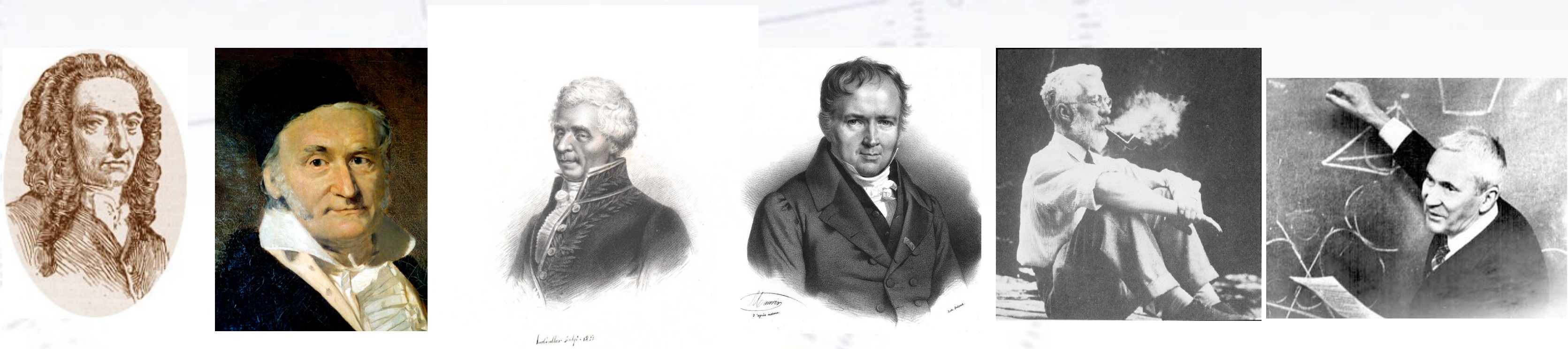
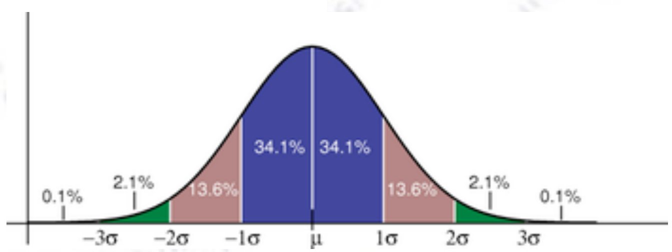
Applied Statistics

