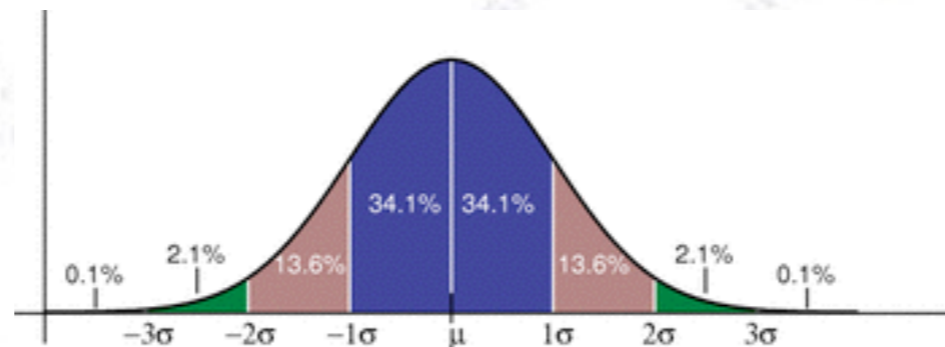


Applied Statistics

Confidence intervals and Limits



Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"



Confidence intervals

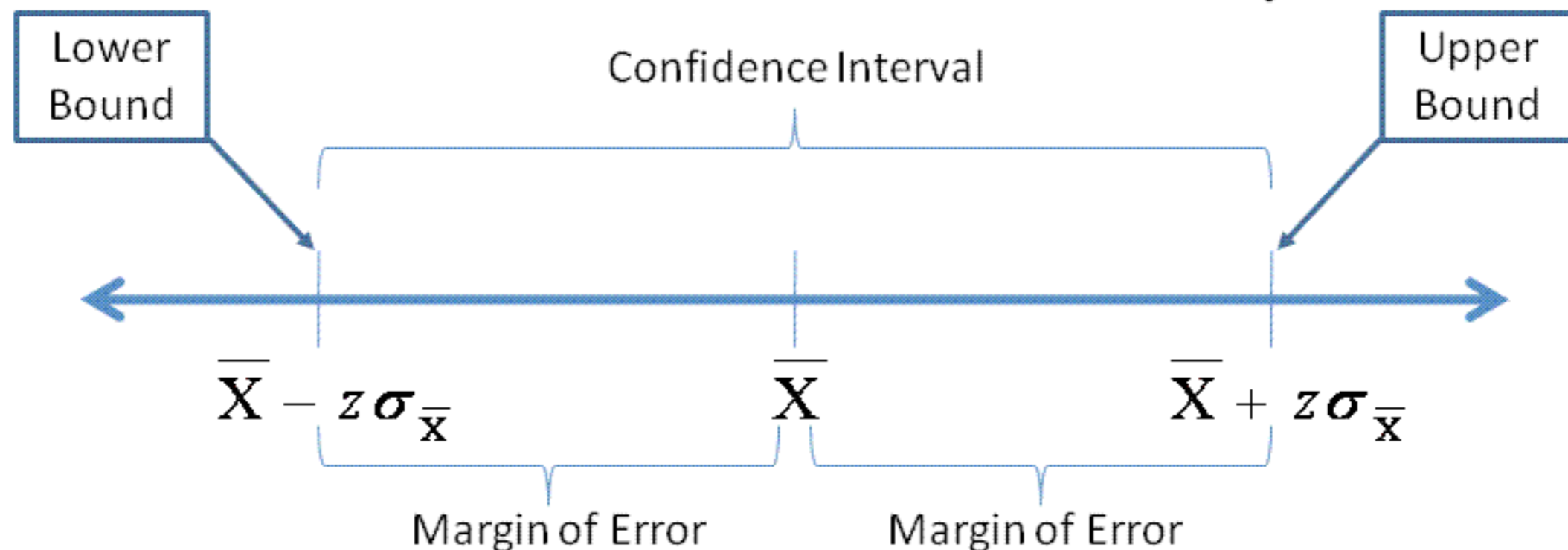
Confidence intervals

“Confidence intervals consist of a range of values (interval) that act as good estimates of the unknown population parameter.”

It is thus a way of giving a range where the true parameter value probably is.

