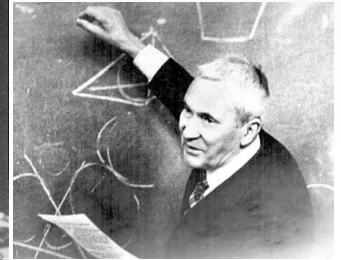
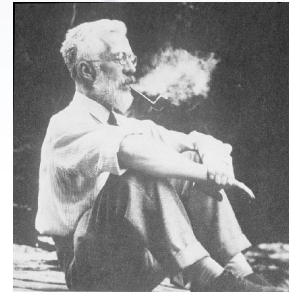
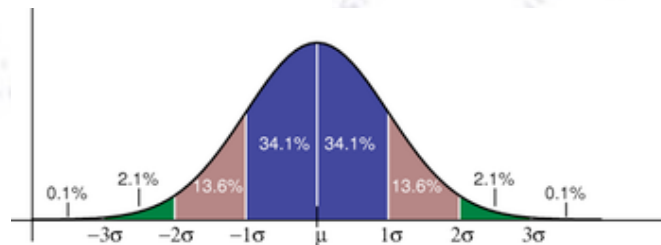


Applied Statistics

Project objectives and evaluation points



Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"

Project objective

The project in Applied Statistics is to **measure the gravitational acceleration,**

g

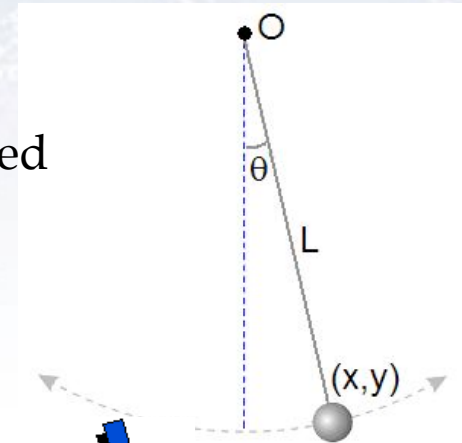
**with the greatest possible
correct precision,
using two different experiments:**

Applied Statistics - Project

The project in Applied Statistics is to **measure the gravitational acceleration, g , with the greatest possible correct precision**, using two different experiments:

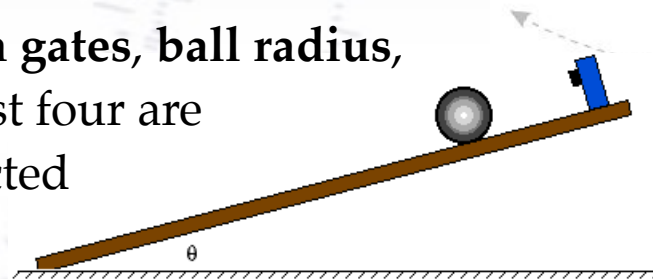
Simple pendulum:

Measure **length** and **period** of the pendulum. Length is measured with a measuring band and a laser, and time by your hand.



Ball rolling down incline:

Measure **incline angle**, **distance between gates**, **ball radius**, **rail distance** and **gate passage times**. First four are measured by hand, while timing is extracted from data files.



The project purpose is to learn how to **extract**, **minimise** and **propagate** errors. Before doing the experiments, please consider through error propagation, which of the measurements are going to be most challenging/limiting.

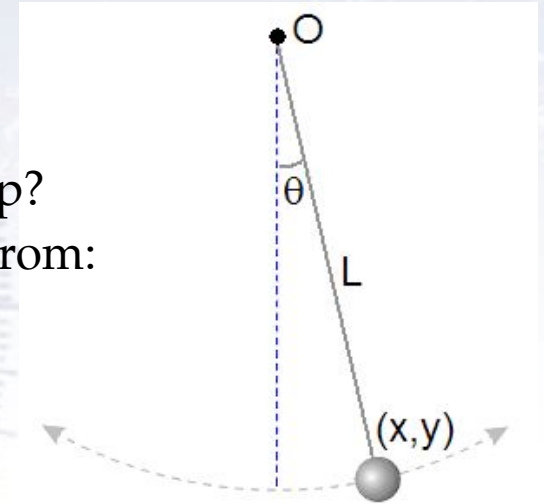
For more information, please look at the [project webpage](#).