Correlations

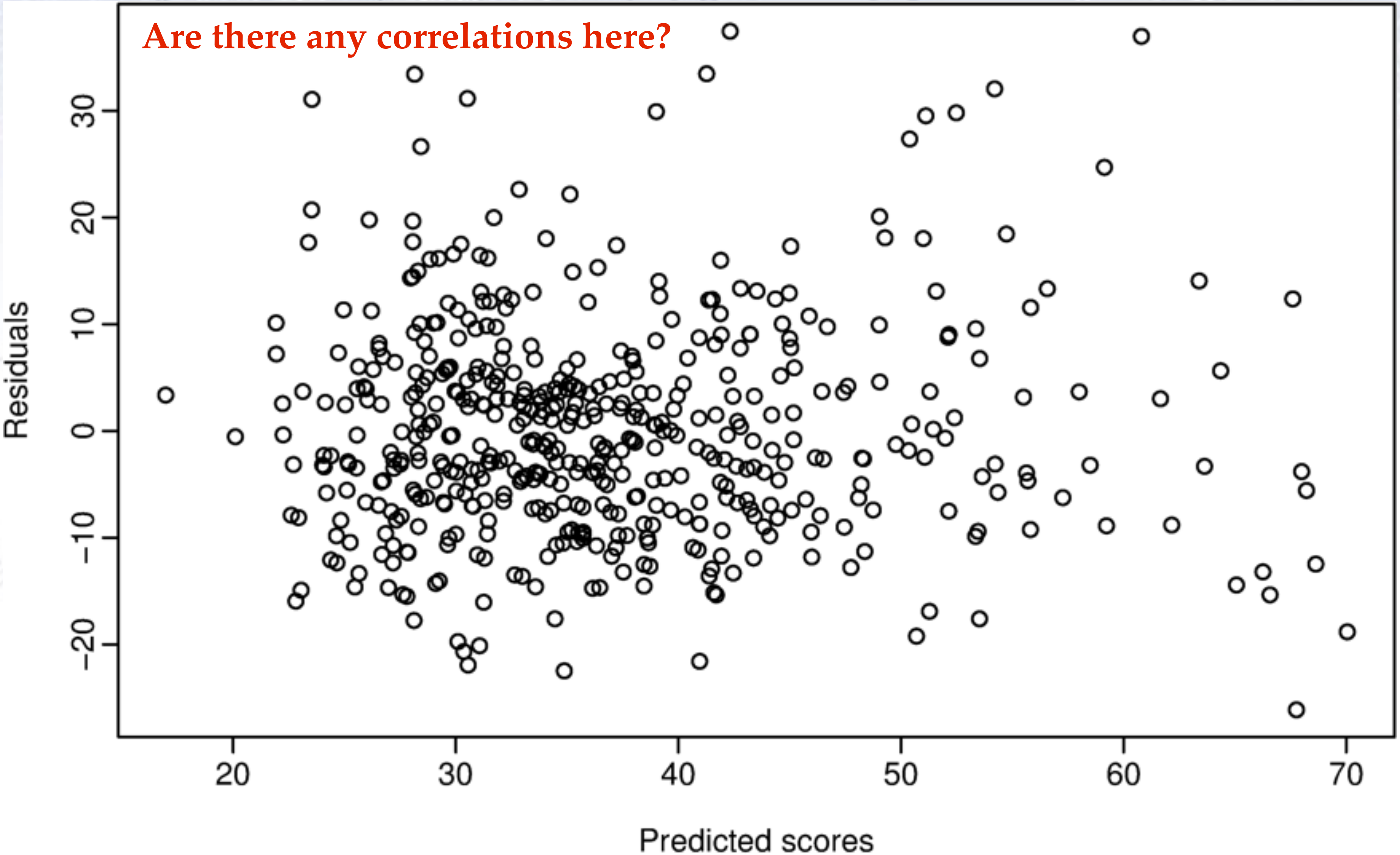


Troels C. Petersen (NBI)

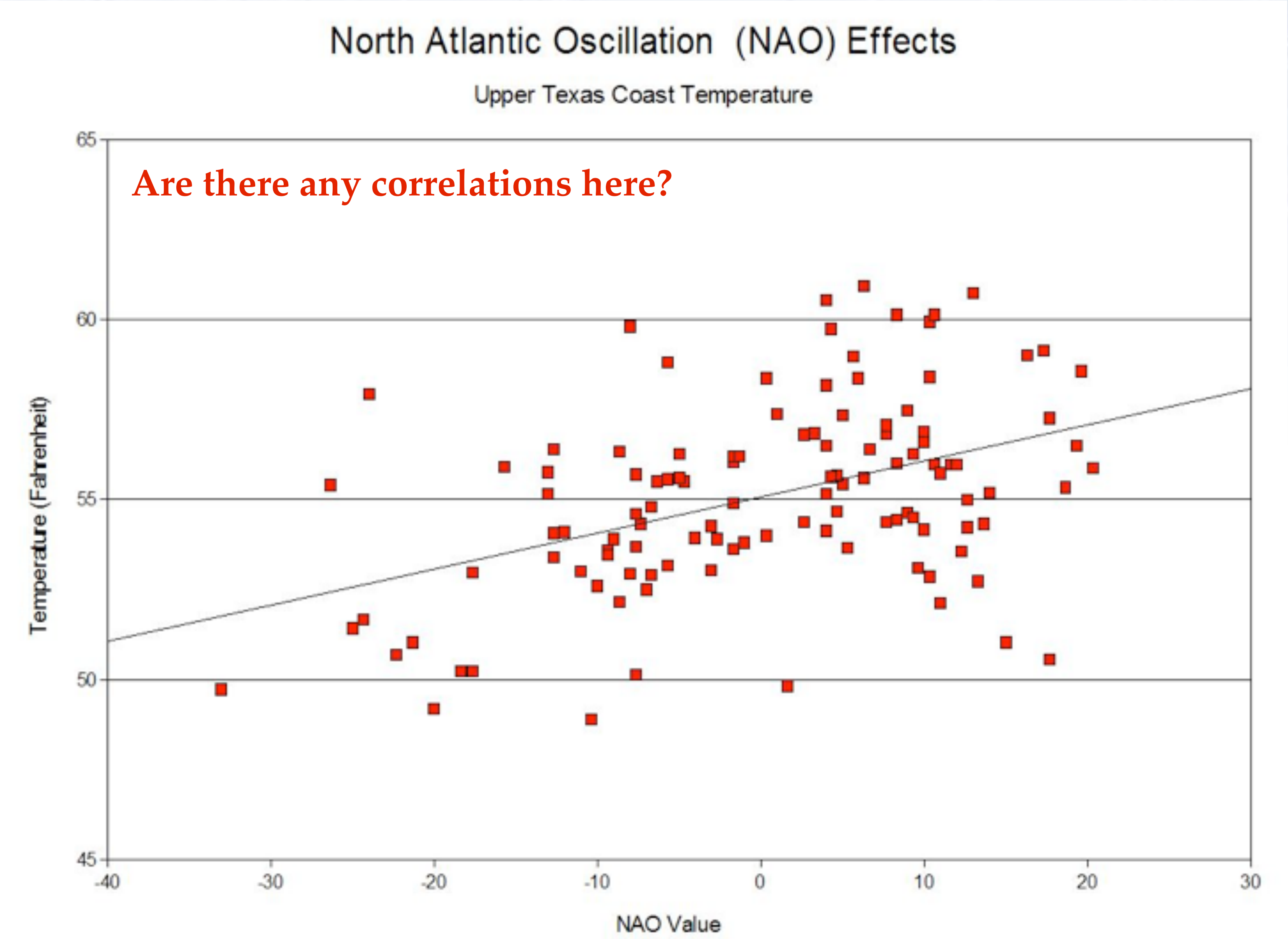


"Statistics is merely a quantisation of common sense"

