Applied Statistics

Project 1 evaluation






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"Statistics is merely a quantisation of common sense"

Project 1 evaluation

Pendulum:

- Did you measure T ± σ(T) correctly? Combine with Chi2 and comments?
- Did you measure L ± σ(L) correctly?
- Did you provide the individual precisions?

Ball on incline:

- T ± σ(T)
- L ± σ(L)

} ⇒ a ± σ(a), with Chi2 and comments.

- θ, Δθ obtained correctly and
- d, R and errors propagated correctly?

Generally:

- **Correctly propagated uncertainties, showing individual contributions.**
- All necessary figures and tables there? 2-3 essential figures needed.
- Text enough to understand results? Clear and fitting captions?
- Comment on result, especially inconsistencies.
- Significant digits.

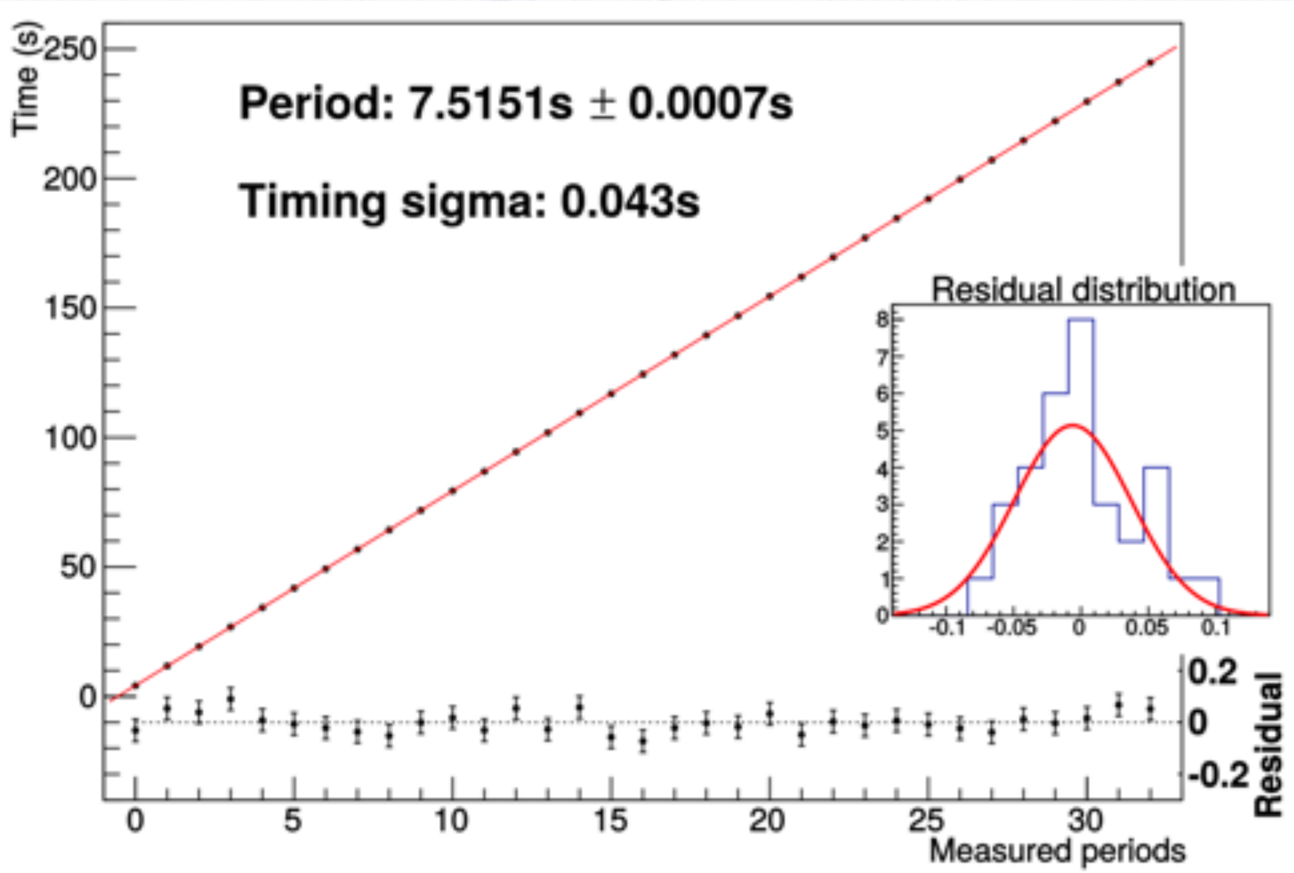
Pendulum - comments

Time measurement:
Many independent measurements, little systematic $\Rightarrow$ Good error estimate

Length measurement:
Some independent measurements but also some systematics $\Rightarrow$ check difference between instruments.

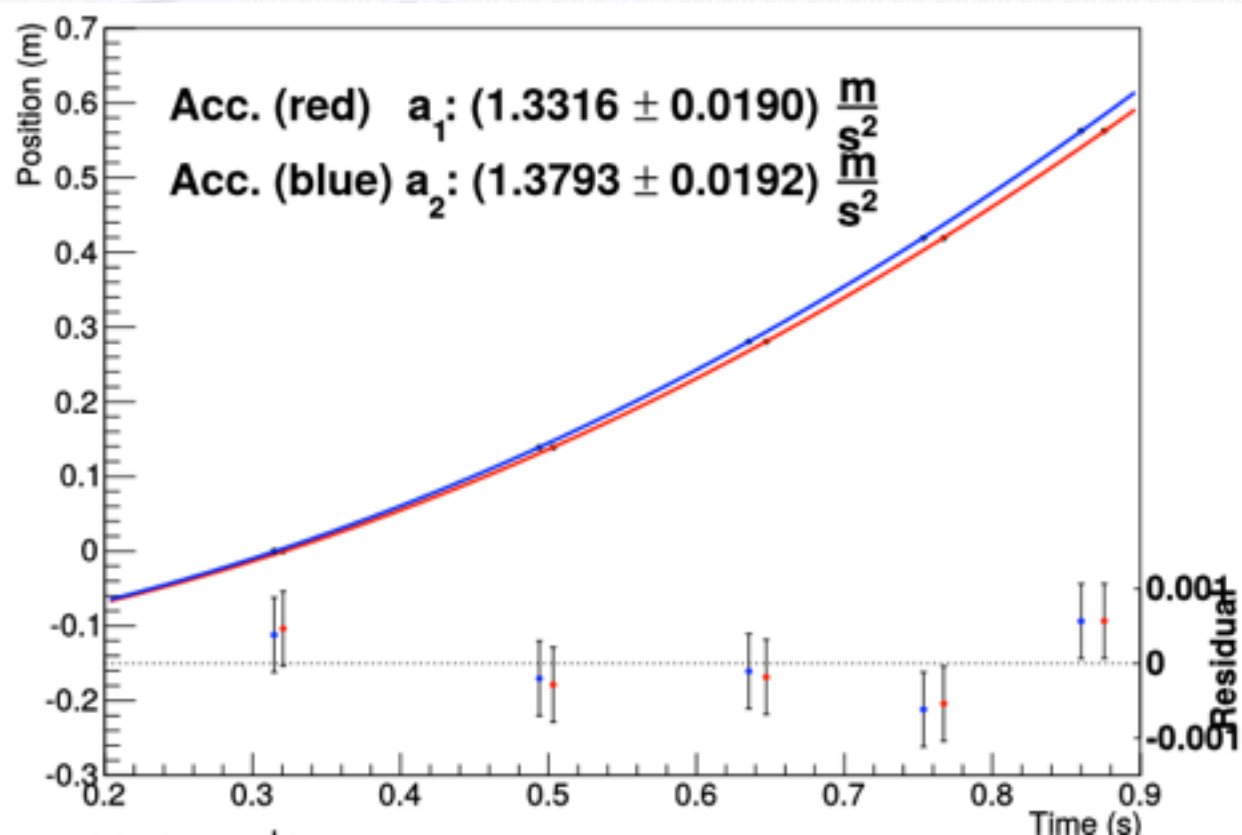
You can not reduce the uncertainty by multiple measurements, if the main limitation is some inherent systematic!