Experiment objectives

In doing these experiments, you should make sure that you answer the following questions:

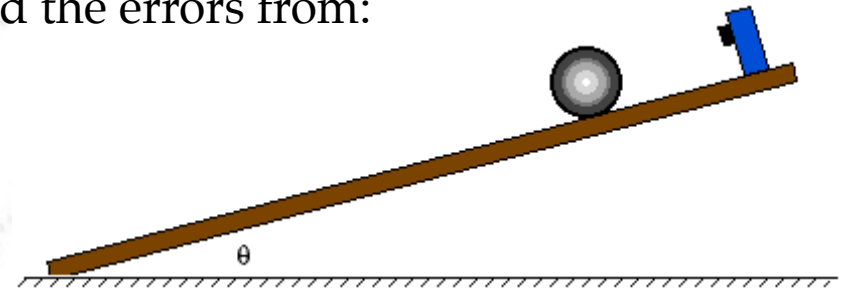
Pendulum:

- What is the timing precision of each person in the group?
- What is the gravitational acceleration g and the errors from:
 - ♦ Length of pendulum.
 - ♦ Period of pendulum.



Ball on an incline:

- What is the angle of the rail θ , and what is the angle of the table, $\Delta\theta$?
- What is the gravitational acceleration g and the errors from:
 - ♦ Timing measurements in the five gates.
 - ♦ Distance between the gates.
 - ♦ Ball radius and rail distance.
 - ♦ Angle(s) of rail.



Finally, perhaps you can eliminate some of your uncertainty by making $\theta = 90^\circ$?

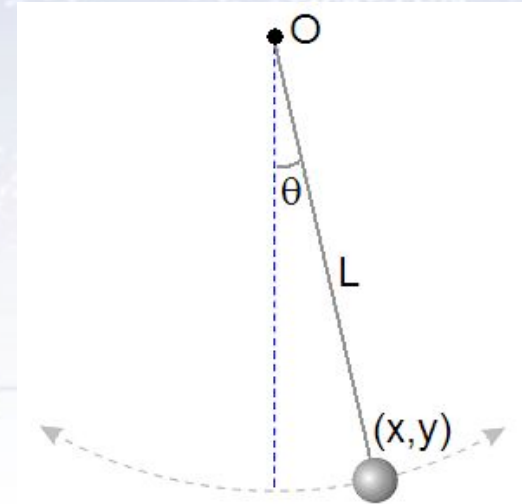
Experiment formulae

The pendulum formula is well known:

$$g = L \left(\frac{2\pi}{T} \right)^2$$

The resulting error formula is easy:

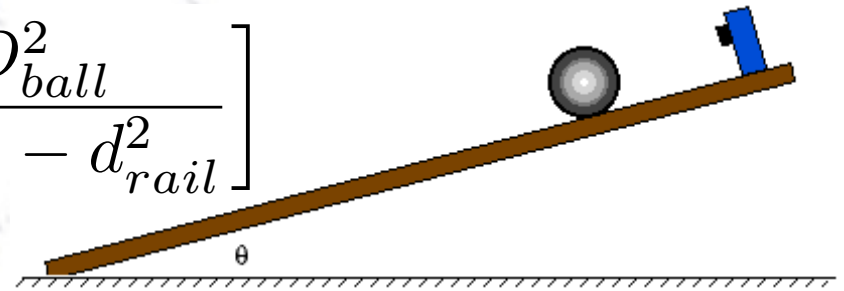
$$\sigma_g^2 = \left(\frac{2\pi}{T} \right)^4 \sigma_L^2 + \left(-2L \frac{(2\pi)^2}{T^3} \right)^2 \sigma_T^2$$



For the ball on incline, the formula is a bit more involved:

$$g = \frac{a}{\sin(\theta \pm \Delta\theta)} \left[1 + \frac{2}{5} \frac{D_{ball}^2}{D_{ball}^2 - d_{rail}^2} \right]$$

The resulting error formula is in this case not that nice, but certainly doable.