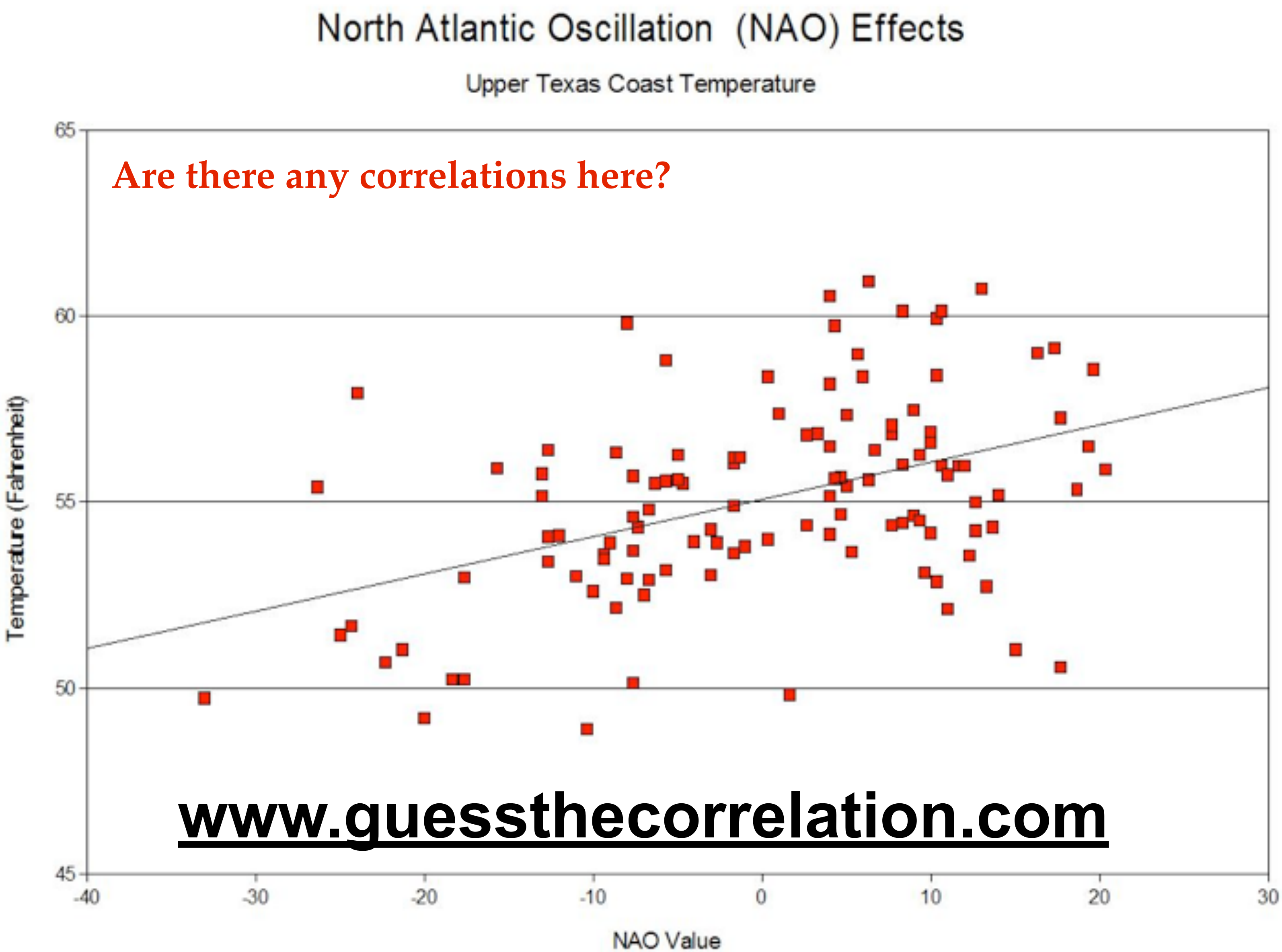
Correlation



Correlation



Correlation



www.guessthecorrelation.com

Correlation

Recall the definition of the Variance, V:

$$V = \sigma^2 = \frac{1}{N}\sum_i^n (x_i - \mu)^2 = E[(x-\mu)^2] = E[x^2] - \mu^2$$

Correlation

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Likewise, one defines the **Covariance, $\mathbf{V_{xy}}$:**

$$V_{xy} = \frac{1}{N}\sum_{i}^{n}(x_i - \mu_x)(y_i - \mu_y) = E[(x_i - \mu_x)(y_i - \mu_y)]$$

Correlation

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"Normalising" by the widths, gives the (linear) correlation coefficient:

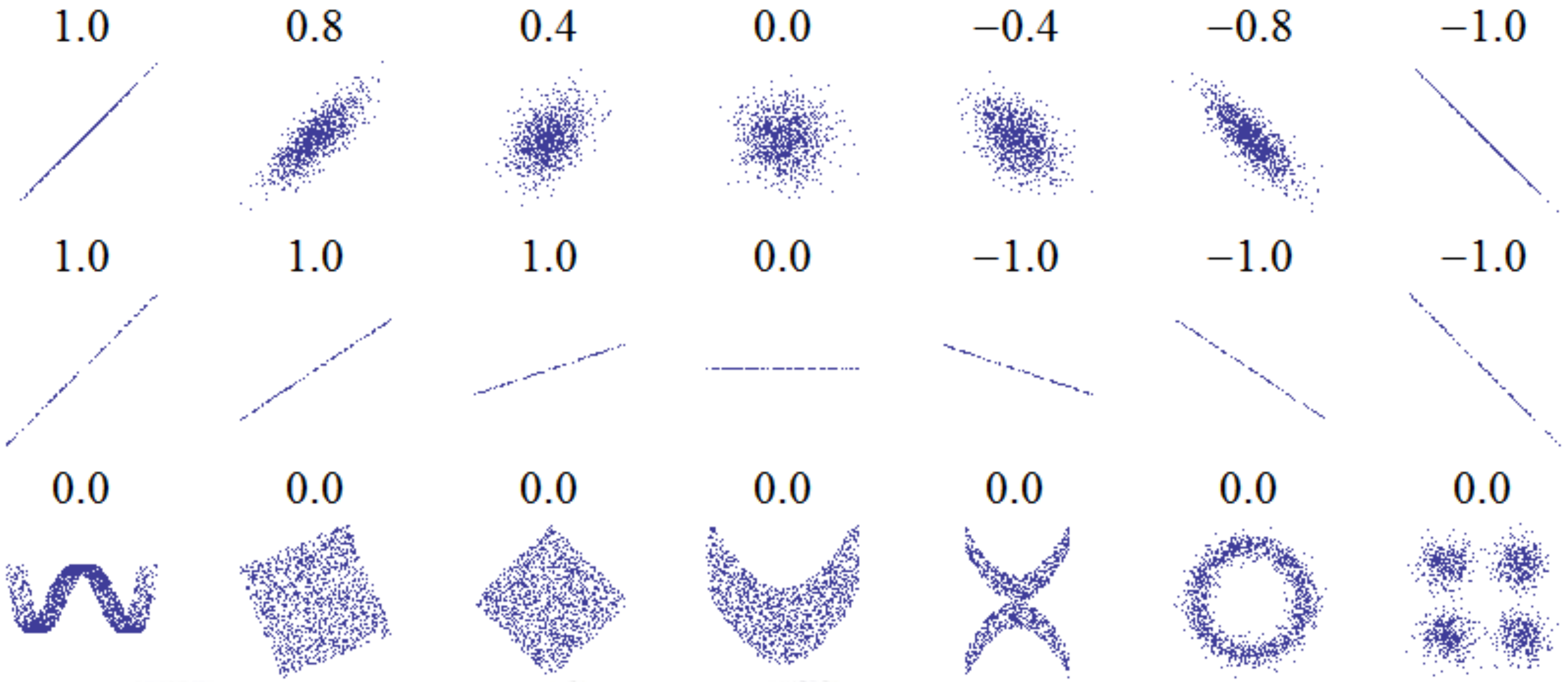
$$\rho_{xy} = \frac{V_{xy}}{\sigma_x \sigma_y}$$

$$-1 < \rho_{xy} < 1$$

$$\sigma(\rho) \simeq \sqrt{\frac{1}{n}(1-\rho^2)^2 + O(n^{-2})}$$

Correlation

Correlations in 2D are in the Gaussian case the "degree of ovalness"!



Note how ALL of the bottom distributions have $\varrho = 0$, despite obvious correlations!

Correlation

The correlation matrix V_{xy} explicitly looks as:

$$V_{xy} = \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 & \cdots & \sigma_{1N}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2N}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_N^2 & \sigma_{N2}^2 & \cdots & \sigma_{NN}^2 \end{bmatrix}$$

Very specifically, the calculations behind are:

$$V = \begin{bmatrix} \mathrm{E}[(X_1-\mu_1)(X_1-\mu_1)] & \mathrm{E}[(X_1-\mu_1)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_1-\mu_1)(X_n-\mu_n)] \\ \mathrm{E}[(X_2-\mu_2)(X_1-\mu_1)] & \mathrm{E}[(X_2-\mu_2)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_2-\mu_2)(X_n-\mu_n)] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{E}[(X_n-\mu_n)(X_1-\mu_1)] & \mathrm{E}[(X_n-\mu_n)(X_2-\mu_2)] & \cdots & \mathrm{E}[(X_n-\mu_n)(X_n-\mu_n)] \end{bmatrix}.$$