A very simple confidence interval for a Gaussian distribution can be constructed as:
(z denotes the number of sigmas wanted)

$$\bar{x} \pm z \frac{s}{\sqrt{n}}$$



Confidence intervals

Confidence intervals are constructed with a certain **confidence level C**, which is roughly speaking the fraction of times (for many experiments) to have the true parameter fall inside the interval:

$$Prob(x_- \leq x \leq x_+) = \int_{x_-}^{x_+} P(x) dx = C$$

Typically, $C = 95\%$ (thus around 2σ), but 90% and 99% are also used occasionally.

There is a choice as follows:

1. Require symmetric interval (x_+ and x_- are equidistant from μ).
2. Require the shortest interval ($x_+ - x_-$ is a minimum).
3. Require a central interval (integral from x_- to μ is the same as from μ to x_+).

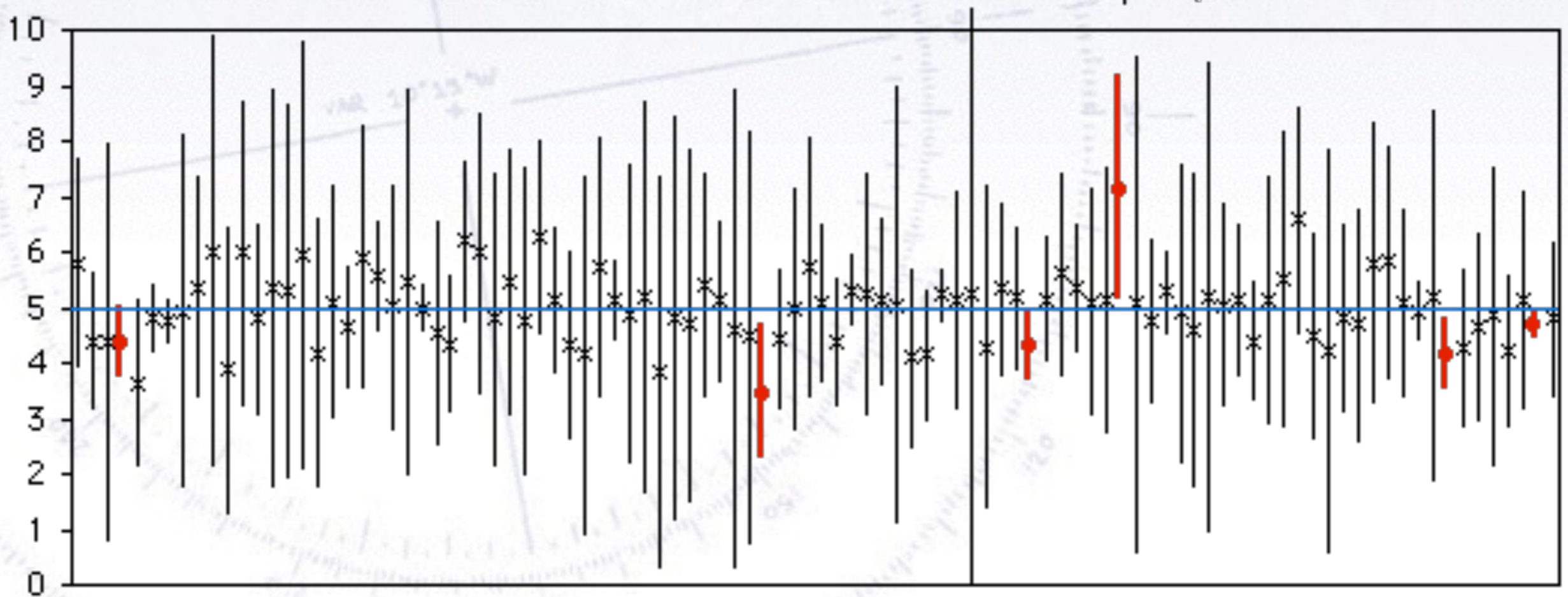
For the Gaussian, the three are equivalent!

Otherwise, 3) is usually used.

Confidence intervals

The confidence interval does not ALWAYS include the true value - only C fraction.

