^2}{\sum 1/\sigma_i^2}$$

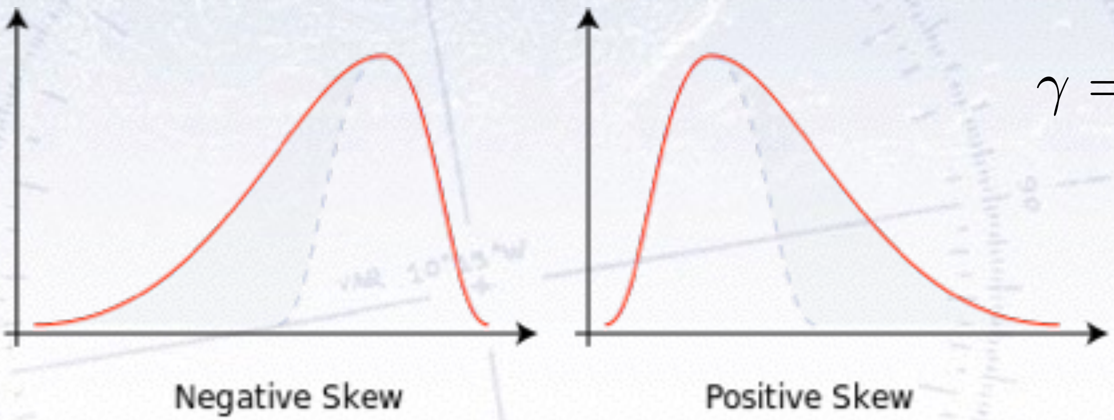
For measurements with varying uncertainty, there is no meaningful RMS!
The uncertainty on the mean is:

$$\hat{\sigma}_\mu = \sqrt{\frac{1}{\sum 1/\sigma_i^2}}$$

Can be understood intuitively, if two persons combine 1 vs. 4 measurements

# Skewness and Kurtosis

Higher moments reveal something about a distributions asymmetry and tails:




$$\gamma = \frac{\frac{1}{N}\sum_i (x_i - \bar{x})^3}{(\frac{1}{N}\sum_i (x_i - \bar{x})^2)^3/2}$$

$$\kappa = \frac{\frac{1}{N}\sum_i (x_i - \bar{x})^4}{(\frac{1}{N}\sum_i (x_i - \bar{x})^2)^2} - 3$$