Correlation and Information

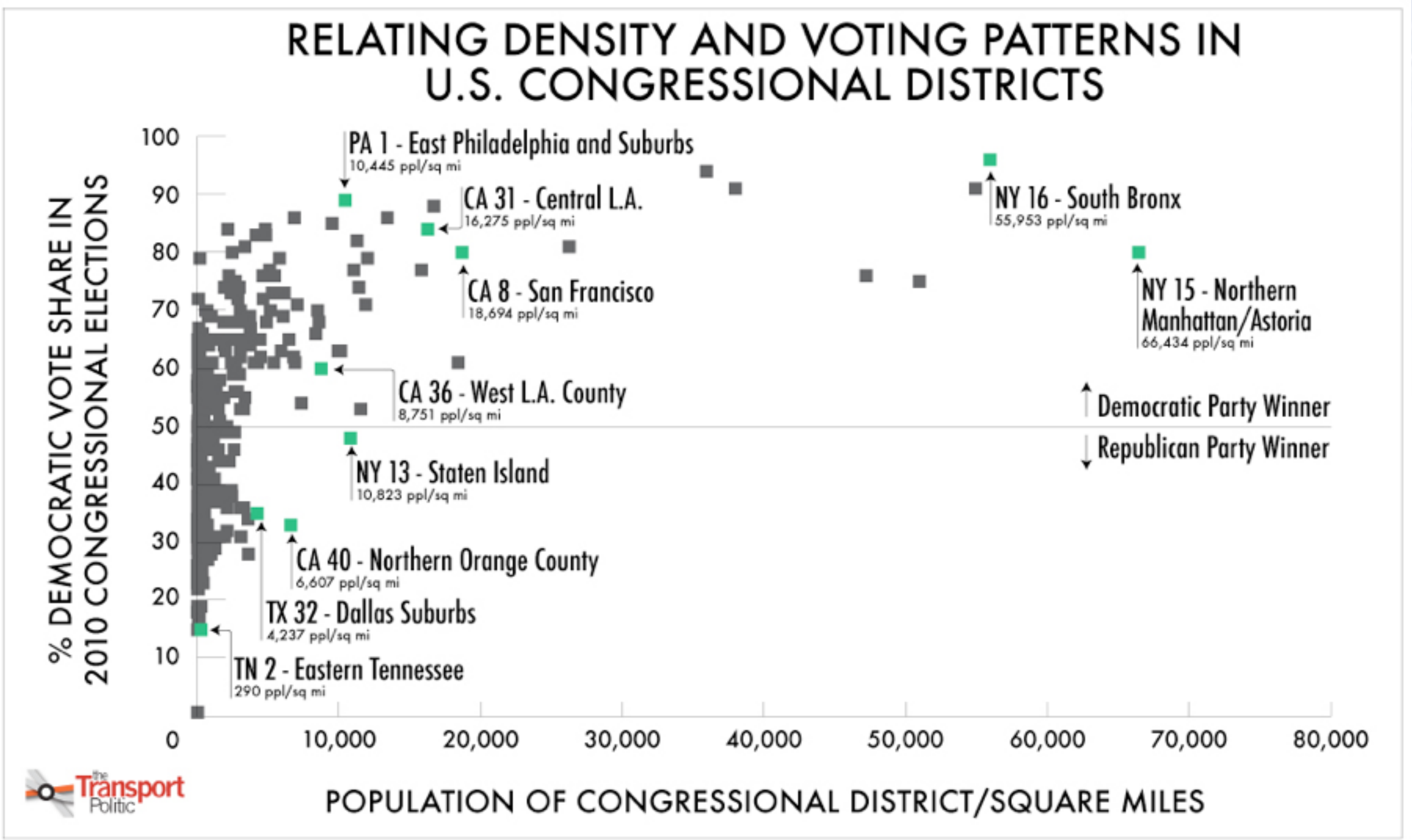
Correlations influence results in complex ways!

They need to be taken into account, for example in **Error Propagation!**

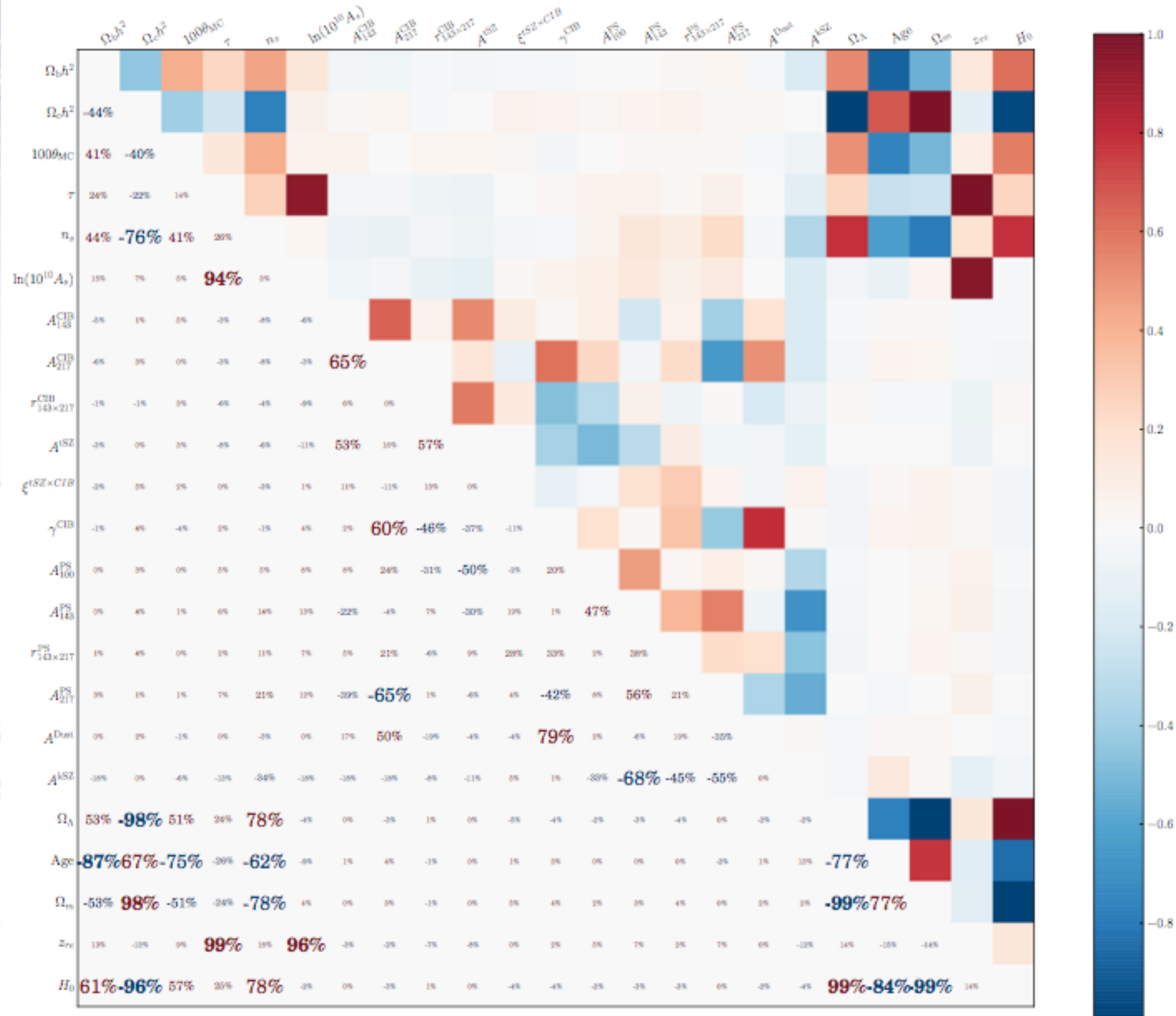
Correlations may contain a significant amount of information.