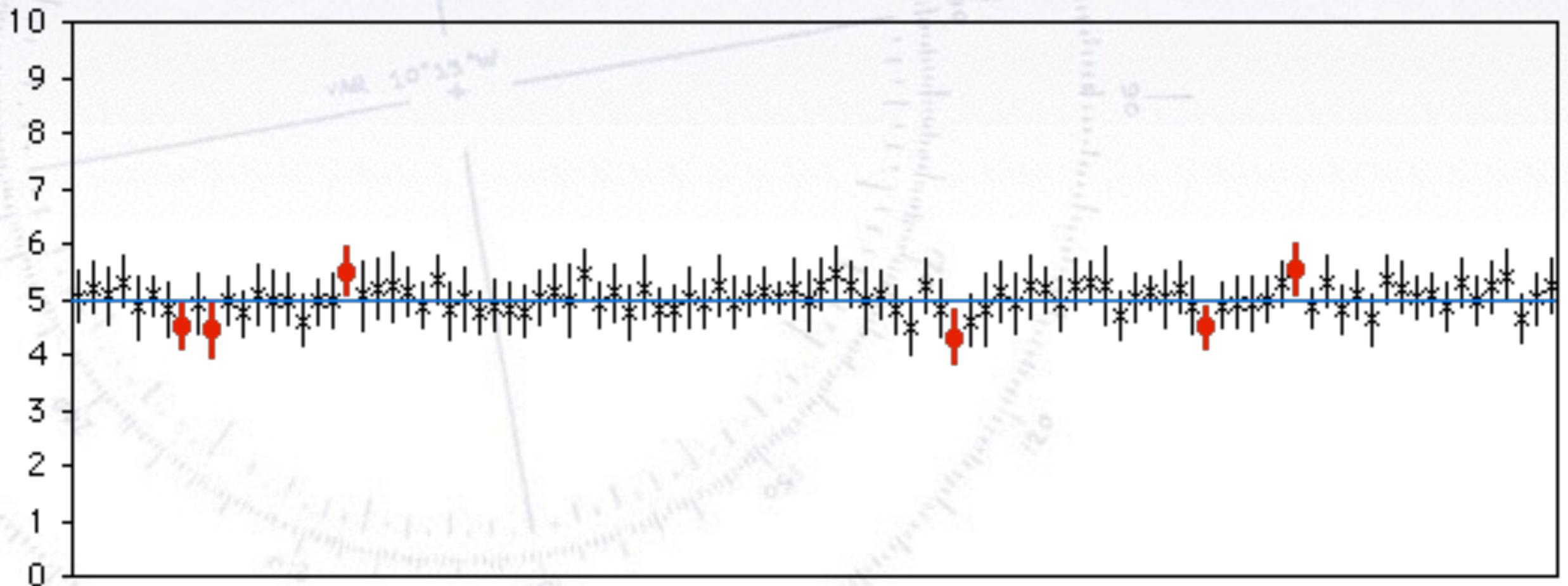
mean and 95% confidence intervals for 100 samples, N=3



Confidence intervals

The confidence interval does not ALWAYS include the true value - only C fraction.

