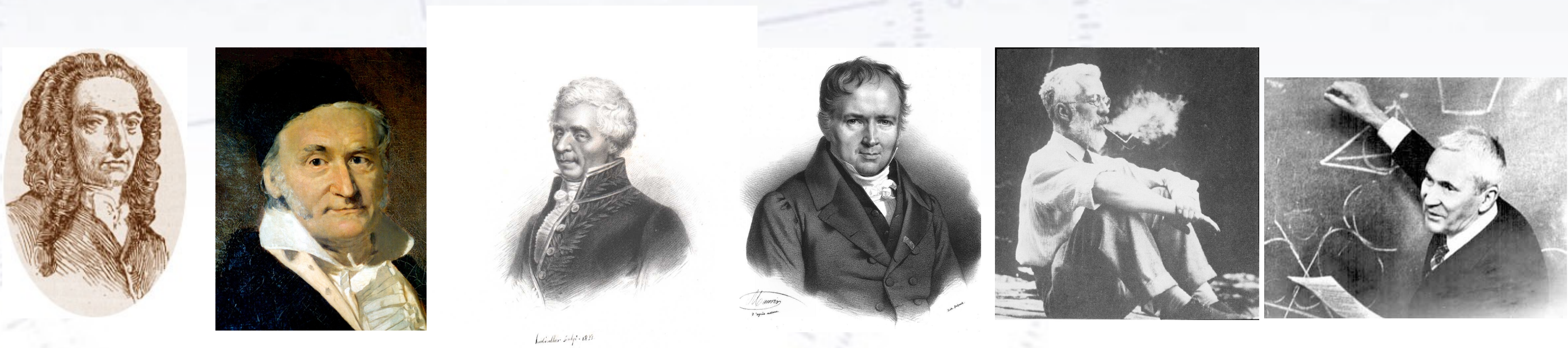
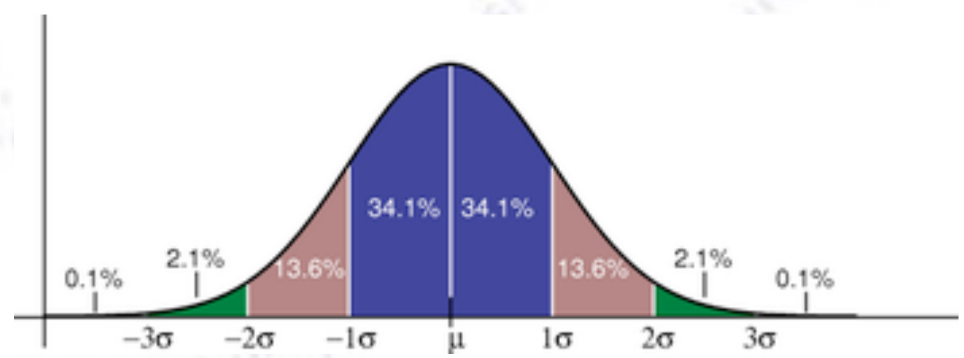
Applied Statistics

Confidence intervals and Limits



Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"