Several groups managed to get uncertainties below 0.1%.



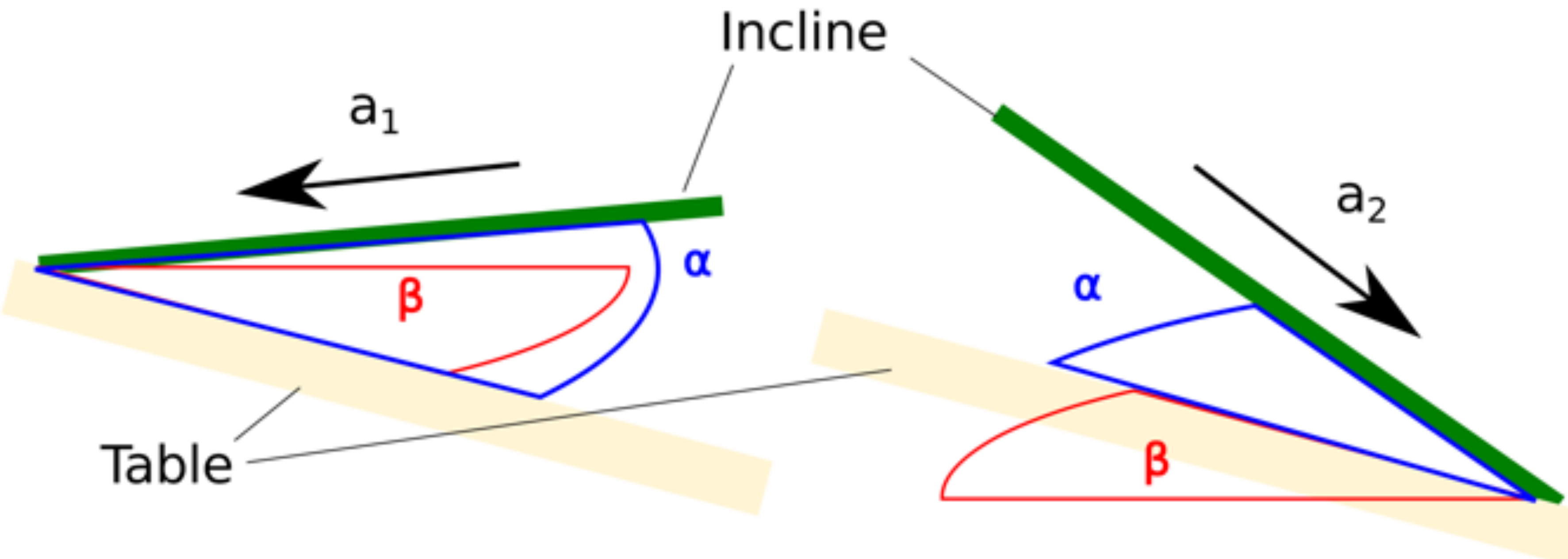
Ball on incline - comments

Time measurement: Insignificant uncertainties, so repetition doesn't help!
Length measurements: Some uncertainties, some systematics, but OK errors.
Note that diode position is not central!
Angle measurement: This is the large challenge! And angle vs. lengths have very different systematics, so they are good to combine!



Ball on incline - comments

Calculate and utilize the table angle:



$$\frac{a_{left}}{a_{right}} = \frac{\sin(\theta + \Delta\theta)}{\sin(\theta - \Delta\theta)}. \qquad \Delta\theta = \frac{(a_{left} - a_{right})\sin(\theta)}{(a_{left} + a_{right})\cos(\theta)}.$$

General comments

- Careful in not combining correlated numbers.
- Never combine inconsistent numbers. (Then you are SURE to make a mistake!)
- Comment (heavily) on inconsistent numbers.
- Careful not to repeat measurements that are small or systematically dominated.
- Always remember correct number of significant digits.
- Plan your plots. Here 3-4 is fitting. At least length vs. time for both experiments.
- Put results and conclusions in abstract. Meant for saving time of reader.
- Be short and precise. Label carefully and refer to these labels.
- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Don’t put definition of mean, RMS, etc. in your text. Considered known to all.
- Be VERY detailed about removing data. Show explicitly why. Give p-values.

Specific comments

<u>Pendulum experiment:</u>

- Pendulum line may be stretching. Requires times and lengths for individual g.
- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Swinging pendulum at 90 degrees is a good check of impact of hook.
- Combine lengths (L) with normal mean, and get uncertainty from RMS.
- Combine periods (T) with weighted mean and use Chi2 to check consistency.

<u>Ball-on-incline experiment:</u>

- Rerunning ball many times to get σ(t_gate) only gives you σ(t_magnet release).
- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Write fit function with constants in front, i.e. $0.5 * a * t^2$. Gives correct errors!
- Different ball size give different results. 1% variation (10, 12.5 and 15mm).
- No, you can’t measure the angle with 0.001 degree precision!!!

Examples from reports

Using different ball sizes gave variations in the result (10, 12.7, and 15 mm).

$$g_{Big} = (9.865 \pm 0.040_{stat} \pm 0.001_{sys})\ \mathrm{m/s^2},$$
$$g_{Medium} = (9.822 \pm 0.042_{stat} \pm 0.001_{sys})\ \mathrm{m/s^2},$$
$$g_{Small} = (9.741 \pm 0.055_{stat} \pm 0.001_{sys})\ \mathrm{m/s^2},$$

Table of (changing) impact is GREAT:

Parameter	Big Ball	Medium Ball	Small Ball
a	0.024	0.024	0.022
θ	0.030	0.030	0.030
$\Delta\theta$	0.0014	0.0014	0.0013
D	0.003	0.007	0.019
d	0.010	0.016	0.035
Total	0.040	0.042	0.055

Examples from reports

Remember to only put 1-2 significant digits on the error, and then the SAME number of digits on the result.