This is one of the cases, when the numerical solution is a good cross check!

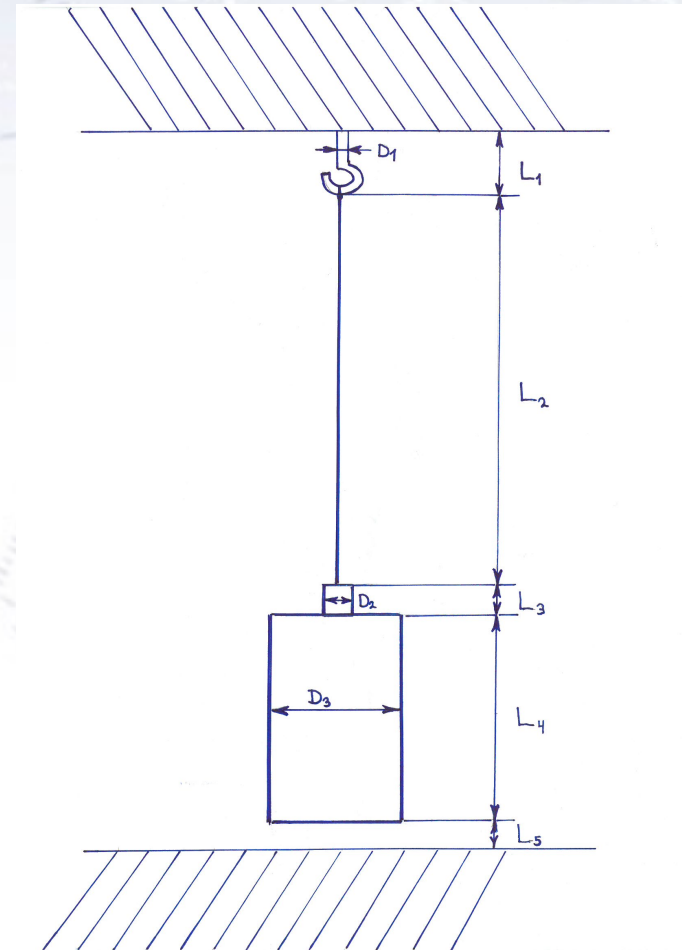
What to measure?

What should you have measured in order to have everything needed for measuring g ?

$$g = L \left(\frac{2\pi}{T} \right)^2$$

The answer is clear from the formula, but each measurement consists of several measurements!

It is worthwhile to make a good drawing ahead of doing the measurements.