Confidence intervals

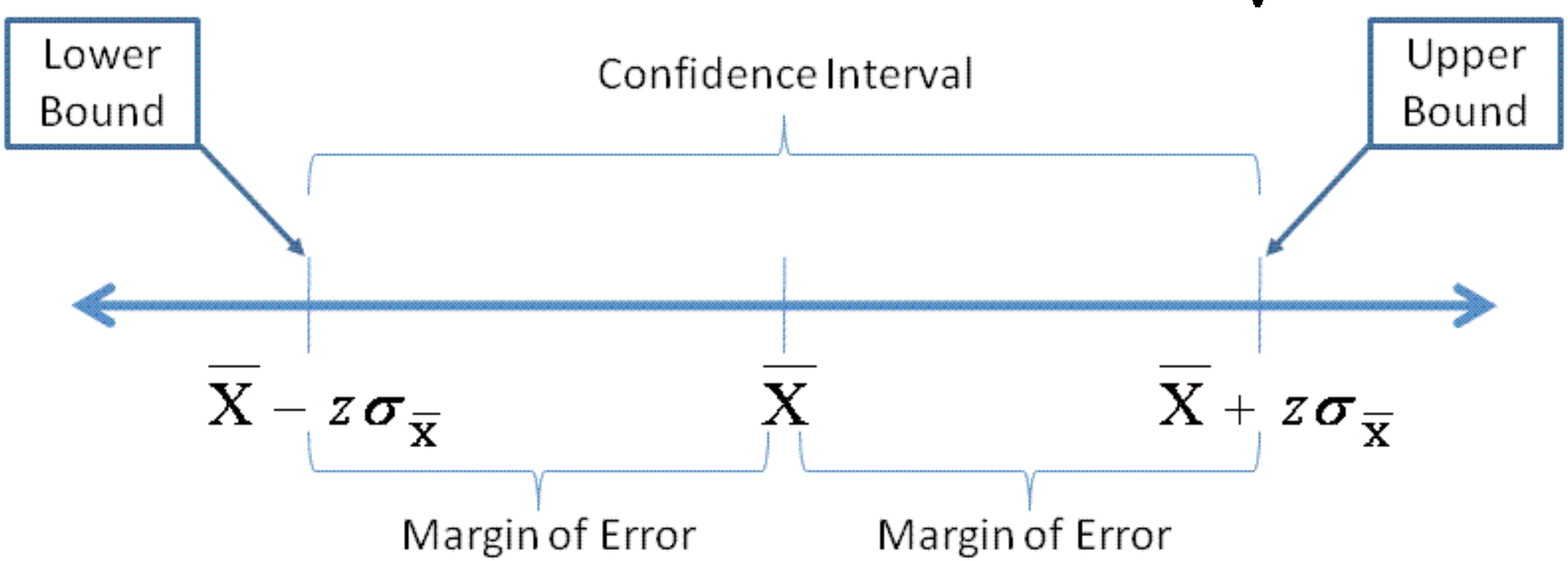
Confidence intervals

"Confidence intervals consist of a range of values (interval) that act as good estimates of the unknown population parameter."

It is thus a way of giving a range where the true parameter value probably is.

A very simple confidence interval for a Gaussian distribution can be constructed as: (z denotes the number of sigmas wanted)

$$\bar{x} \pm z \frac{s}{\sqrt{n}}$$



Confidence intervals

Confidence intervals are constructed with a certain **confidence level C**, which is roughly speaking the fraction of times (for many experiments) to have the true parameter fall inside the interval:

$$Prob(x_- \leq x \leq x_+) = \int_{x_-}^{x_+} P(x)dx = C$$

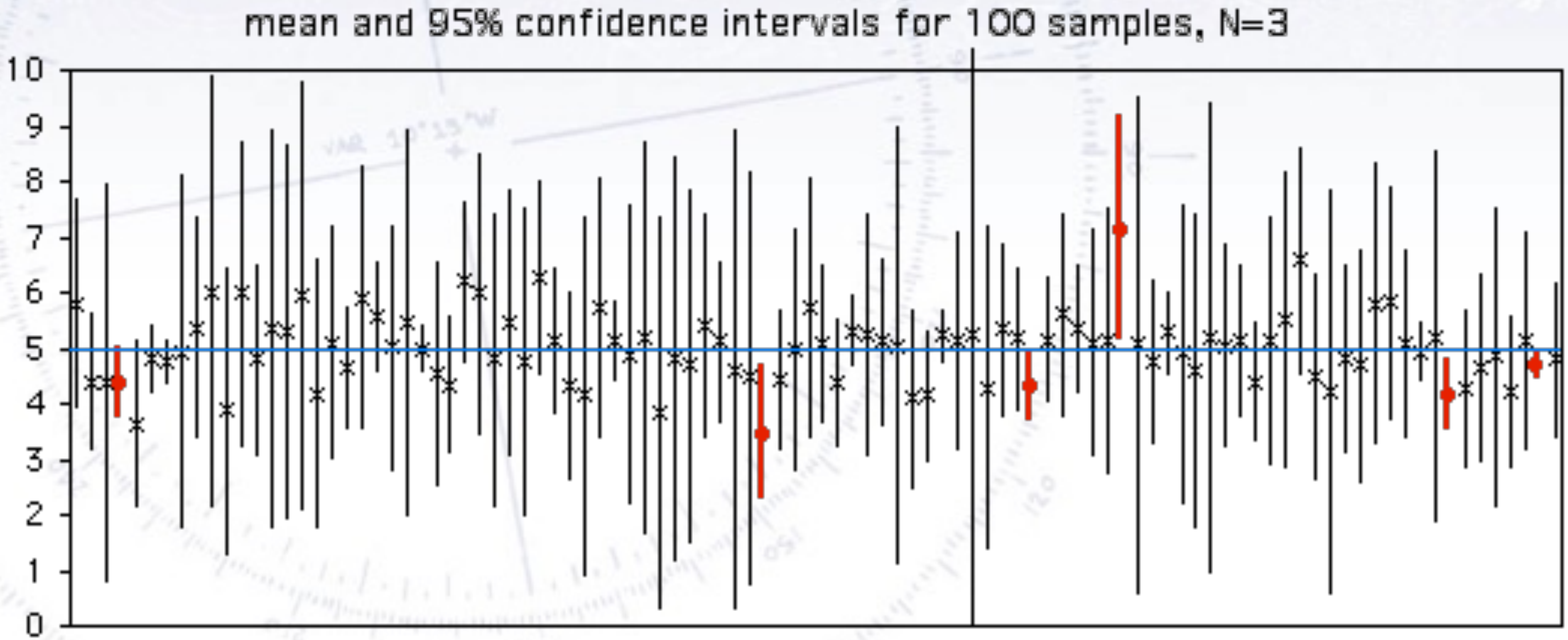
Typically, C = 95% (thus around 2σ), but 68%, 90% and 99% are also used often.