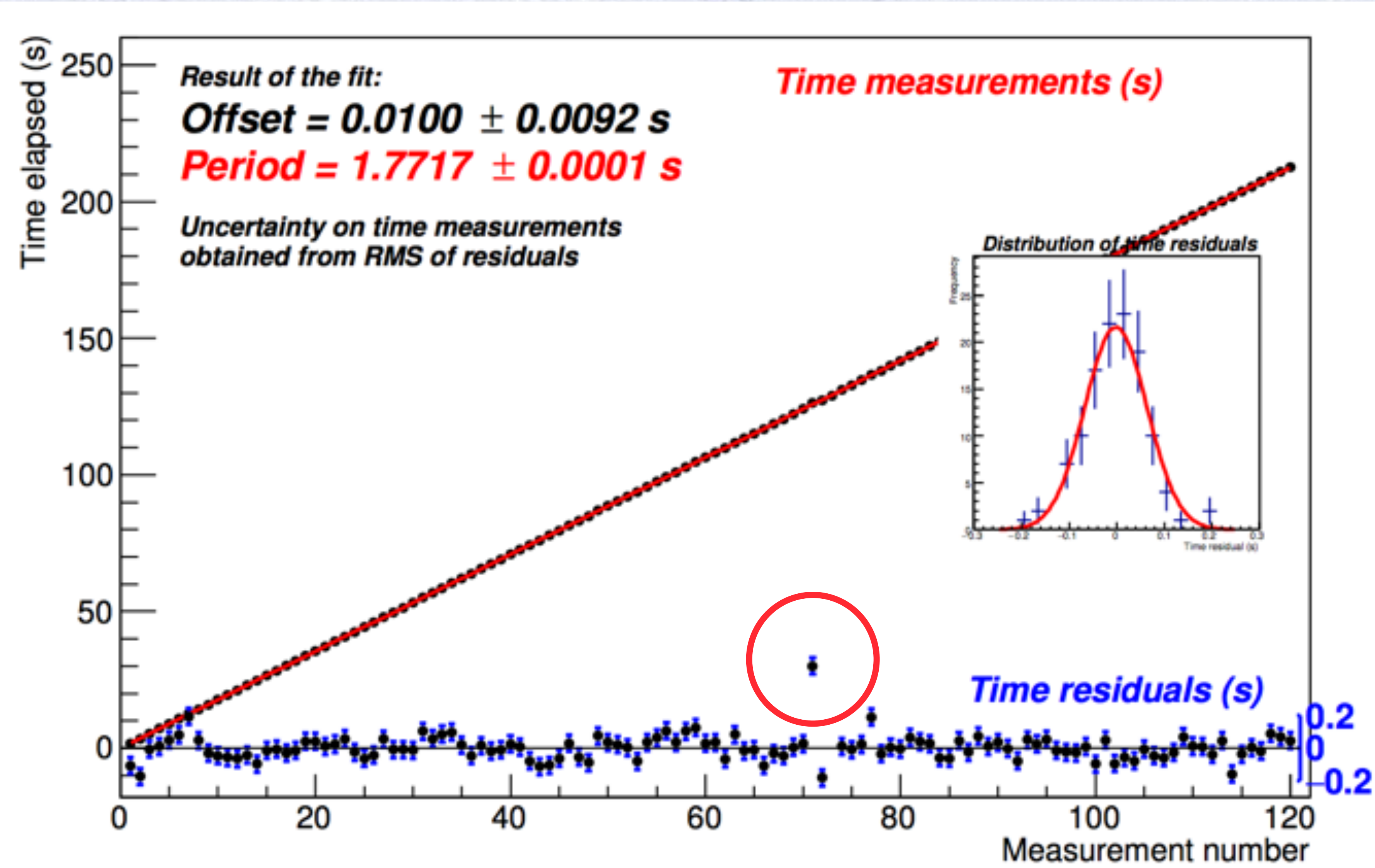
Exp.	a_h	σ_{ah}	a_v	σ_{av}
1	1.541 m/s^2	0.01170 m/s^2	1.473 m/s^2	0.008515 m/s^2
2	1.543 m/s^2	0.01176 m/s^2	1.472 m/s^2	0.008517 m/s^2
3	1.543 m/s^2	0.01173 m/s^2	1.475 m/s^2	0.008550 m/s^2
4	1.541 m/s^2	0.01172 m/s^2	1.474 m/s^2	0.008533 m/s^2
5	1.539 m/s^2	0.01175 m/s^2	1.474 m/s^2	0.008544 m/s^2
6	1.545 m/s^2	0.01177 m/s^2	1.473 m/s^2	0.008521 m/s^2
7	1.543 m/s^2	0.01173 m/s^2	1.471 m/s^2	0.008541 m/s^2
8	1.540 m/s^2	0.01171 m/s^2	1.474 m/s^2	0.008513 m/s^2
9	1.543 m/s^2	0.01175 m/s^2	1.471 m/s^2	0.008498 m/s^2
10	discarded	discarded	1.475 m/s^2	0.008529 m/s^2
Result	**1.54** m/s^2	**0.0039** m/s^2	**1.47** m/s^2	**0.0027** m/s^2
χ^2/Ndf	0.20/8		0.27/9	
Prob.	1		1	

Tool	Height	Statistical error	Systematic error
Tomme	313.29 cm	0.1331 cm	0.05 cm
Lineal	311.88 cm	0.1664 cm	0.05 cm
Vinkel	311.89 cm	0.1464 cm	0.05 cm
Result	**312.46 cm**	**0.085 cm**	0.0162 cm
χ^2/Ndf	66.19/2		
Prob.	$4.234 \cdot 10^{-15}$		

Furthermore, remember to COMMENT on your Chi2 probabilities!
This is very important, because a test is worthless, if not acted upon.

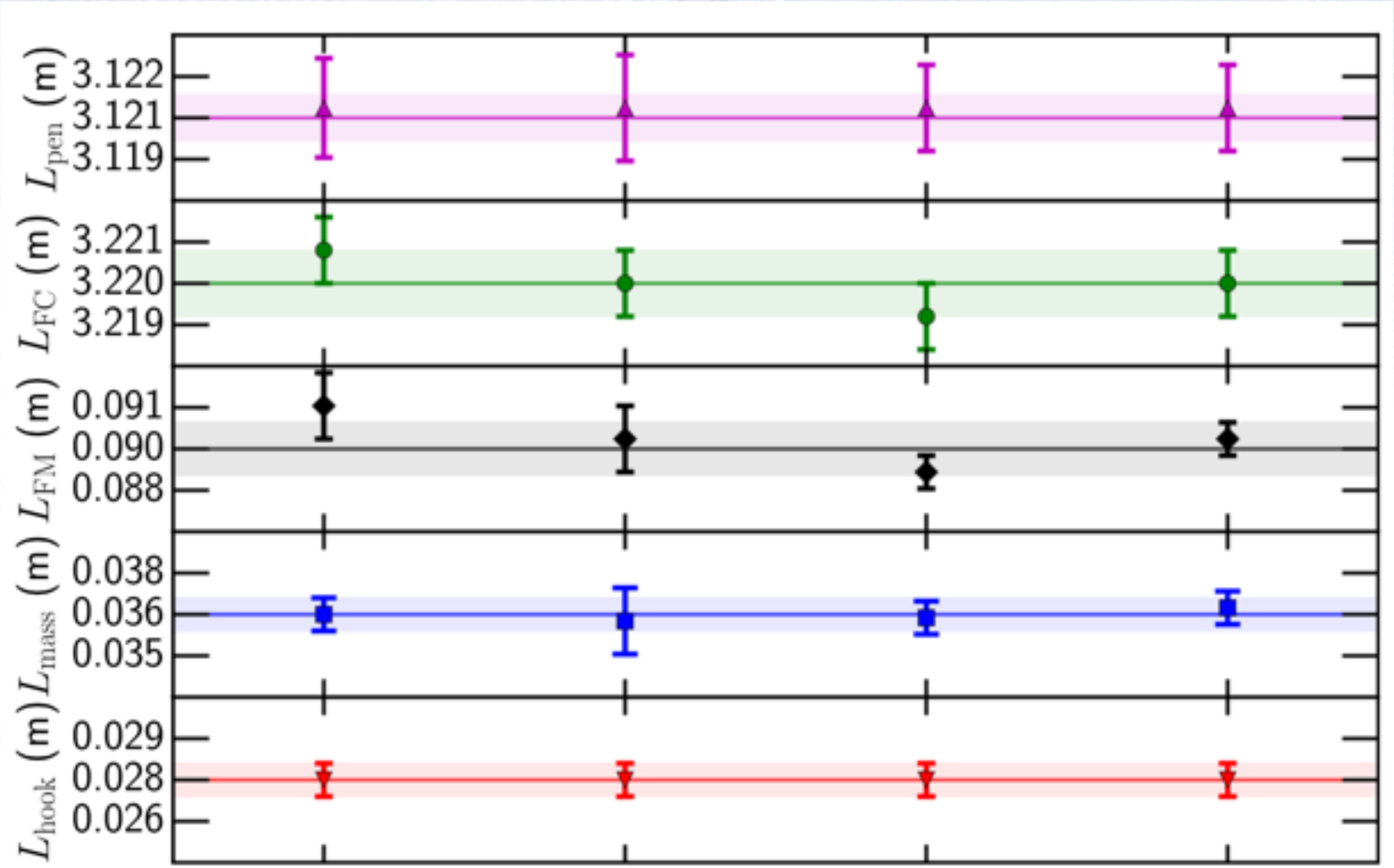
Examples from reports

Sometimes, single measurement are clearly off… consider/comment on these!