mean and 95% confidence intervals for 100 samples, N=20

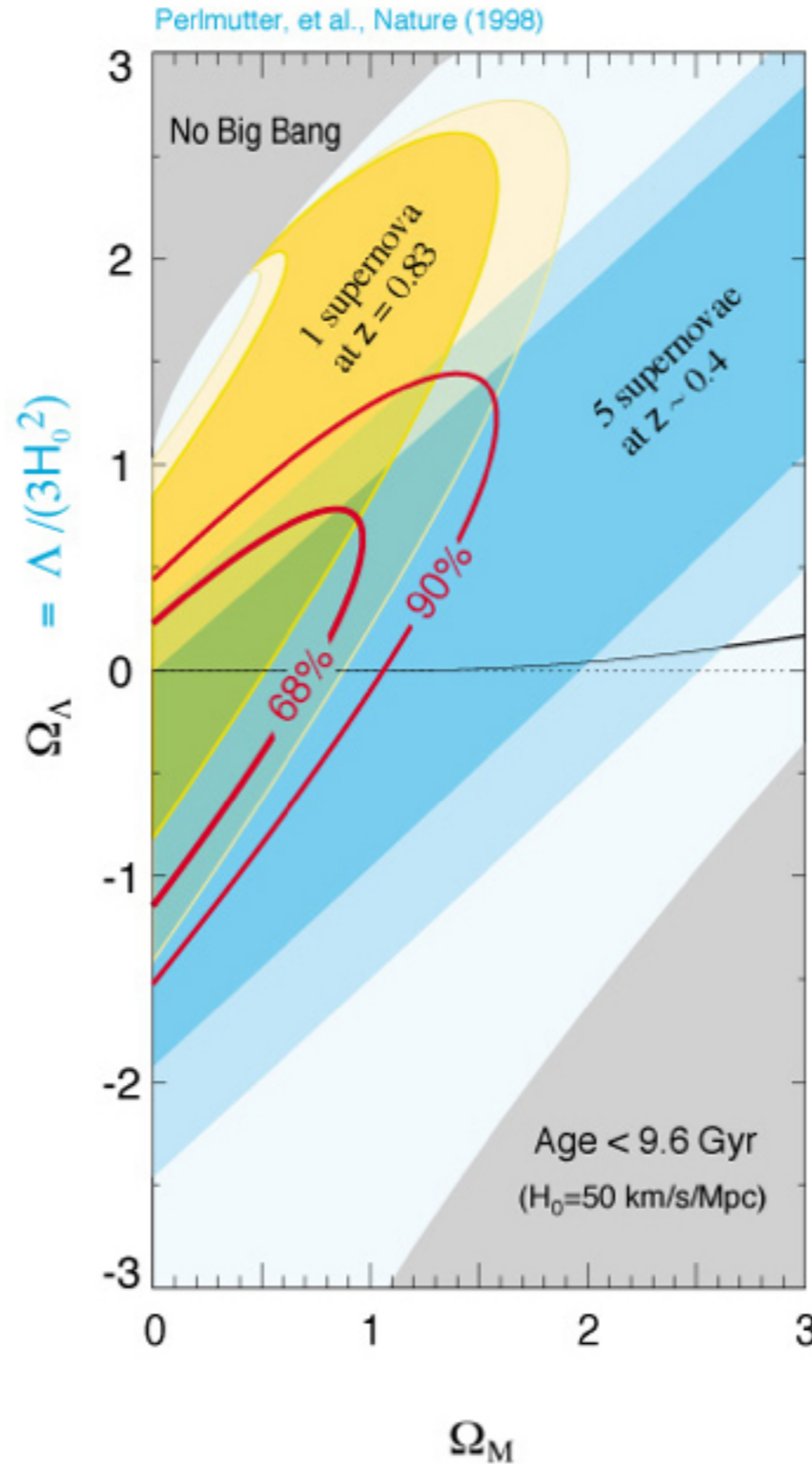


...and higher statistics does not help you!