We will consider this more when we play with multivariate analysis.

Correlation example



Planck example



Rank correlations

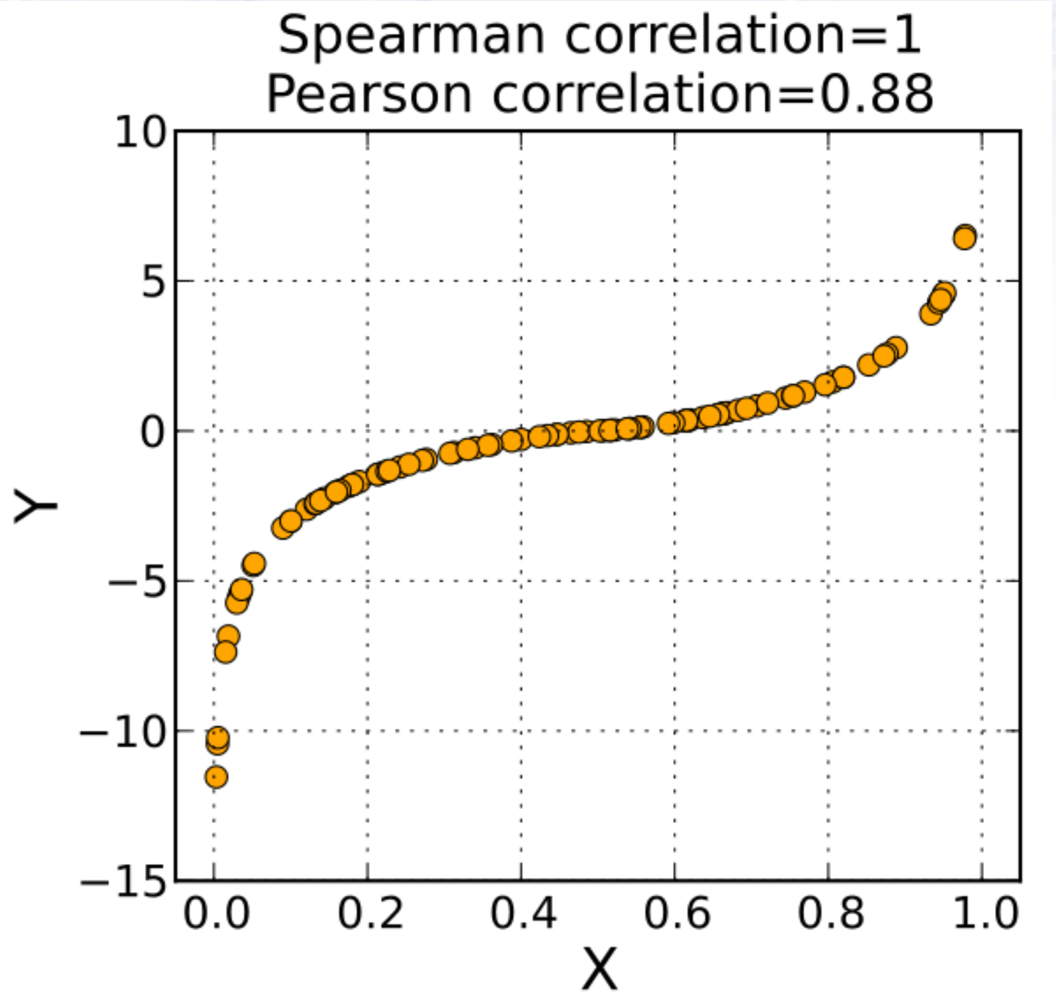
Sometimes, variables are perfectly correlated, just not linearly:

In this case the Pearson correlation is not the best measure.

Rank correlation compares the ranking between the two sets, and therefore gets a good measure of the correlation (see figure).

