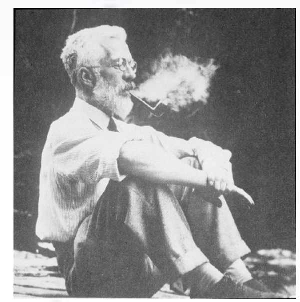
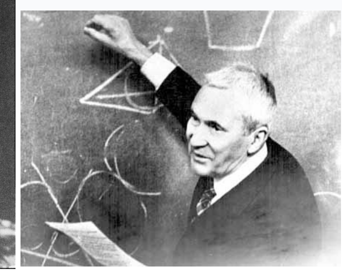
Applied Statistics

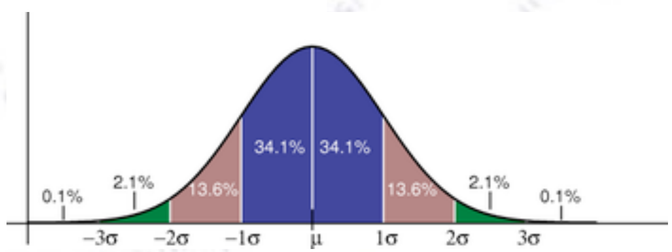
Supplementary on the ChiSquare





Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"

Defining the Chi-Square

Problem Statement: Given N data points (x,y), adjust the parameter(s) θ of a model, such that it fits data best.

The best way to do this, given uncertainties σ_i on y_i is by minimising:

$$\chi^2(\theta) = \sum_i^N \frac{(y_i - f(x_i, \theta))^2}{\sigma_i^2}$$

The power of this method is hard to overstate!

Not only does it provide a simple, elegant and unique way of fitting data, but more importantly it provides a **goodness-of-fit measure**.

This is the Chi-Square test!

Example: Weighted mean & Chi2

Example data (from pendulum experiment) could be (in mm):

dhook = [[17.8, 0.5], [18.1, 0.3], [17.7, 0.5], [17.7, 0.2]]

An example of output from the above data is (many digits for check only):

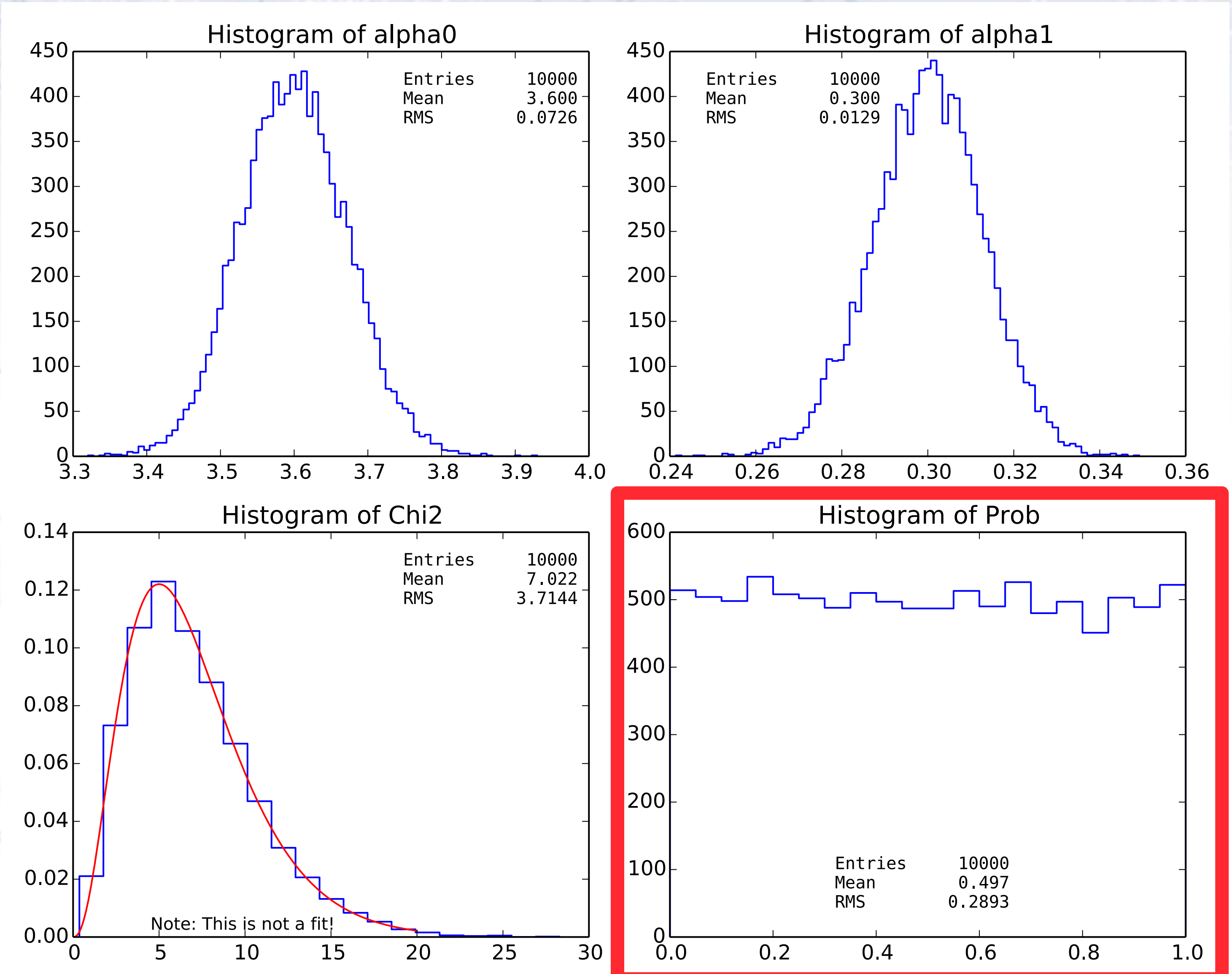
Mean	= 17.8098 mm
Error on mean	= 0.15057 mm
ChiSquare	= 1.28574
Ndof	= 3
Probability	= 0.7325213