<u>There is a choice as follows:</u>
1. Require symmetric interval (x+ and x- are equidistant from μ).
2. Require the shortest interval (x+ - x- is a minimum).
3. Require a central interval (integral from x- to μ is the same as from μ to x+).

For the Gaussian, the three are equivalent!
Otherwise, 3) is usually used.

Confidence intervals

The confidence interval does not ALWAYS include the true value - only C fraction.



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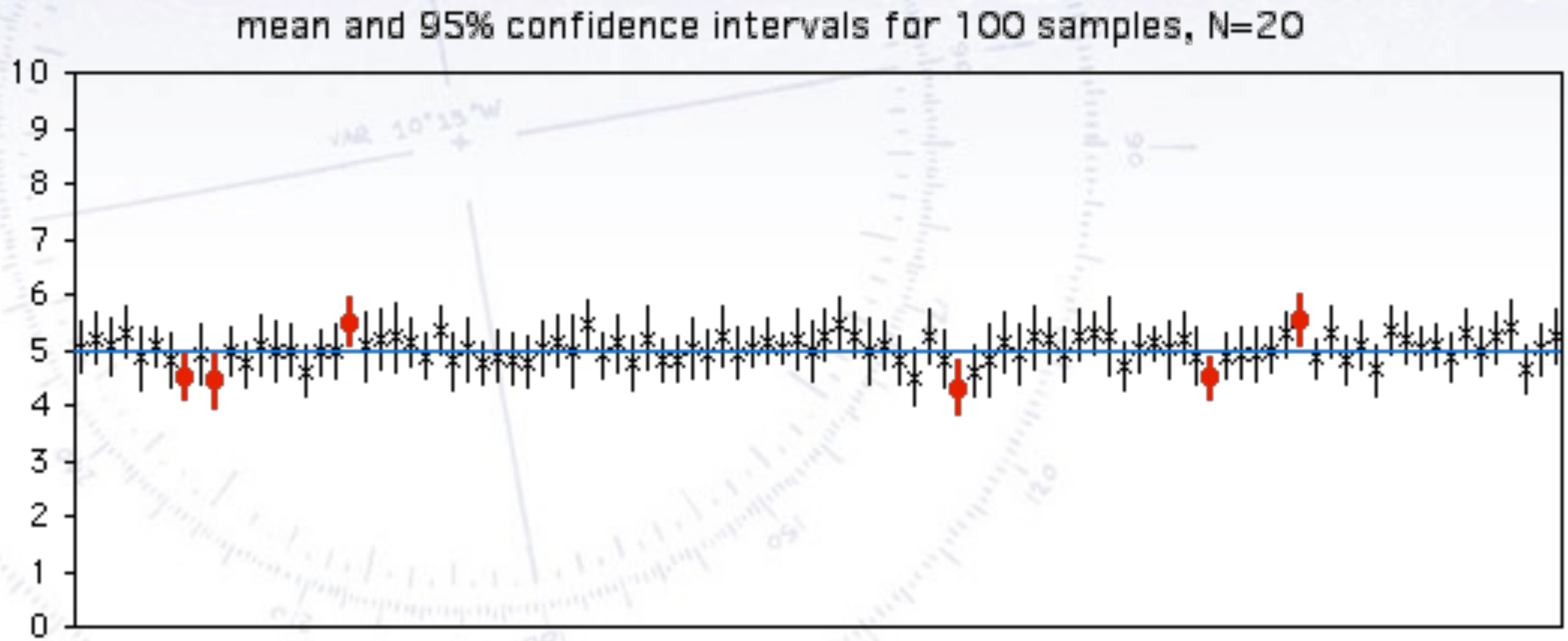
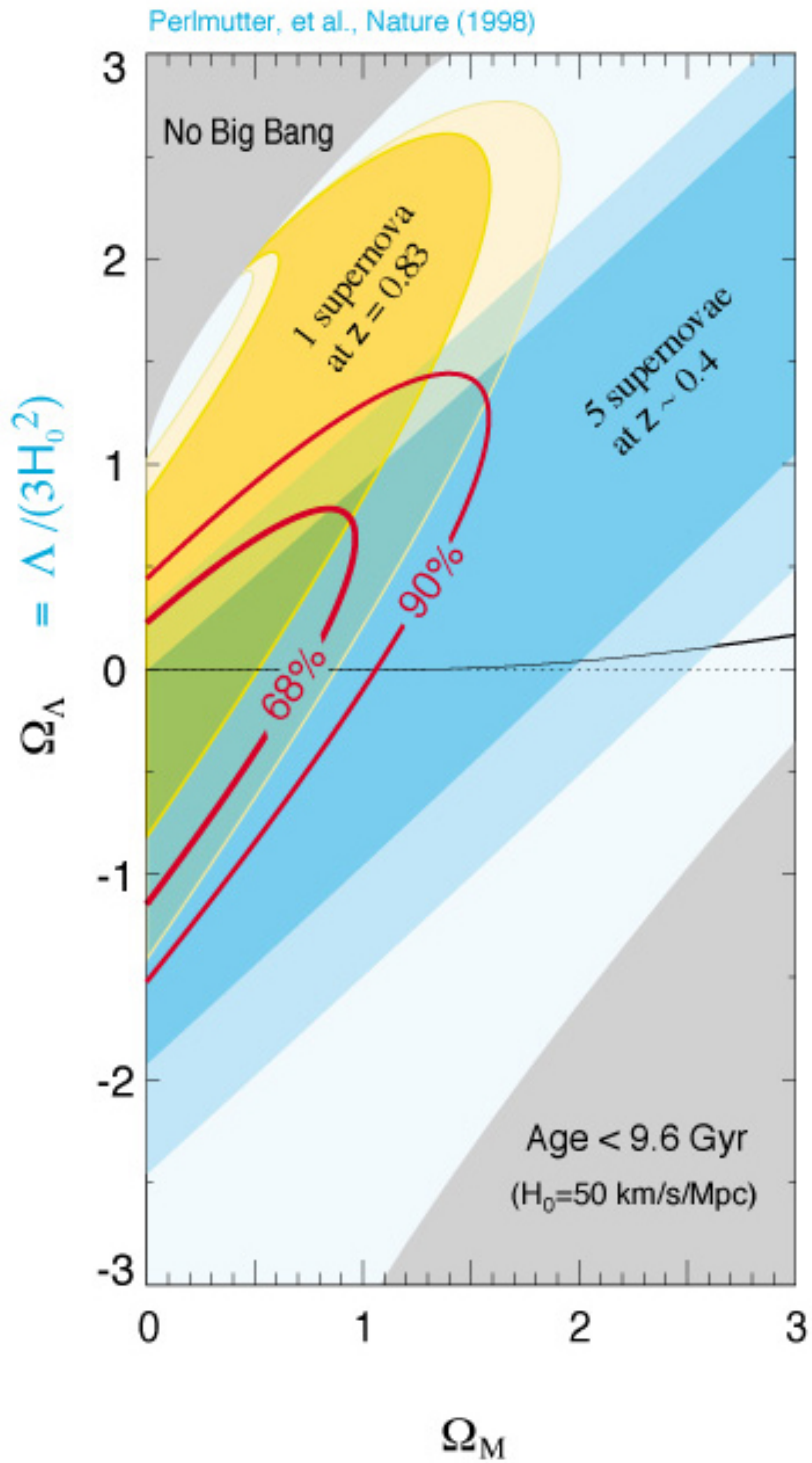