Examples from reports

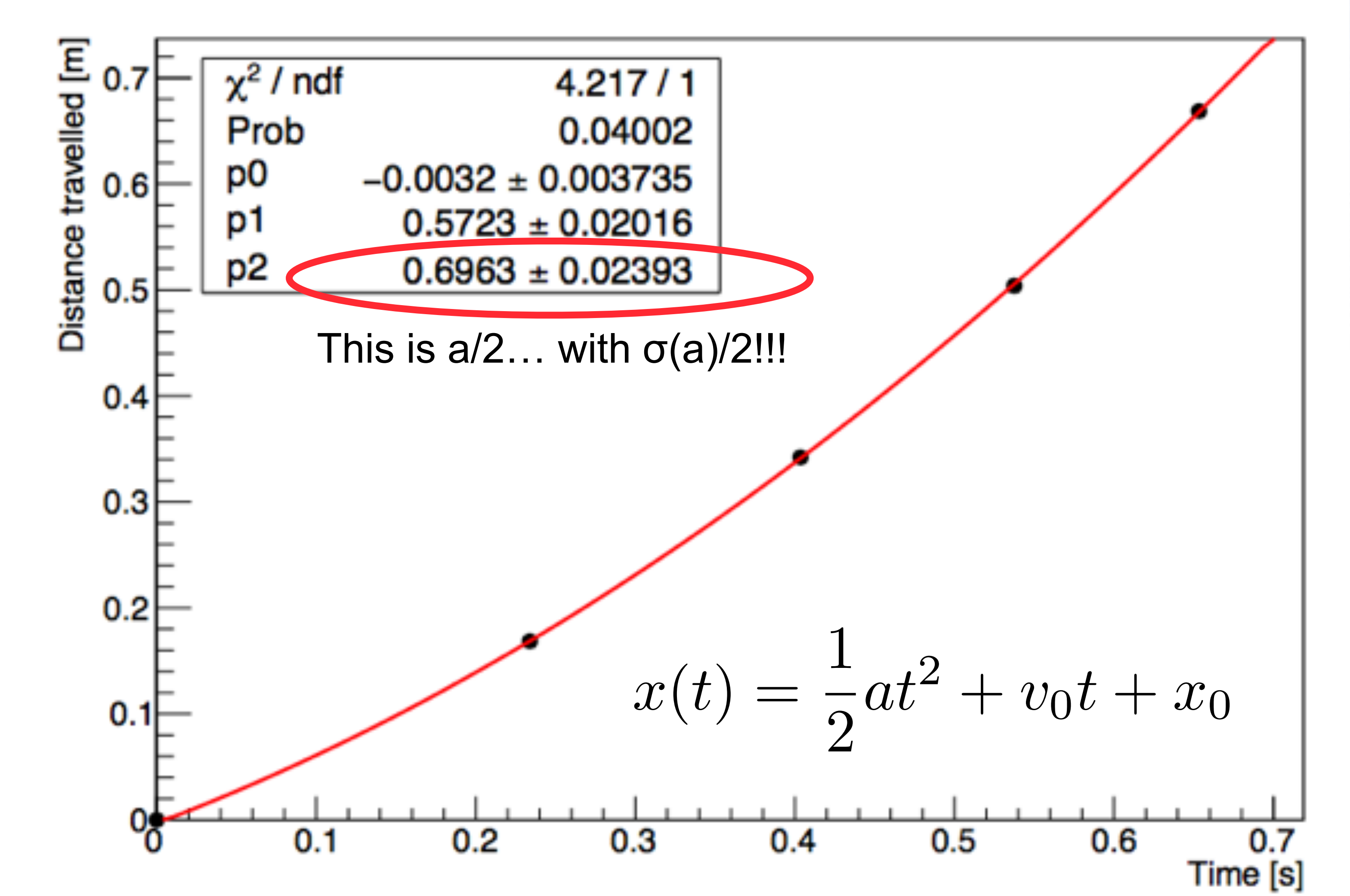
Simple but illustrative figure, showing correspondence and uncertainties.



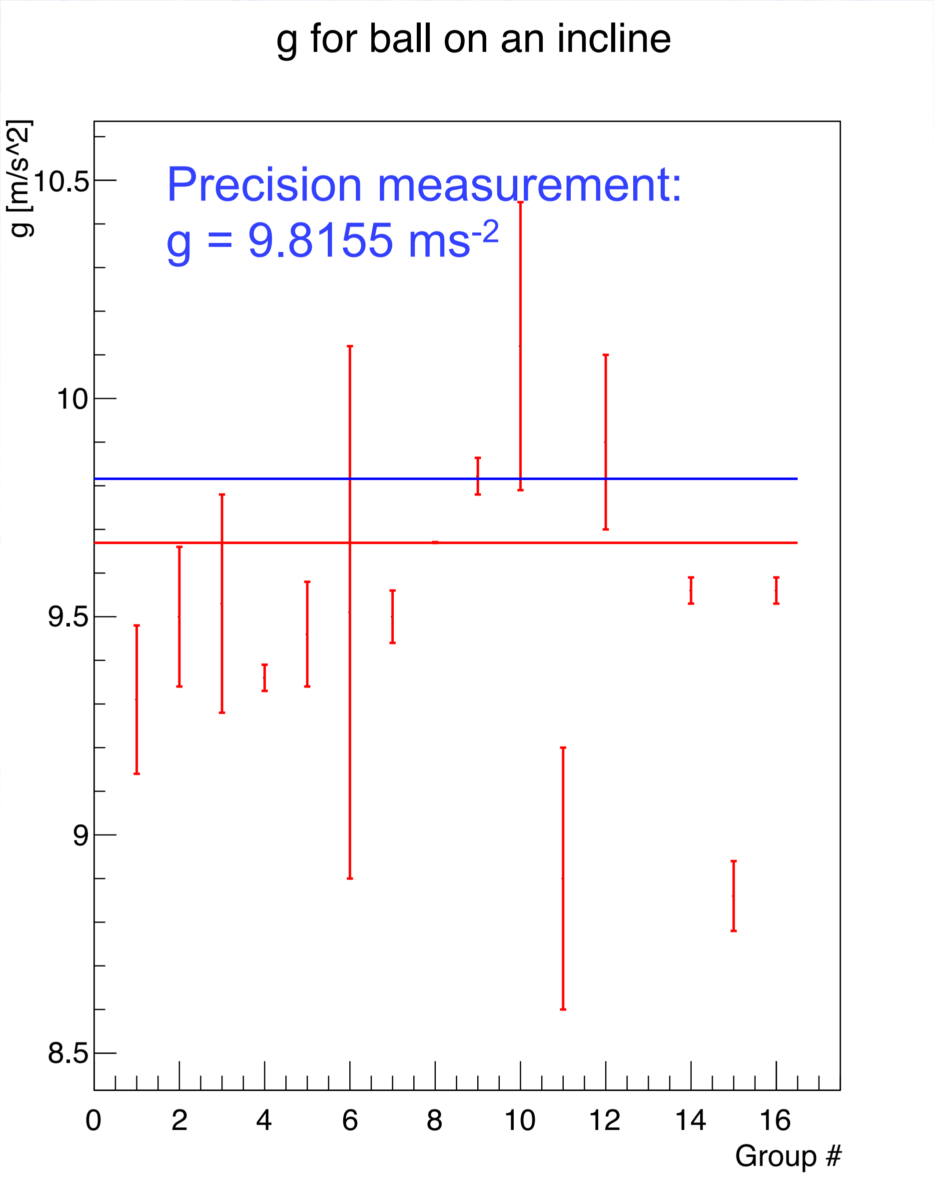
Of course, since there are errors on all estimates, a Chi2 would have been in place!