NOTE: This seems a very nice (and precise) result, and it may very well be.

BUT, it might also be, that we all four estimated it from the same photo or similarly, which could be biased by an angled view. Then we would be fooling ourselves.
We will discuss such "Systematic Uncertainties" more on Friday.

Why is p(Chi2, Ndof) flat?

A lot (all?) wondered why the probability distribution (should) end up flat!



Bonus slides

Chi-Square probability calculation

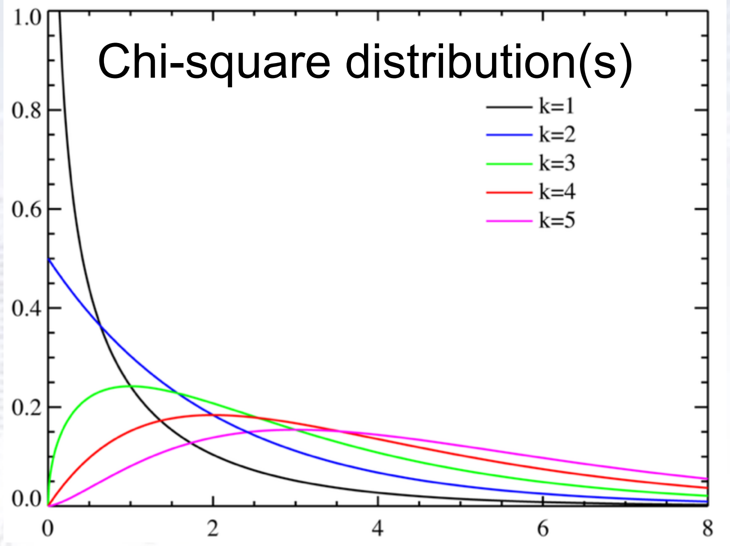
Given a **Chi-square value** and a **number of degrees of freedom** (Ndof), one can obtain a **"goodness-of-fit"**.

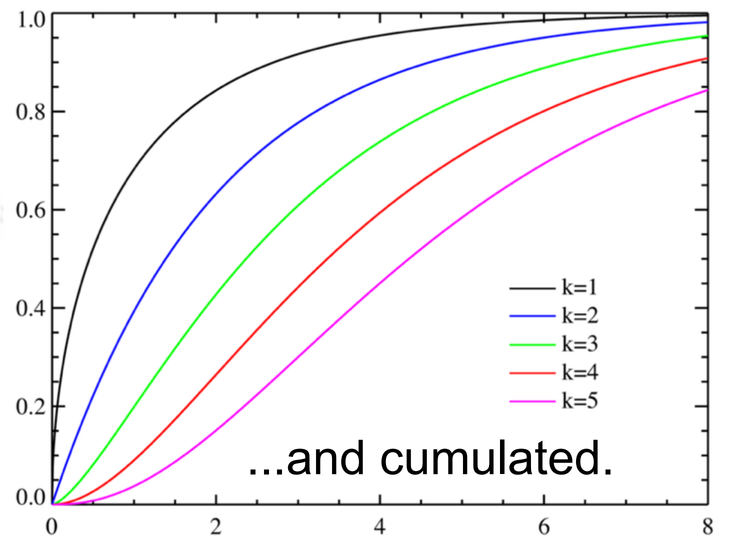
It is known, what Chi-square values to expect given the Ndof. One can therefore compare to this (Chi-square) distribution, and see...

what is the probability of getting this Chi-square value or something worse!

<u>Example:</u>

A fit gave the Chi-square 7.1 with 5 dof. The chance of getting this Chi-square or worse is... (reading the pink bottom curve (Ndof = k = 5) at 7.1)...





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<u>Example:</u>
A fit gave the Chi-square 7.1 with 5 dof. The chance of getting this Chi-square or worse is... (reading the pink bottom curve (Ndof = k = 5) at 7.1)... 1 - 0.78 = **22%**