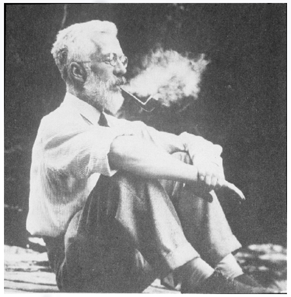
Applied Statistics

Project 1 experimental objectives





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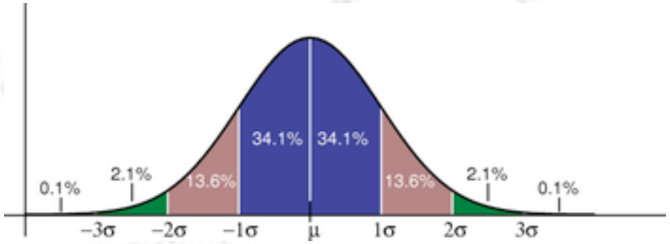
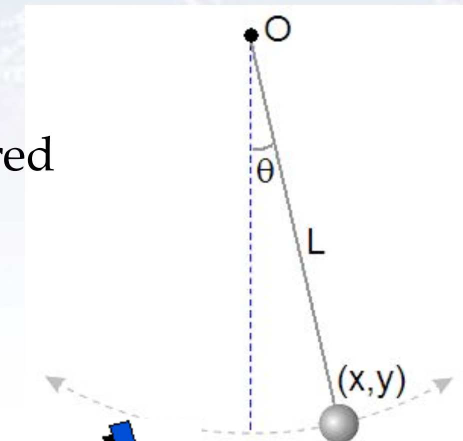


"Statistics is merely a quantisation of common sense"

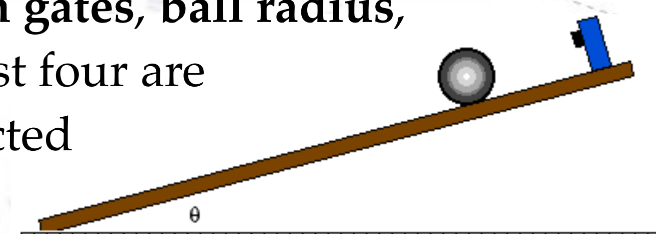
Applied Statistics - Project 1

The first project in Applied Statistics is to measure the gravitational acceleration, g, with the greatest possible precision, using two different experiments:

Simple pendulum:
Measure **length** and **period** of the pendulum. Length is measured with a measuring band and a laser, and time by your hand.



Ball rolling down incline:
Measure **incline angle**, **distance between gates**, **ball radius**, **rail distance** and **gate passage times**. First four are measured by hand, while timing is extracted from data files.

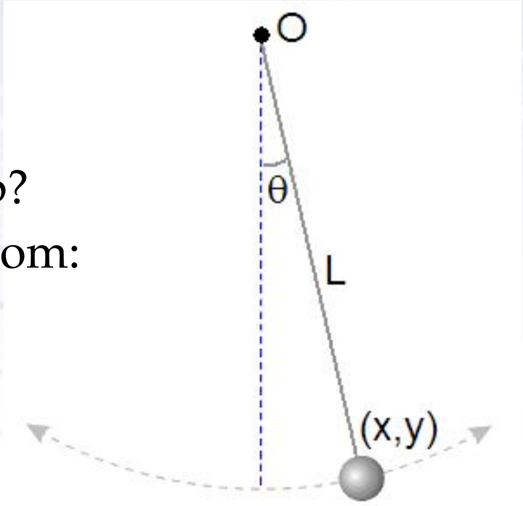


The project purpose is to learn how to **extract**, **minimise** and **propagate** errors. Before doing the experiments, please consider through error propagation, which of the measurements are going to be most challenging/limiting.

For more information, please look at the project 1 webpage.

Experiment objectives

In doing these experiments, you should make sure that you answer the following questions:

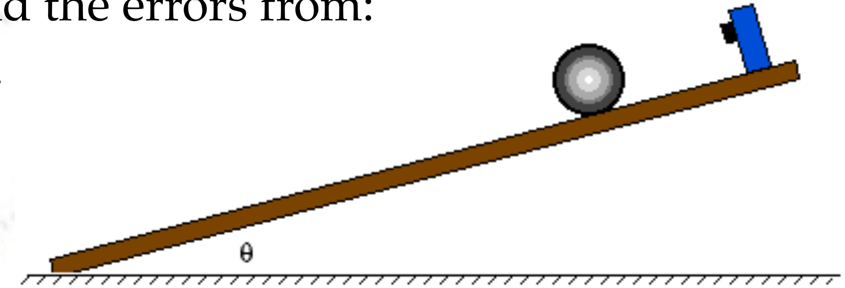


Pendulum:

- What is the timing precision of each person in the group?
- What is the gravitational acceleration g and the errors from:
  - Length of pendulum.
  - Period of pendulum.