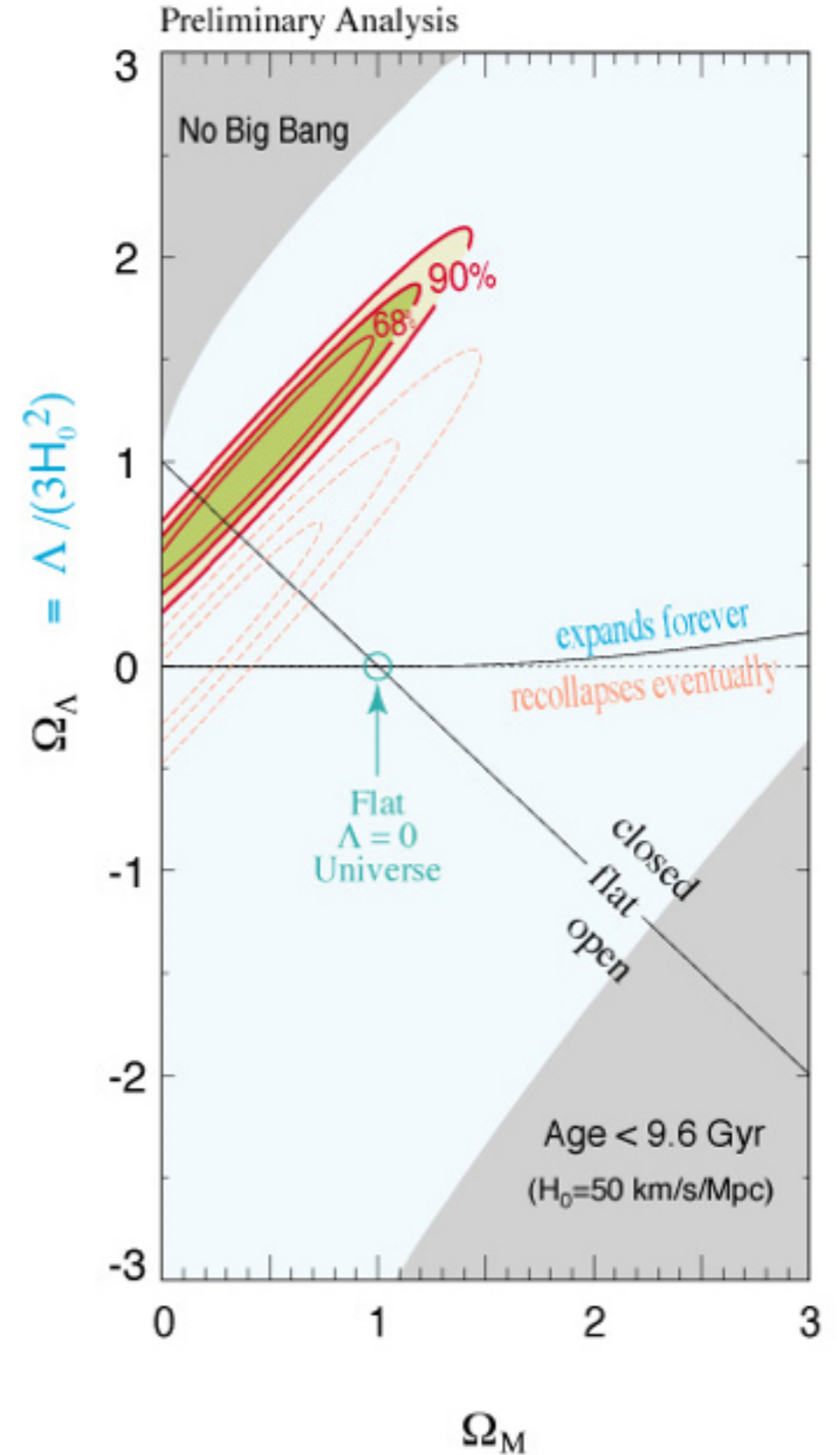
Example

from cosmology

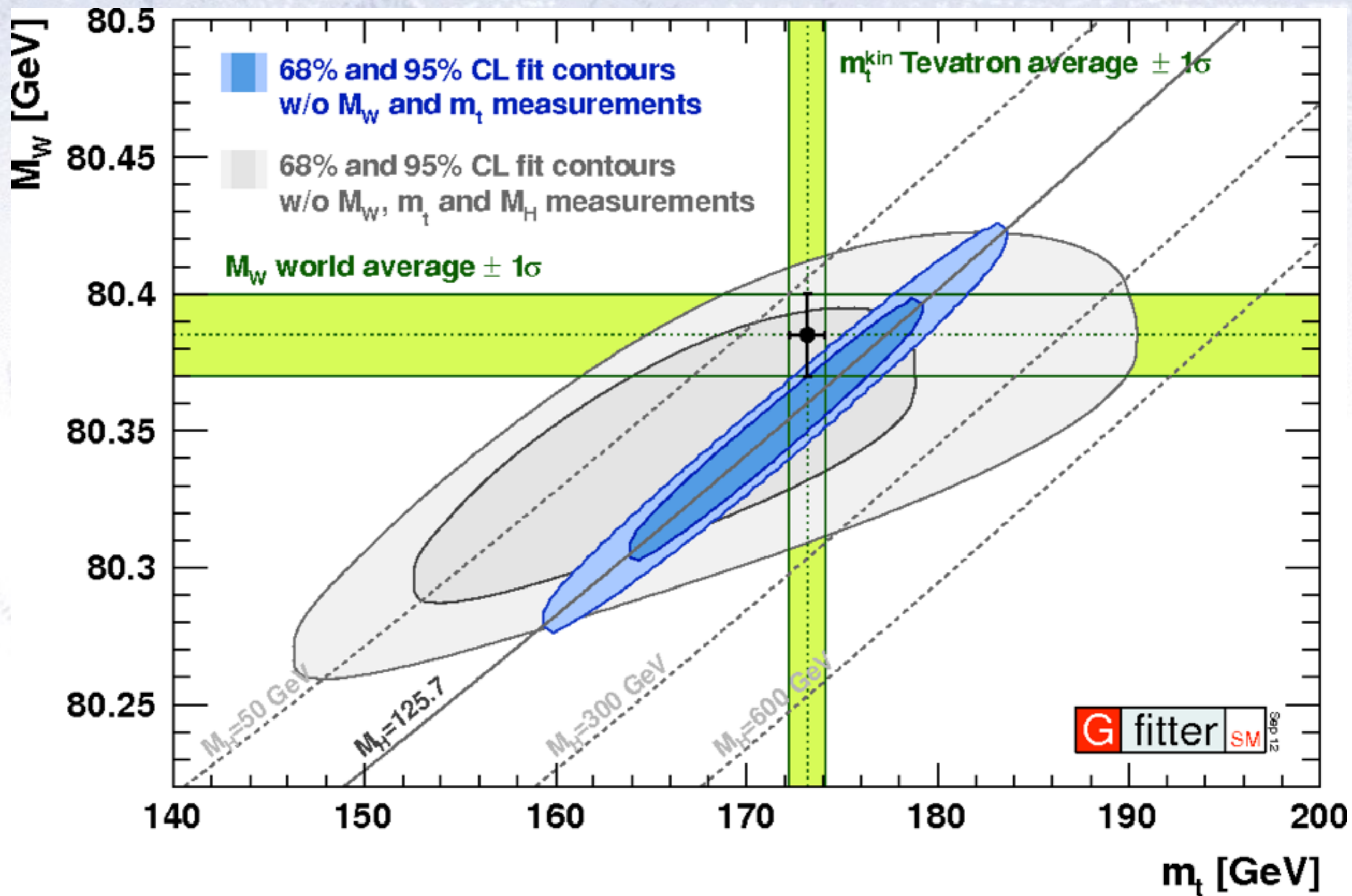
Results: Ω vs Λ
from 6 supernovae



Results: Ω vs Λ
from 40 supernovae



Example from particle physics



A background map of the North Atlantic Ocean showing bathymetry contours and a red confidence limit boundary. The map includes labels for 'MADAGASCAR' and 'ICE BITTER END YACHT CLUB'. A red line with a dashed center and solid edges forms a roughly rectangular shape, indicating a confidence limit. A star symbol is located at approximately 10°15'W, 30°N, with the label 'VAR 10°15'W' next to it. The map also shows various depth contours and latitude/longitude lines.

Confidence limits

Confidence limits

Imagine that you do an experiment to search for an unknown but predicted phenomenon (aether, planet Vulcan, dark energy, Higgs particle, etc.), and that find nothing!