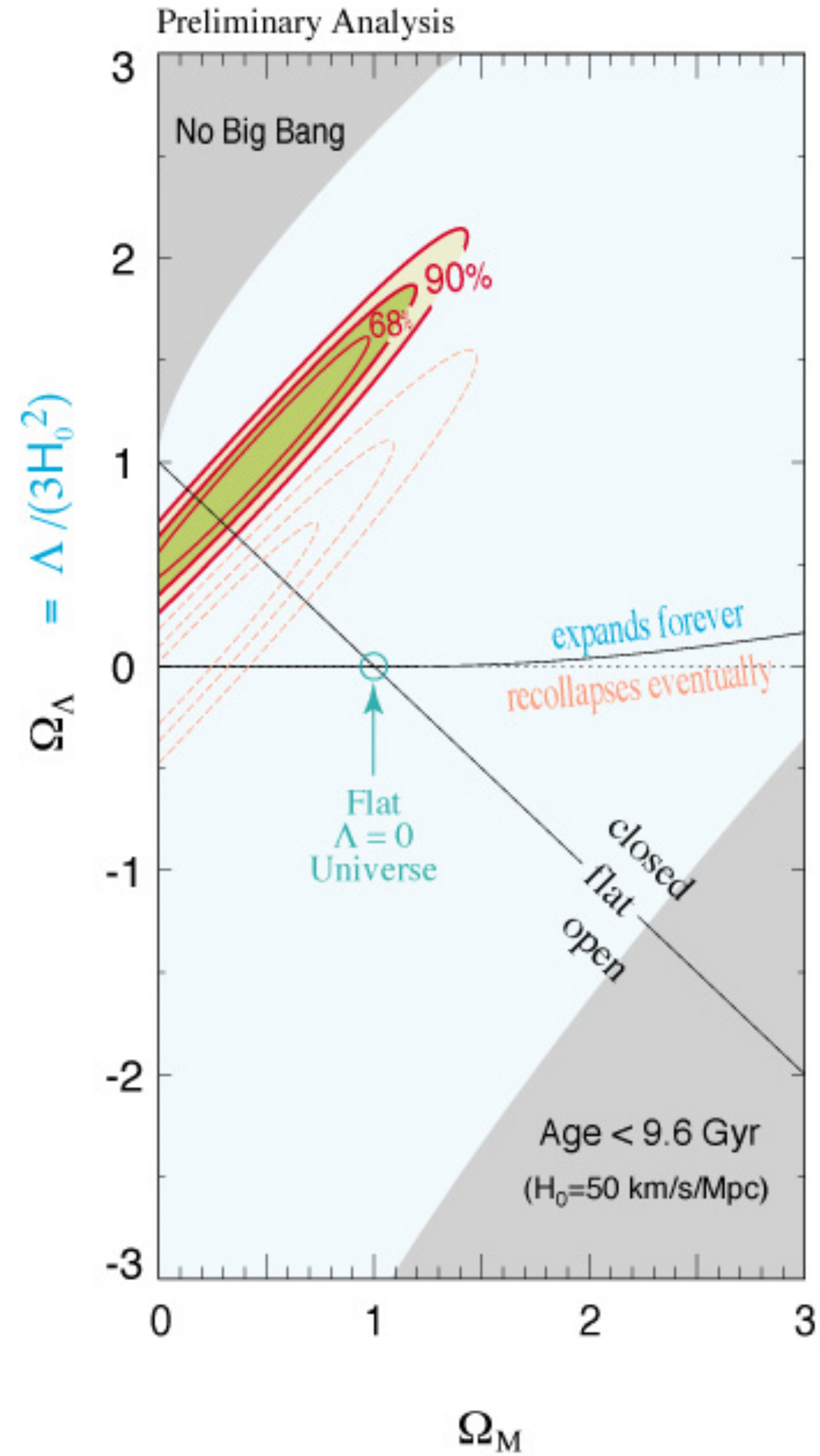
...and higher statistics does not help you!