Ball on an incline:

- What is the angle of the rail θ, and what is the angle of the table, Δθ?
- What is the gravitational acceleration g and the errors from:
  - Timing measurements in the five gates.
  - Distance between the gates.
  - Ball radius and rail distance.
  - Angle of rail.

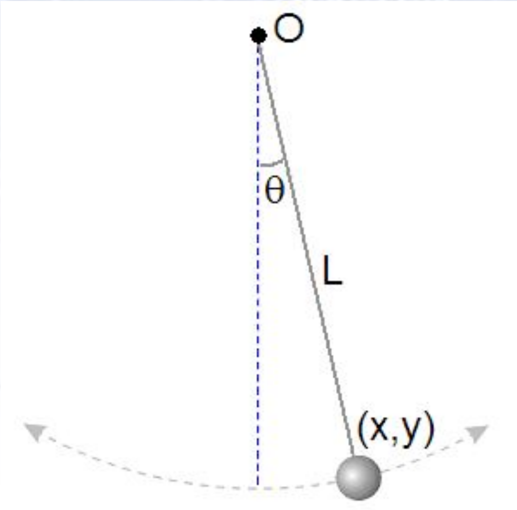


Finally, perhaps you can eliminate some of your uncertainty by making $\theta = 90^o$?

Experiment formulae

The pendulum formula is well known:

$$g = L\left(\frac{2\pi}{T}\right)^2$$

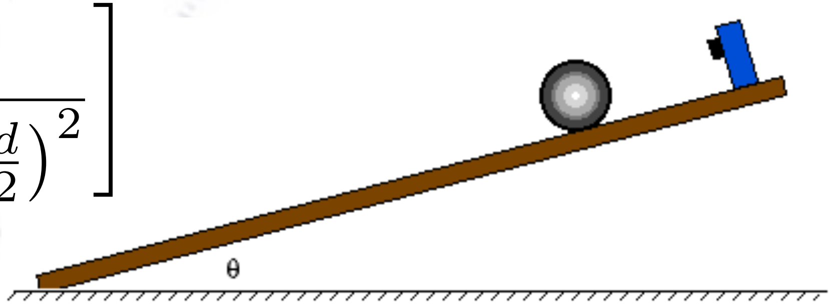


The resulting error formula is easy:

$$\sigma_g^2 = \left(\frac{2\pi}{T}\right)^4 \sigma_L^2 + \left(-2L\frac{(2\pi)^2}{T^3}\right)^2 \sigma_T^2$$

For the ball on incline, the formula is a bit more involved:

$$g = \frac{a}{\sin(\theta \pm \Delta\theta)}\left[1 + \frac{2}{5}\frac{R^2}{R^2 - \left(\frac{d}{2}\right)^2}\right]$$



The resulting error formula is in this case not very nice, but certainly doable.
However, this is clearly one of the cases, when the numerical solution is attractive!

Measurement situation

There are four possible situations in experimental measurements of a quantity:

One measurement, no error:

X = 3.14

Situation: You are f***ed!
You have no clue about uncertainty, and you can not obtain it!

Several measurements, no errors:

X1 = 3.14
X2 = 3.21
X3 = …

Situation: You are OK
You can combine the measurements, and from RMS get error on mean.

One measurement, with error:

X = 3.14 ± 0.13

Situation: You are OK
You have a number with error, which you can continue with.

Several measurements, with errors:

X1 = 3.14 ± 0.13
X2 = 3.21 ± 0.09
X3 = …

Situation: You are on top of things!
You can both combine to a weighted average and check with a chi-square.

Combining measurements

Given repeated measurements (by individual group members) of several quantities, that can be combined, what is the best way forward?

Combine at the end of analysis:

<u>Measurements:</u>
L1 = 3.543 ± 0.002 m
T1 = 3.942 ± 0.002 s
⇒ g1 = 9.821 ± 0.005 m/s^2

L2 = 3.545 ± 0.003 m
T2 = 3.940 ± 0.003 s
⇒ g2 = 9.827 ± 0.007 m/s^2

L3 = 3.523 ± 0.002 m
T3 = 3.944 ± 0.003 s
⇒ g3 = 9.771 ± 0.006 m/s^2

<u>Combination:</u>
g = 9.806 ± 0.004 m/s^2
Chi2 = 28.3, Ndof = 2
Prob(Chi2,Ndof) = 7.5 x 10^{-7}

Combine each quantity first:

<u>Measurements:</u>
L1 = 3.543 ± 0.002 m
L2 = 3.545 ± 0.003 m
L3 = 3.523 ± 0.002 m
⇒ L = 3.537 ± 0.002 m

Chi2 = 30.8, Ndof = 2
Prob(Chi2,Ndof) = 2.1 x 10^{-7}

T1 = 3.942 ± 0.002 s
T2 = 3.940 ± 0.003 s
T3 = 3.944 ± 0.003 s
⇒ T = 3.942 ± 0.002 s

Chi2 = 1.3, Ndof = 2
Prob(Chi2,Ndof) = 0.52

<u>Combination:</u>
g = 9.806 ± 0.004 m/s^2