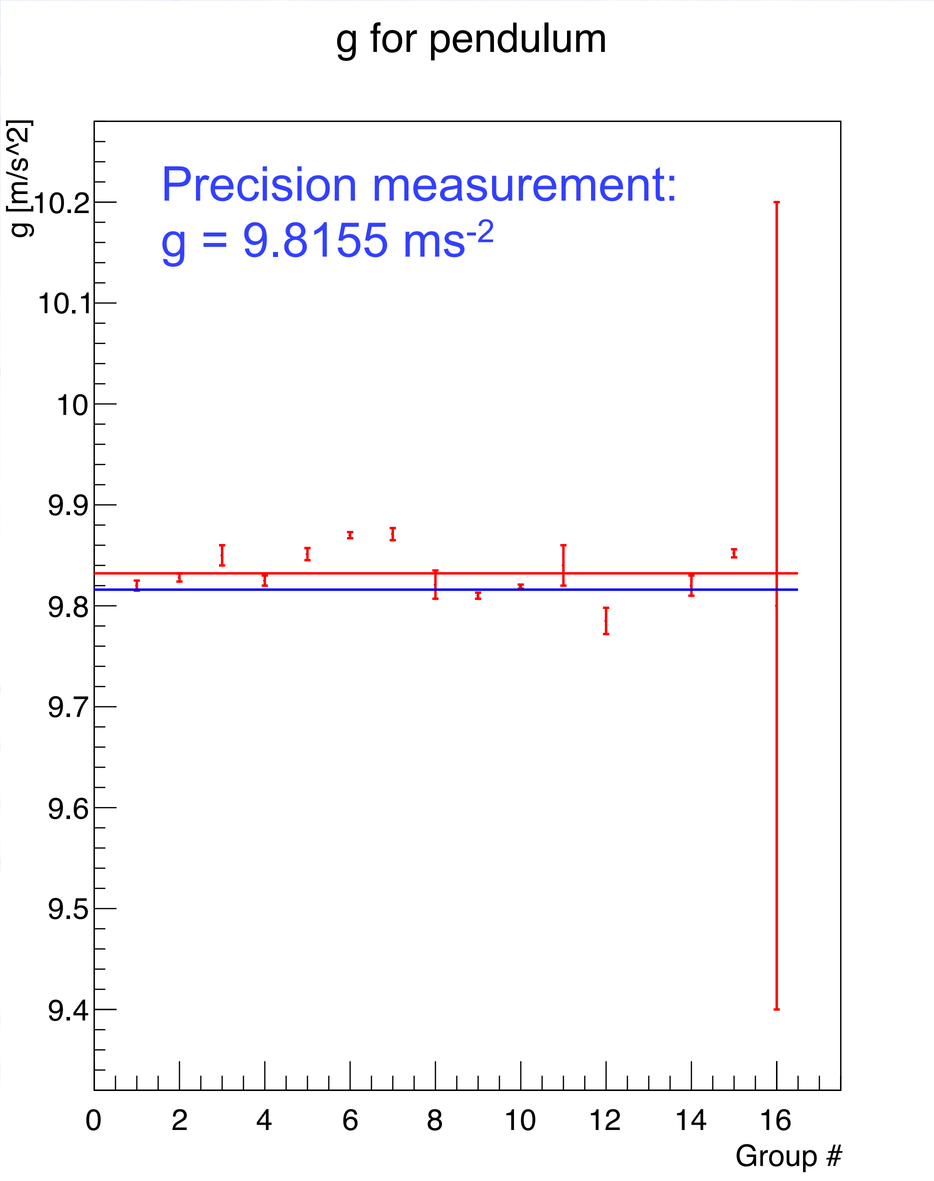
Examples from reports

Think (a lot) about constraining points in the fit (i.e. by giving 0.0 in error).
Also, it is a good idea to fit with all constants in place in formula, i.e. factor 1/2.



Your results





Various uncertainties

Group:	sT (s)	sL (m)	g_pend	sAcc.	sTheta	sDtheta	g_ball
1:	0.0005	0.0014	9.820+-0.005	0.016	0.03	0.003	9.31+-0.17
2:	0.0016	0.0012	9.828+-0.004	0.02	0.14	1	9.5X+-0.16
3:	0.0008	0.0022	9.85 +-0.01	0.004	0.17	0.03	9.53+-0.25
4:	0.0018	0.001	9.825+-0.005	0.005	0.014	0.14	9.36+-0.03
5:	0.00001	0.00004	9.8512+-0.006	0.003	0.01	0.02	9.46+-0.12
6:	0.0003	0.0009	9.87X+-0.003	0.011	0.03	0.12	9.51+-0.61
7:	0.0006	0.0017	9.871+-0.006	0.006	0.014	0.18	9.50+-0.06
8:	0.0008	0.035	9.821+-0.014	0.002	0.001	0.00004	9.670+-0.001
9:	0.0030	0.0020	9.810+-0.003	0.010	0.04	0.0017	9.822+-0.042
10:	0.0001	0.0010	9.819+-0.002	0.0015	0.18	-9.999	10.12+-0.33
11:	0.0008	0.005	9.840+-0.020	0.030	0.001	0.03	8.9+-0.3
12:	0.0014	0.0005	9.785+-0.013	0.040	-9.999	-9.999	9.9+-0.2
14:	0.0011	0.03	9.82 +-0.01	0.003	0.11	-9.999	9.56+-0.03
15:	0.000008	0.0011	9.852+-0.004	0.04	0.05	0.15	8.86+-0.08
16:	0.0003	0.002	9.8 +-0.4	0.010	0.02	0.05	9.56+-0.03

My best estimates of a "minimum" uncertainty:

0.0002s 0.0005m 0.005 0.20 0.02

General considerations

Be careful with error estimation and propagation:

- Systematic errors do not decrease with repetition!
- Propagation of error needs consideration - not always in quadrature.
- Be conservative, if you are uncertain about your errors.

Always show what you're doing:

- Make good plots.
- Give measurement tables.
- Compare the difference between results and methods.

Also:

- Put the result in the abstract.
- Explain your symbols in formulae.

Additional ideas

It is possible to “leave” the pendulum swinging between to sets of measurements. This maximises the period over which you measure, without requiring your activity all the time…