Example
from cosmology

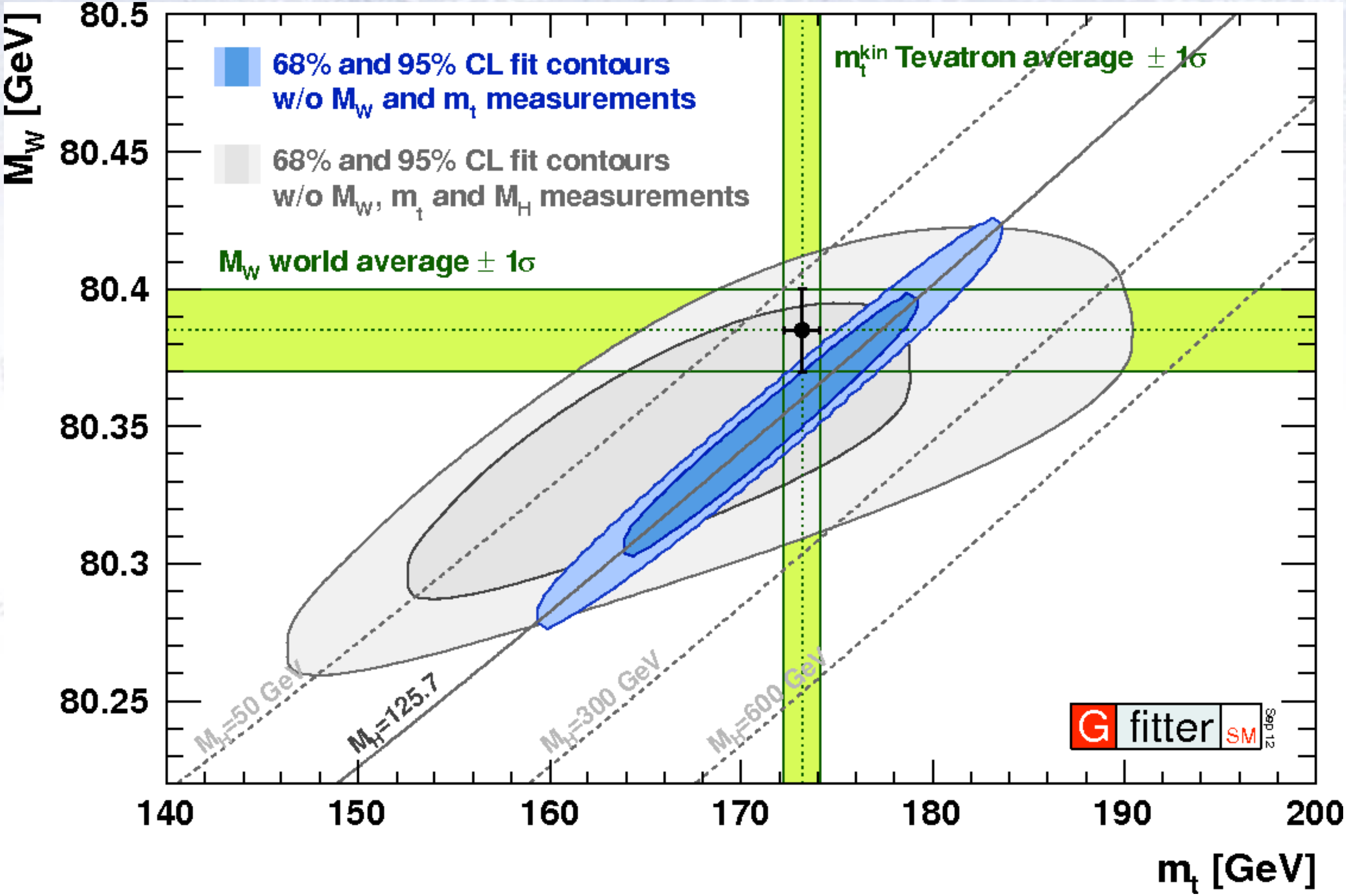
Results: Ω vs Λ
from 6 supernovae



Results: Ω vs Λ
from 40 supernovae



Example from particle physics



Confidence limits

Confidence limits

Imagine that you do an experiment to search for an unknown but predicted phenomenon (aether, planet Vulcan, dark energy, Higgs particle, etc.), and that find

…**nothing**!