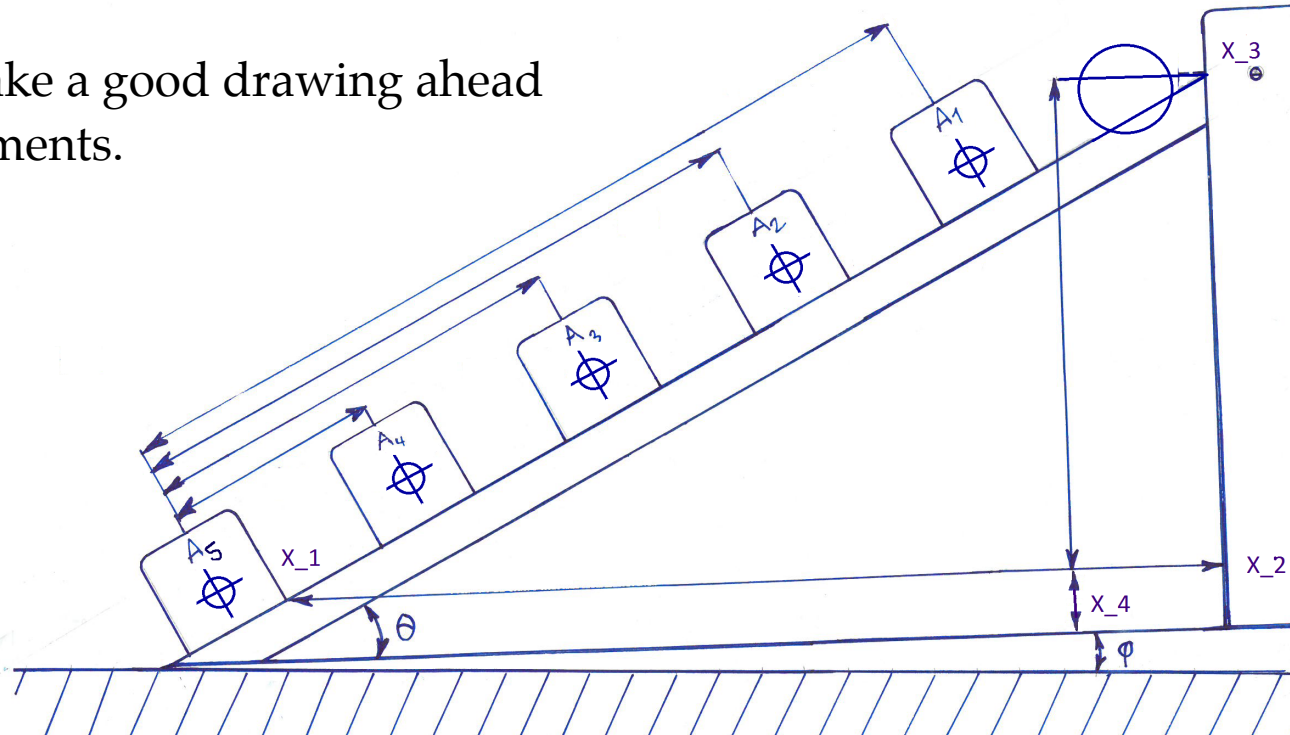
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The answer is clear from the formula, but each measurement consists of several measurements!

It is worthwhile to make a good drawing ahead of doing the measurements.



Measurement situation

There are four possible situations in experimental measurements of a quantity:

One measurement, no error:

$$X = 3.14$$

Situation: You are f***ed!

You have no clue about uncertainty, and you can not obtain it!

Several measurements, no errors:

$$X_1 = 3.14$$

$$X_2 = 3.21$$

$$X_3 = \dots$$

Situation: You are OK

You can combine the measurements, and from RMS get error on mean.

One measurement, with error:

$$X = 3.14 \pm 0.13$$

Situation: You are OK

You have a number with error, which you can continue with.

Several measurements, with errors:

$$X_1 = 3.14 \pm 0.13$$

$$X_2 = 3.21 \pm 0.09$$

$$X_3 = \dots$$

Situation: You are on top of things!

You can both combine to a weighted, average and check with a chi-square.

Combining measurements

Given repeated measurements (by individual group members) of several quantities, that can be combined, what is the best way forward?

Combine at the end of analysis:

Measurements:

$$L1 = 3.543 \pm 0.002 \text{ m}$$

$$T1 = 3.942 \pm 0.002 \text{ s}$$

$$\Rightarrow \mathbf{g1 = 9.821 \pm 0.005 \text{ m/s}^2}$$

$$L2 = 3.545 \pm 0.003 \text{ m}$$

$$T2 = 3.940 \pm 0.003 \text{ s}$$

$$\Rightarrow \mathbf{g2 = 9.827 \pm 0.007 \text{ m/s}^2}$$

$$L3 = 3.523 \pm 0.002 \text{ m}$$

$$T3 = 3.944 \pm 0.003 \text{ s}$$

$$\Rightarrow \mathbf{g3 = 9.771 \pm 0.006 \text{ m/s}^2}$$

Combination:

$$\mathbf{g = 9.806 \pm 0.004 \text{ m/s}^2}$$

$$\text{Chi2} = 28.3, \text{Ndof} = 2$$

$$\text{Prob}(\text{Chi2}, \text{Ndof}) = 7.5 \times 10^{-7}$$

Combine each quantity first:

Measurements:

$$L1 = 3.543 \pm 0.002 \text{ m}$$

$$L2 = 3.545 \pm 0.003 \text{ m}$$

$$L3 = 3.523 \pm 0.002 \text{ m}$$

$$\Rightarrow \mathbf{L = 3.537 \pm 0.002 \text{ m}}$$

$$\text{Chi2} = 30.8, \text{Ndof} = 2$$

$$\text{Prob}(\text{Chi2}, \text{Ndof}) = 2.1 \times 10^{-7}$$

$$T1 = 3.942 \pm 0.002 \text{ s}$$

$$T2 = 3.940 \pm 0.003 \text{ s}$$

$$T3 = 3.944 \pm 0.003 \text{ s}$$

$$\Rightarrow \mathbf{T = 3.942 \pm 0.002 \text{ s}}$$

$$\text{Chi2} = 1.3, \text{Ndof} = 2$$

$$\text{Prob}(\text{Chi2}, \text{Ndof}) = 0.52$$

Combination:

$$\mathbf{g = 9.806 \pm 0.004 \text{ m/s}^2}$$

NOTE: These two approaches leads to **different** results (but near, see appendix).



Project evaluation

Project evaluation

Pendulum:

- Did you measure $T \pm \sigma(T)$ correctly? Combine with Chi2 and comments?
- Did you measure $L \pm \sigma(L)$ correctly? Combine and check correctly?
- Did you provide the individual T and L precisions/uncertainties on g?
- Did you measure each team members timing precision and submit these?

Ball on incline:

- $T \pm \sigma(T)$
 - $L \pm \sigma(L)$
- } $\Rightarrow a \pm \sigma(a)$, with Chi2 and comments.
- $\theta, \Delta\theta$ obtained correctly and
 - d, R and errors propagated correctly?

Generally:

- **Correctly propagated uncertainties, showing individual contributions.**
- **Using Chi2 and its probability, whenever possible.**
- All necessary figures and tables there? 2-3 essential figures needed.
- Text enough to understand results? Clear and fitting captions?
- Comment on result (especially inconsistencies) and correct significant digits.

Collect results: Pendulum (T, L, g) and Ball on Incline (T, L, a, θ , $\Delta\theta$, d, R)

Project challenge

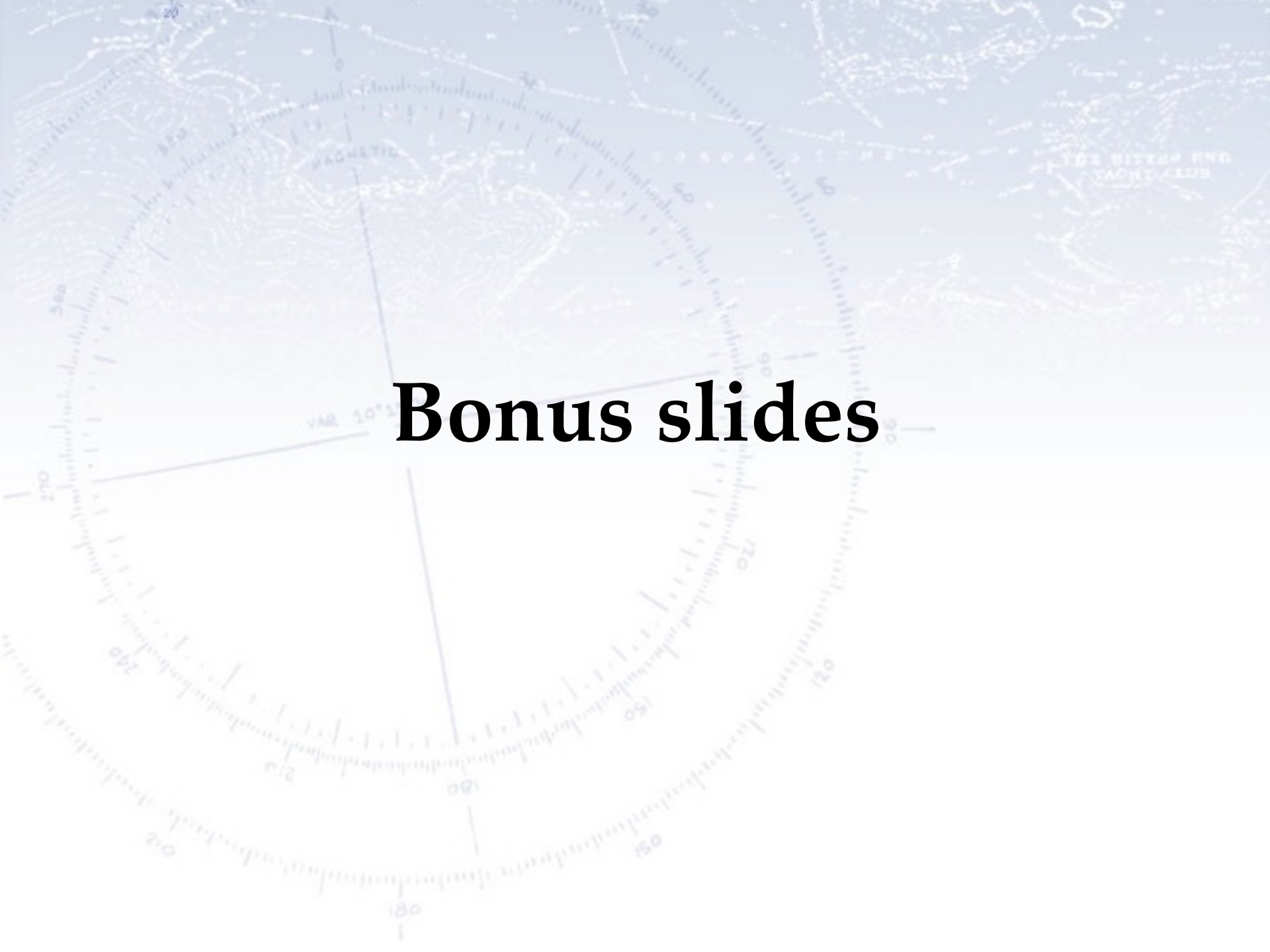
The project consist of experiments and data analysis, which well resembles those in real life.