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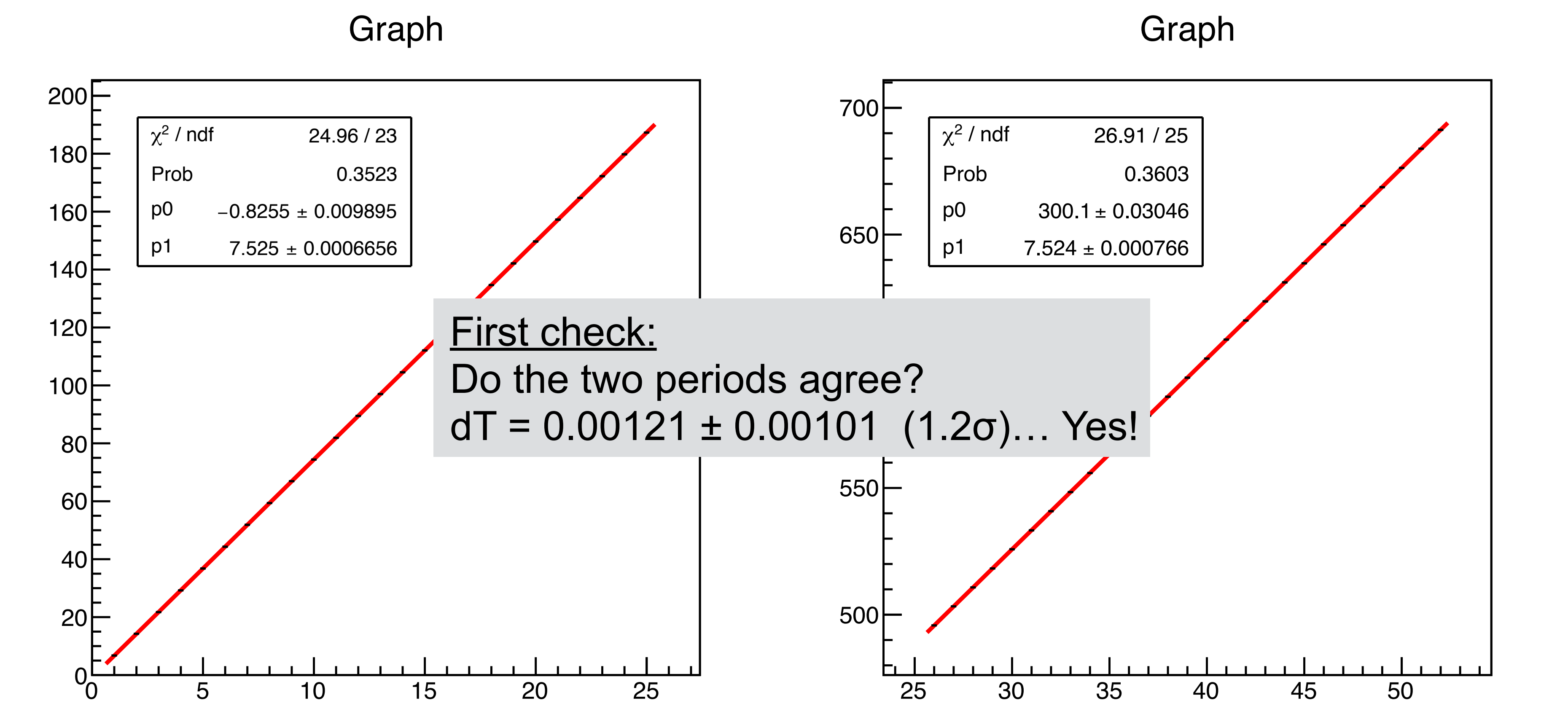
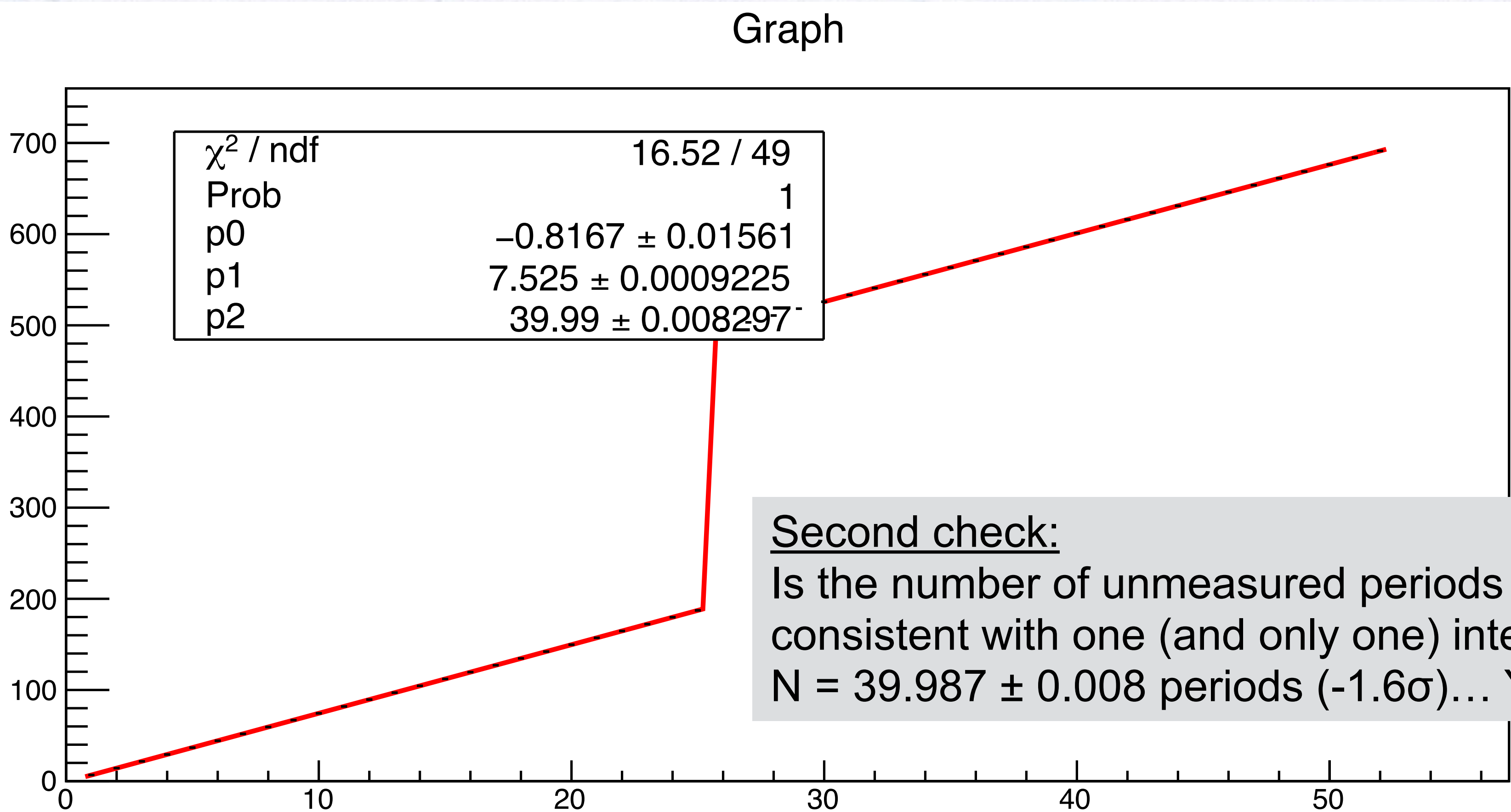


<u>First check:</u>
Do the two periods agree?
dT = 0.00121 ± 0.00101 (1.2σ)… Yes!

Additional ideas

It is possible to "leave" the pendulum swinging between to sets of measurements. This maximises the period over which you measure, without requiring your activity all the time…



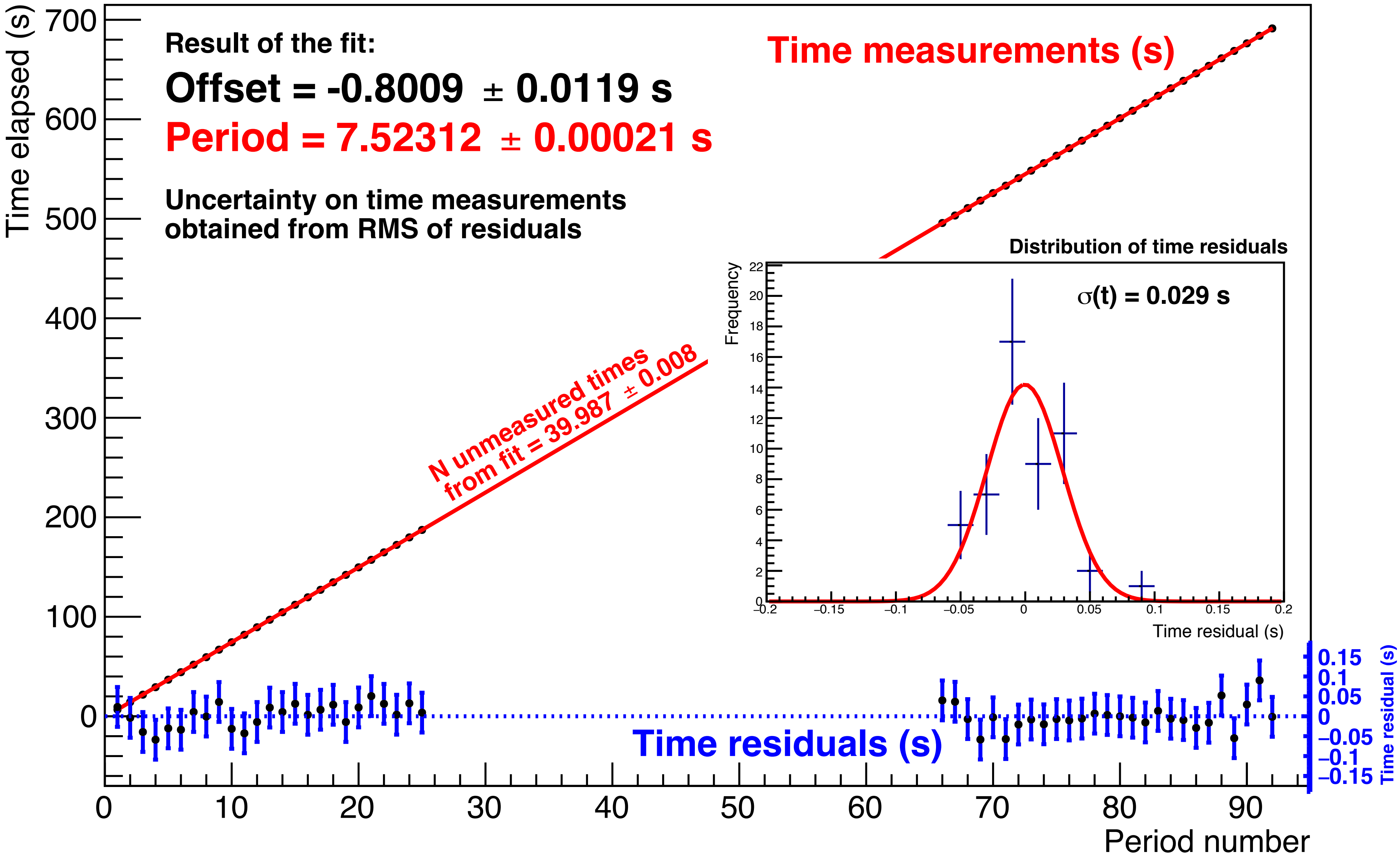
Second check:

Is the number of unmeasured periods consistent with one (and only one) integer?