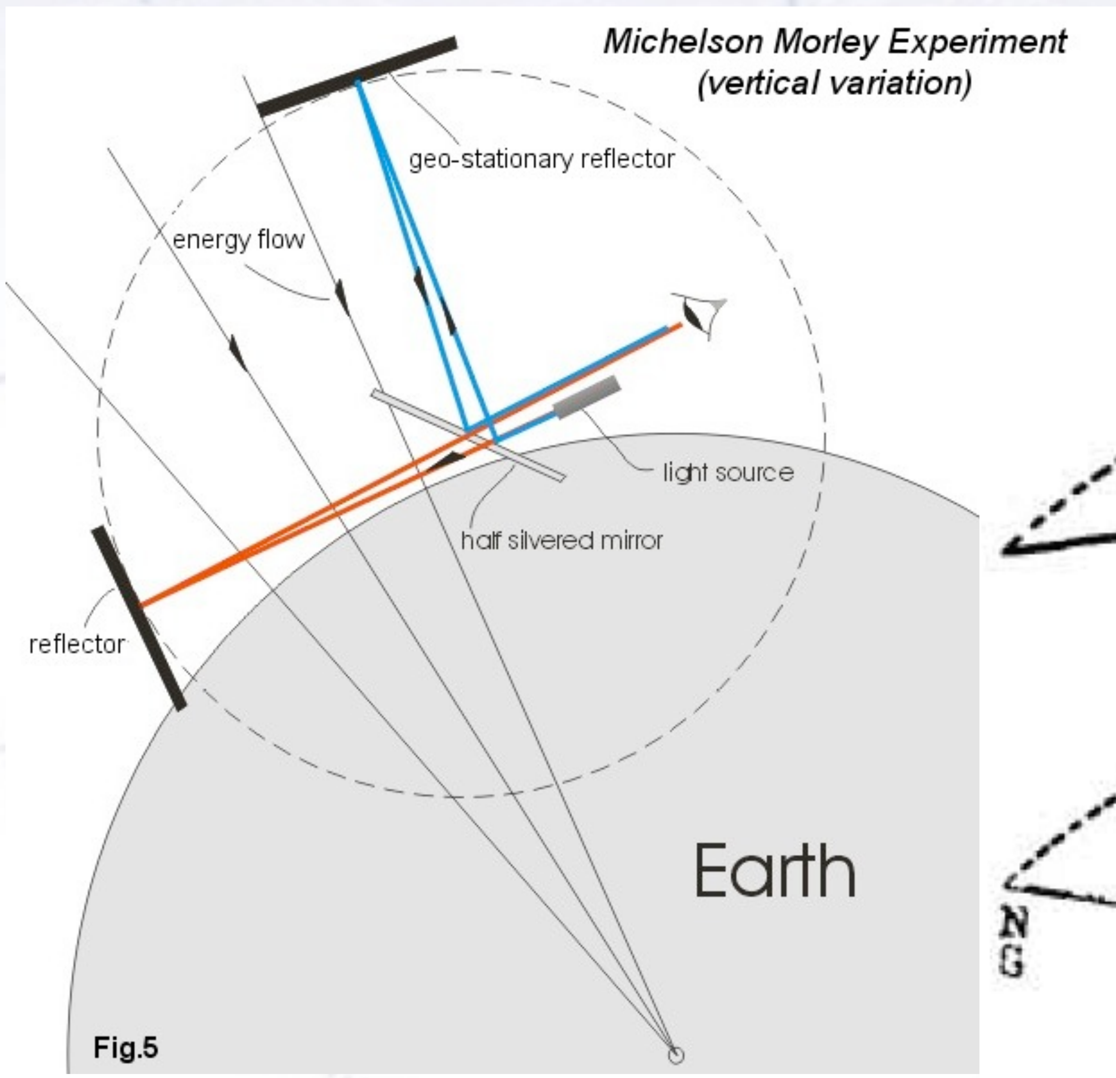
Reporting this result, you wish to state ***what you would have discovered, if it had been there***, i.e. something along the lines:

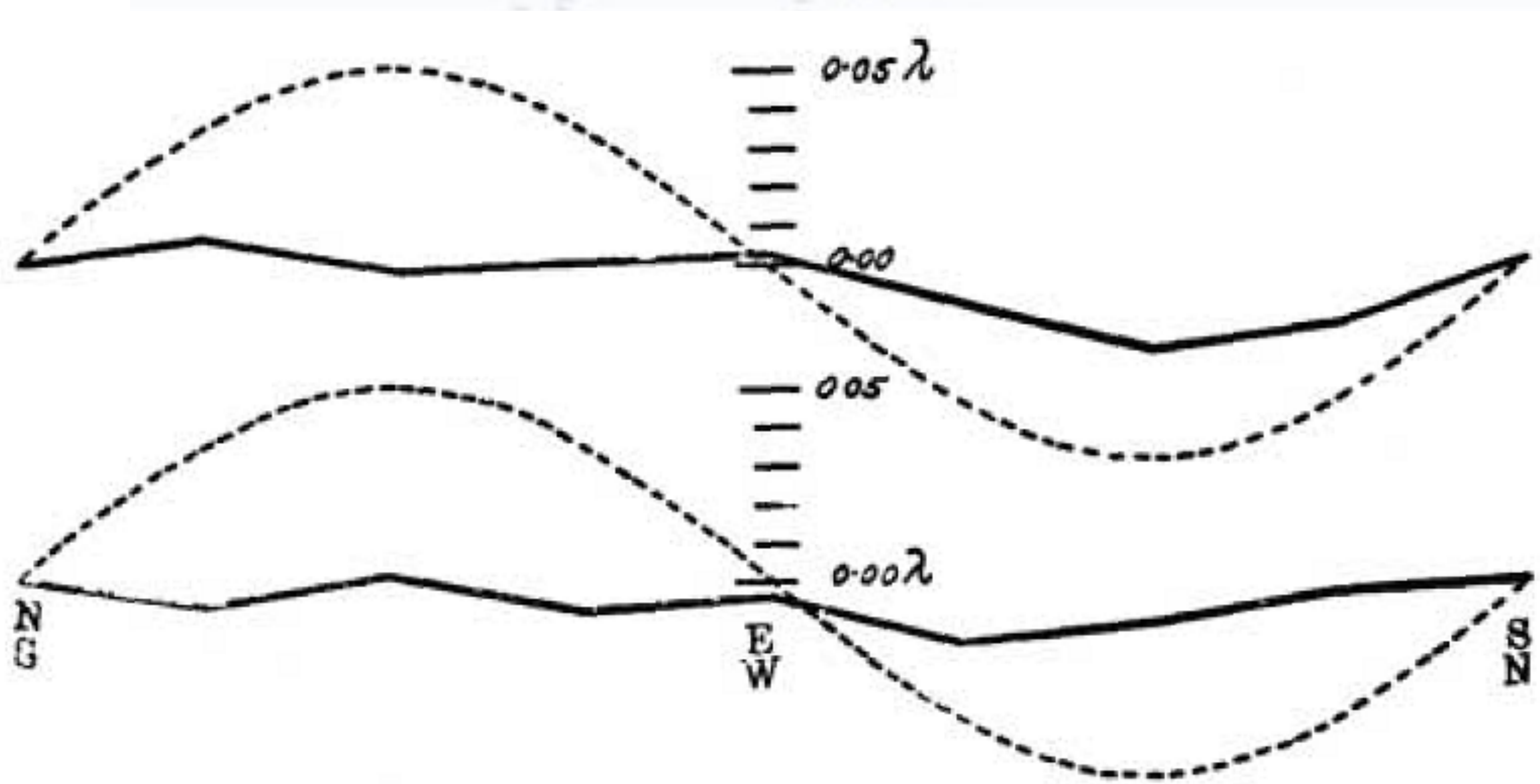
"If the aether had affected the speed of light by X%, we would have seen the effect with 95% confidence".

This is a **confidence limit** (much like a one-sided confidence interval).

Confidence intervals

In the case of Michelson-Morley, a limit could be set on the “degree of dragging” of the aether (though they didn’t do this, as statistics was still in its infancy!).





Confidence limits - Poisson

Poisson statistics is a neat special case, perhaps best explained by numbers:

Example:
If you in a day observe 0 red cars on Blegdamsvej, you can with 95% confidence say that there are less than 3.00 pr. day, and with 90% confidence say that there are less than 2.30 pr. day.
If you in a day observe 2 red cars, you can say at 95% CL that there are more than 0.355 and less than 6.30 red cars.

	$1-\alpha=90\%$		$1-\alpha=95\%$	
n	μ_{lo}	μ_{up}	μ_{lo}	μ_{up}
0	–	2.30	–	3.00
1	0.105	3.89	0.051	4.74
2	0.532	5.32	0.355	6.30
3	1.10	6.68	0.818	7.75
4	1.74	7.99	1.37	9.15
5	2.43	9.27	1.97	10.51
6	3.15	10.53	2.61	11.84
7	3.89	11.77	3.29	13.15
8	4.66	12.99	3.98	14.43
9	5.43	14.21	4.70	15.71
10	6.22	15.41	5.43	16.96