There is TONS of experience to gather from these!!!

For this reason, we give 1-2 extra points to persons/ groups, who manage the following:

- Pendulum measurement better than $1/1000$ with full and correct data analysis and error propagation consistent with g .
- Ball on incline measurement better than $1/100$ with full and correct data analysis and error propagation consistent with g .

It is perfectly alright NOT to do this, and one is of course allowed to continue in person, and just submit a personal addition.



Bonus slides

Different equation versions

$$g = \frac{a}{\sin(\theta \pm \Delta\theta)} \left[1 + \frac{2}{5} \frac{R^2}{R^2 - \left(\frac{d}{2}\right)^2} \right]$$

$$g = \frac{a}{\sin(\theta \pm \Delta\theta)} \left[1 + \frac{2}{5} \frac{r_{ball}^2}{r_{ball}^2 - (d_{rail}/2)^2} \right]$$

$$g = \frac{a}{\sin(\theta \pm \Delta\theta)} \left[1 + \frac{2}{5} \frac{D_{ball}^2}{D_{ball}^2 - d_{rail}^2} \right]$$

Same data - different results

As the below example shows, combining the same data in different ways (either getting g first and then combining, or getting T and L first, and then one g), leads to different results, though very similar:

```
###simple example
import numpy as np
L1=10
L2=10.1
VarL1=0.005
VarL2=0.004
T1=5
T2=5.1
VarT1=0.005
VarT2=0.004
g1=(((L1/VarL1)+(L2/VarL2))/((1/VarL1)+(1/VarL2)))*(2*np.pi/(((T1/VarT1)+(T2/VarT2))/((1/VarT1)+(1/VarT2))))**2
print(g1)
g01 = L1*(2*np.pi/T1)**2
g02=L2*(2*np.pi / T2)**2
varg01=(2*np.pi / T1)**4*VarL1 + (2*L1*(2*np.pi)**2/T1**3)**2*VarT1
varg02=(2*np.pi / T2)**4*VarL2 + (2*L2*(2*np.pi)**2/T2**3)**2*VarT2
gcc = (((g01/varg01)+(g02/varg02))/((1/varg01)+(1/varg02)))

print()
print(gcc)
```