N = 39.987 ± 0.008 periods (-1.6σ)… Yes!

Additional ideas

After checks, fit the entire time span to get "insanely great" precision.



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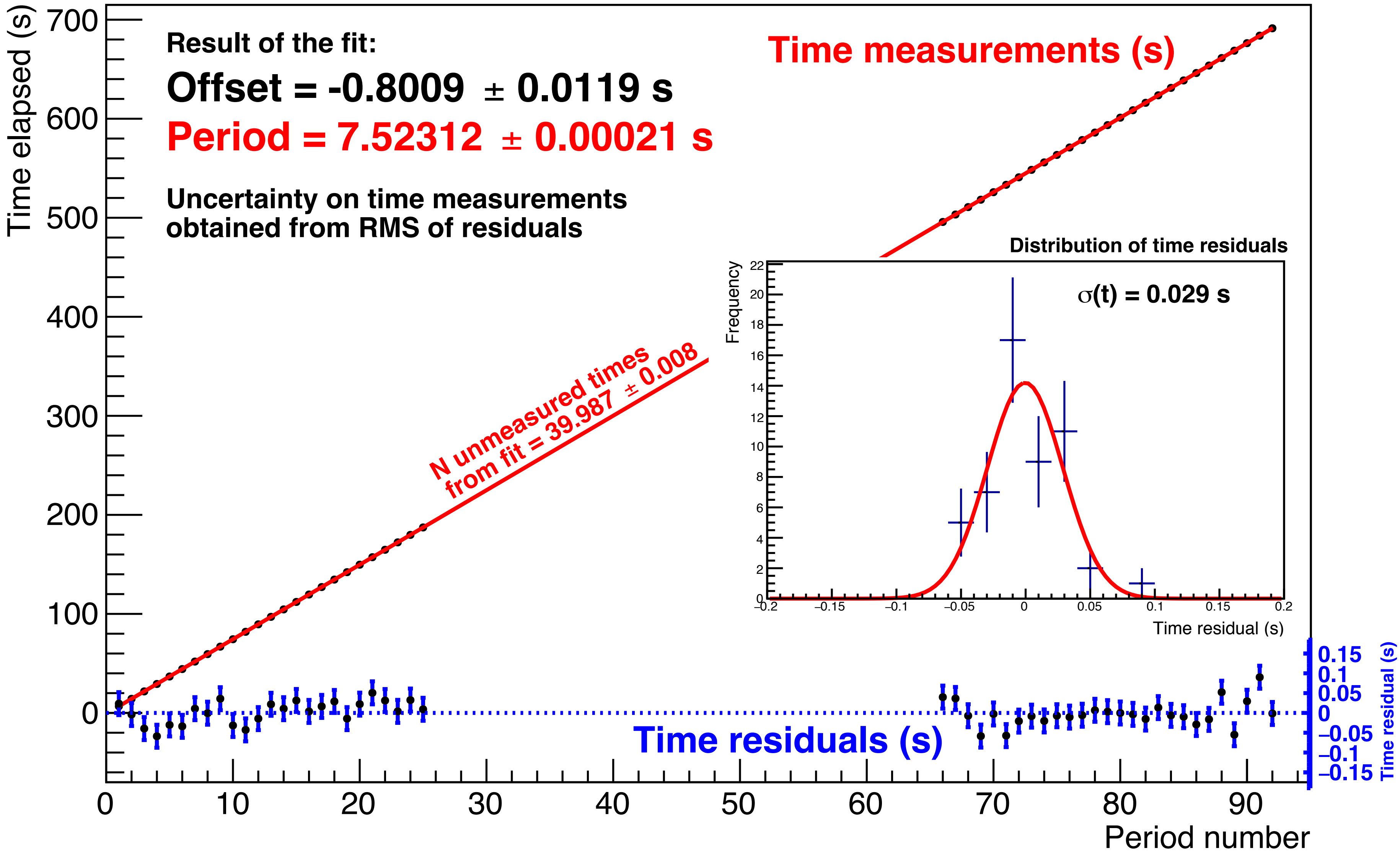
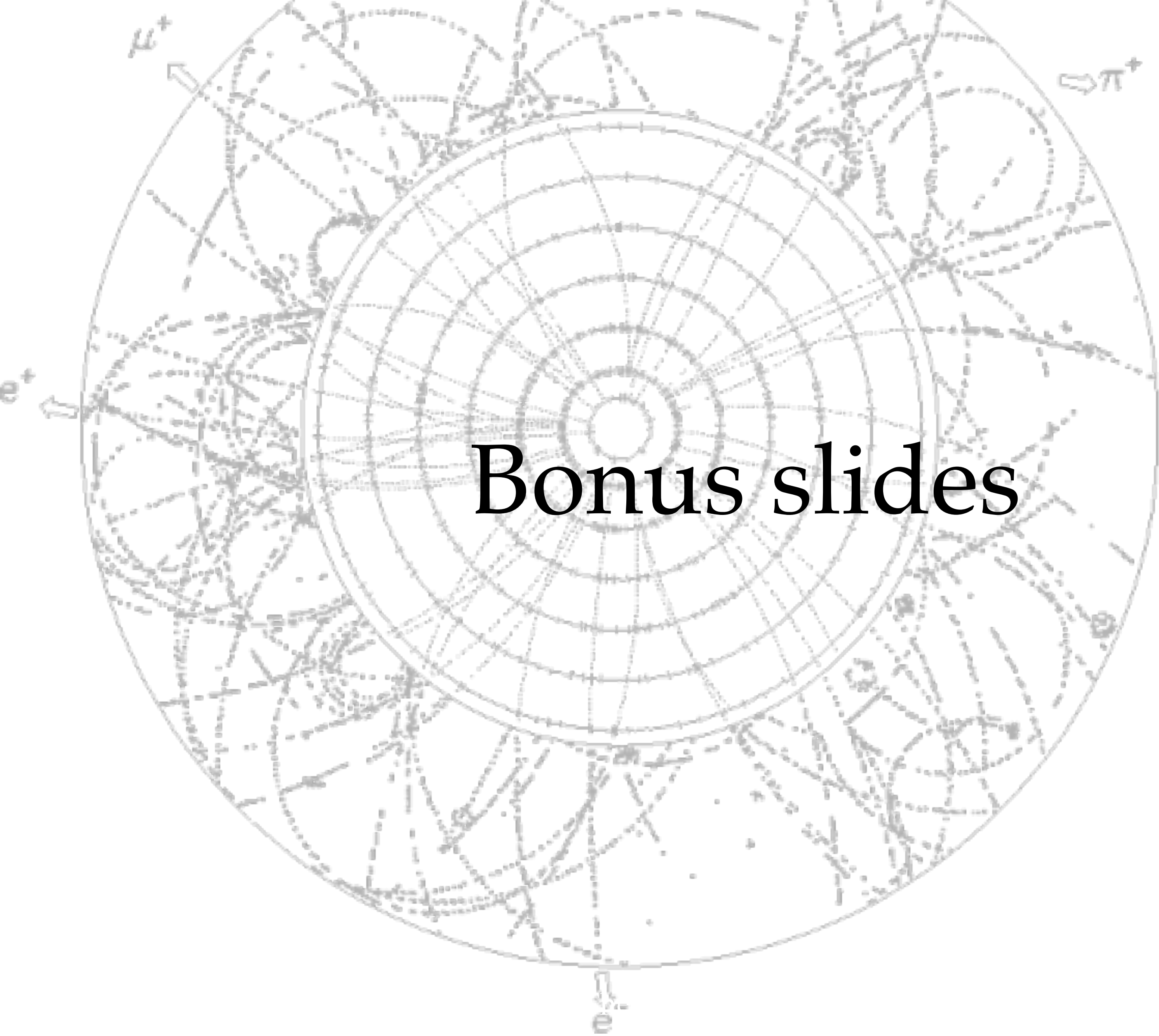
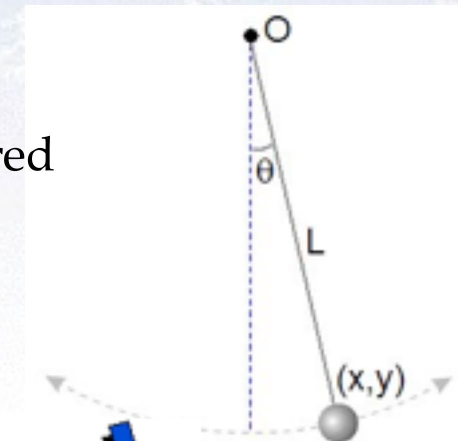