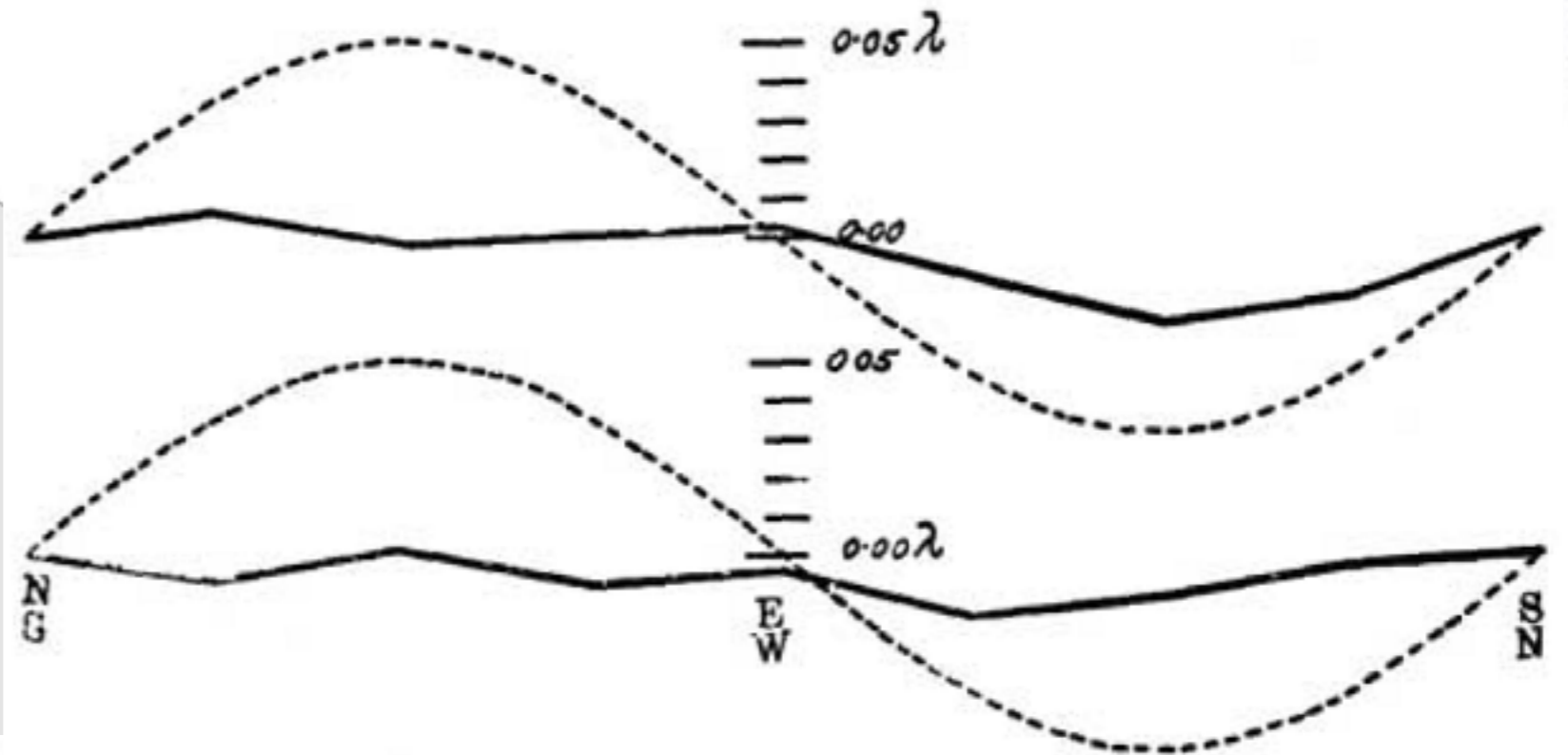
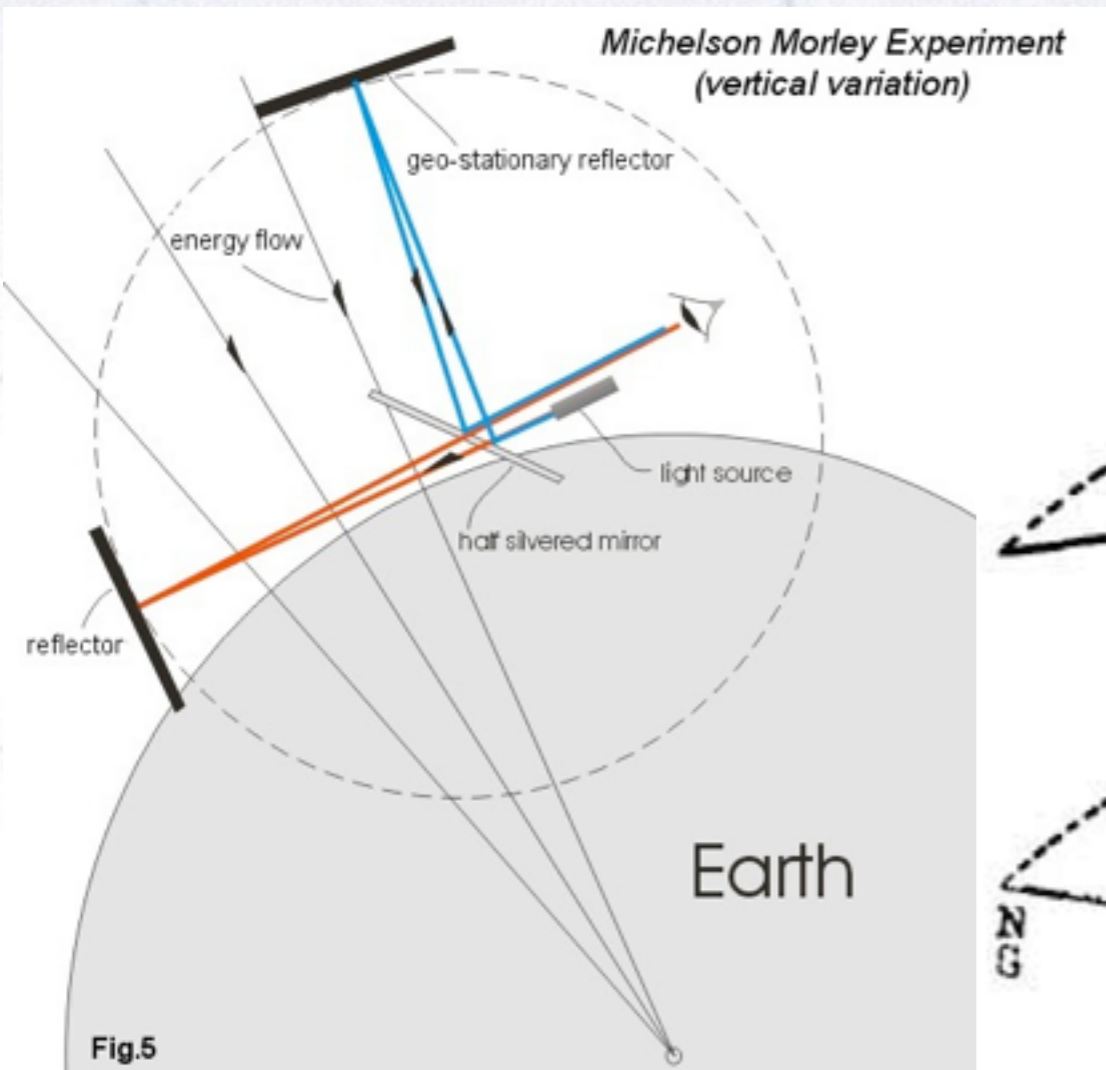
Reporting this result, you wish to state *what you would have discovered, if it had been there*, i.e. something along the lines:

“If the aether had affected the speed of light by $X\%$, we would have seen the effect with 95% confidence”.

This is a **confidence limit** (much like a one-sided confidence interval).

Confidence intervals

In the case of Michelson-Morley, a limit could be set on the “degree of dragging” of the aether (though they didn’t do this, as statistics was still in its infancy!).



Confidence limits - Poisson

Poisson statistics is a neat special case, perhaps best explained by numbers:

Table 9.3 Poisson lower and upper limits for n_{obs} observed events.

n_{obs}	lower limit a			upper limit b		
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.01$	$\beta = 0.1$	$\beta = 0.05$	$\beta = 0.01$
0	-	-	-	2.30	3.00	4.61
1	0.105	0.051	0.010	3.89	4.74	6.64
2	0.532	0.355	0.149	5.32	6.30	8.41
3	1.10	0.818	0.436	6.68	7.75	10.04
4	1.74	1.37	0.823	7.99	9.15	11.60
5	2.43	1.97	1.28	9.27	10.51	13.11
6	3.15	2.61	1.79	10.53	11.84	14.57
7	3.89	3.29	2.33	11.77	13.15	16.00
8	4.66	3.98	2.91	12.99	14.43	17.40
9	5.43	4.70	3.51	14.21	15.71	18.78
10	6.22	5.43	4.13	15.41	16.96	20.14

Example: If you in a day observe 0 red cars on Blegdamsvej, you can with 95% confidence say that there are fewer than 3.00 pr. day, and with 90% confidence say that there are fewer than 2.30 pr. day.