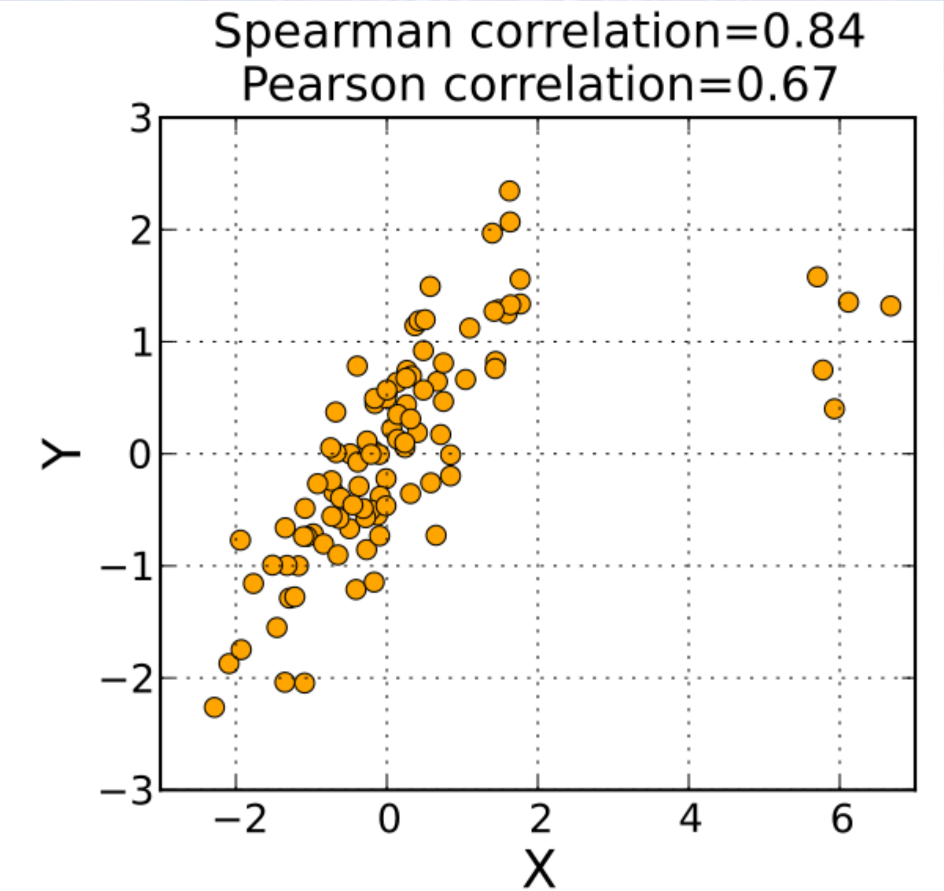
The two main cases of rank correlations are:

- Spearman's rho
- Kendall's tau



Rank correlations

An additional advantage is, that the rank correlation is less sensitive to outliers:



The two rank correlations are special cases of a more general rank correlation.
Typically, Spearman's rank correlation is used.

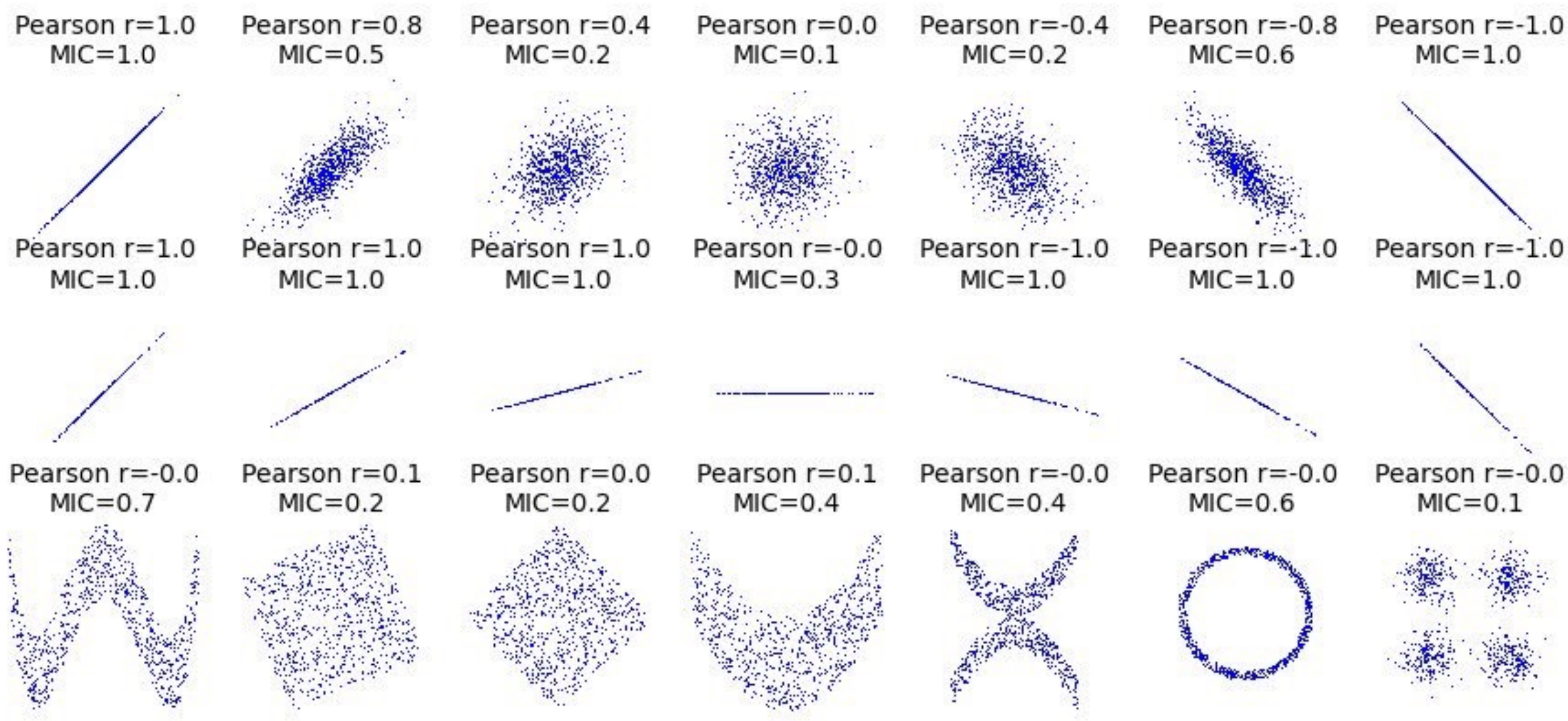
The definition is:

$$\rho = 1 - 6 \sum_i (r_i - s_i)^2 / (n^3 - n)$$

where r_i and s_i is the rank of the i'th element.