Bonus slides



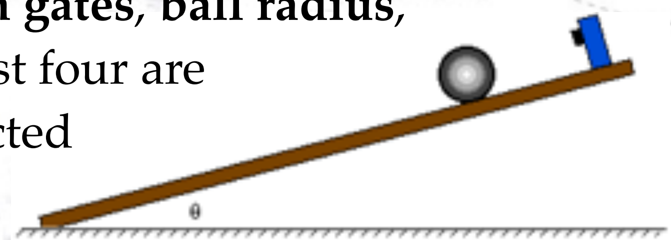
Applied Statistics - Project 1

The first project in Applied Statistics is to measure the gravitational acceleration, g, with the greatest possible precision, using two different experiments:



Simple pendulum:
Measure **length** and **period** of the pendulum. Length is measured with a measuring band and a laser, and time by your hand.

Ball rolling down incline:
Measure **incline angle**, **distance between gates**, **ball radius**, **rail distance** and **gate passage times**. First four are measured by hand, while timing is extracted from data files.

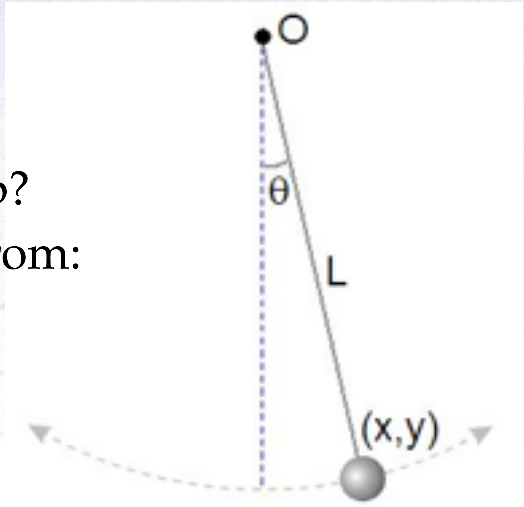


The project purpose is to learn how to **extract**, **minimise** and **propagate** errors. Before doing the experiments, please consider through error propagation, which of the measurements are going to be most challenging/limiting.

For more information, please look at the project 1 webpage.

Experiment objectives

In doing these experiments, you should make sure that you answer the following questions:

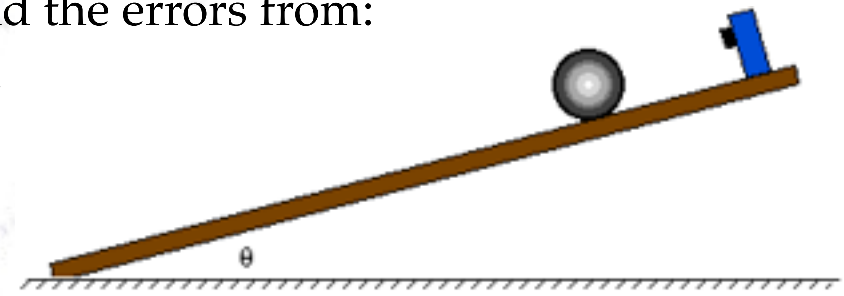


Pendulum:

- What is the timing precision of each person in the group?
- What is the gravitational acceleration g and the errors from:
  - Length of pendulum.
  - Period of pendulum.

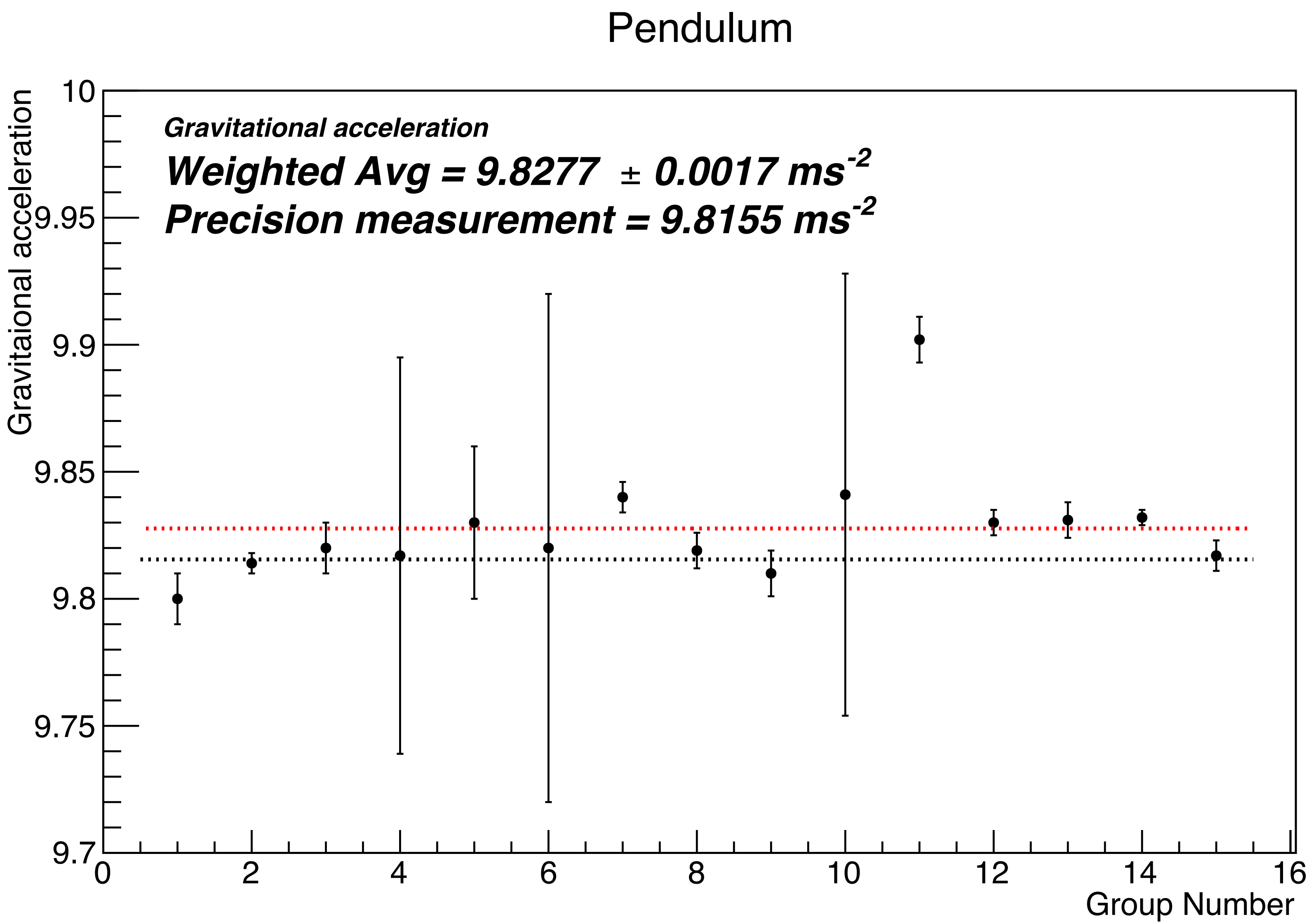
Ball on an incline:

- What is the angle of the rail, and what is the angle of the table?
- What is the gravitational acceleration g and the errors from:
  - Timing measurements in the five gates.
  - Distance between the gates.
  - Ball radius and rail distance.
  - Angle of rail.



Finally, perhaps you can eliminate some of your uncertainty by making $\theta = 90^o$?

Results 2015



Results 2015