Non-linear correlations

Non-linear correlations (associations) are harder to measure, but possible. One measure is the Maximal Information Coefficient (MIC, outside course scope), defined in the reference below, and well explained on Wikipedia.

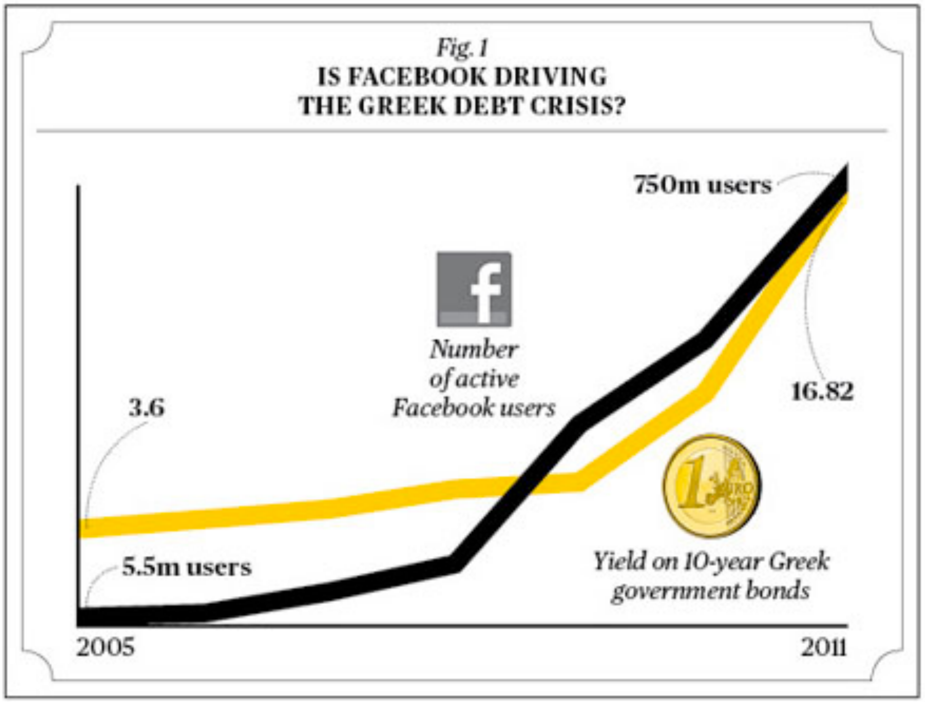


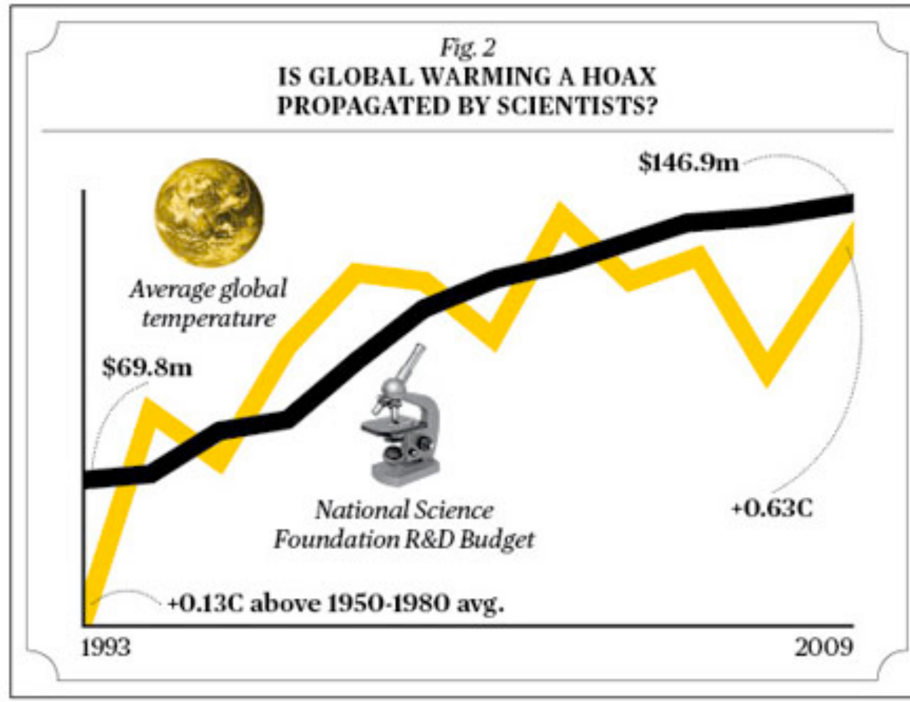
Original paper: "Detecting Novel Associations in Large Data Sets" (2011). Science 334 (6062): 1518–1524.

Correlation Vs. Causation

"Com hoc ergo propter hoc"

(with this, therefore because of this)





It is a common mistake to think that correlation proves causation…

Correlation Vs. Causation

"Com hoc ergo propter hoc"

(with this, therefore because of this)