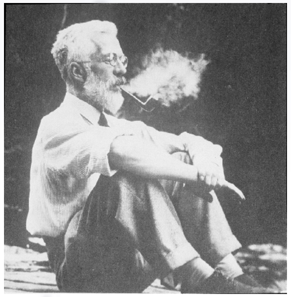
Applied Statistics

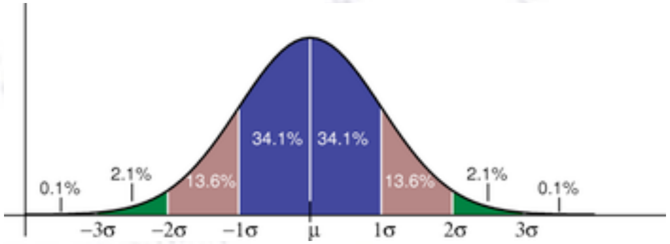
Project evaluation








Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"

Project evaluation

Pendulum:

- Did you measure $T \pm \sigma(T)$ correctly? Combine with Chi2 and comments?
- Did you measure $L \pm \sigma(L)$ correctly?
- Did you provide the individual precisions?

Ball on incline:

- $T \pm \sigma(T)$
- $L \pm \sigma(L)$

$\Rightarrow a \pm \sigma(a)$, with Chi2 and comments.

- θ, $\Delta\theta$ obtained correctly and
- d, R and errors propagated correctly?

Generally:

- **Correctly propagated uncertainties, showing individual contributions.**
- All necessary figures and tables there? 2-3 essential figures needed.
- Text enough to understand results? Clear and fitting captions?
- Comment on result, especially inconsistencies.
- Significant digits.

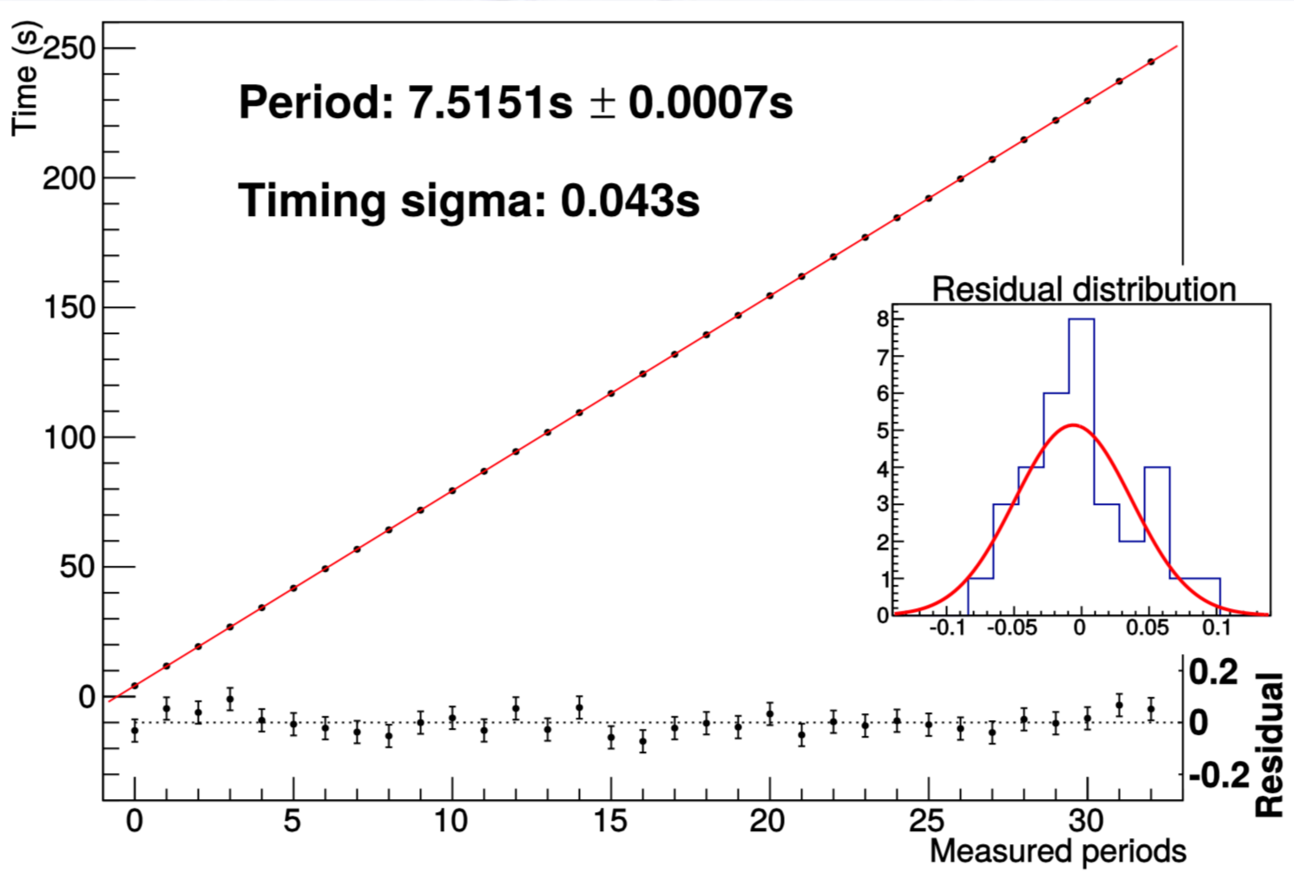
Pendulum - comments

Time measurement:
Many independent measurements, little systematic $\Rightarrow$ Good error estimate

Length measurement:
Some independent measurements but also some systematics $\Rightarrow$ check difference between instruments.

You can not reduce the uncertainty by multiple measurements, if the main limitation is some inherent systematic!

Several groups managed to get uncertainties below 0.1%.



Pendulum - comments

Time measurement:

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Length measu

Some indepen ference between instruments.

> Very importantly:
> RMS of residuals = **your single measurement** precision.
> Uncertainty on fit slope = **pendulum period** precision.
> The latter is MUCH smaller!

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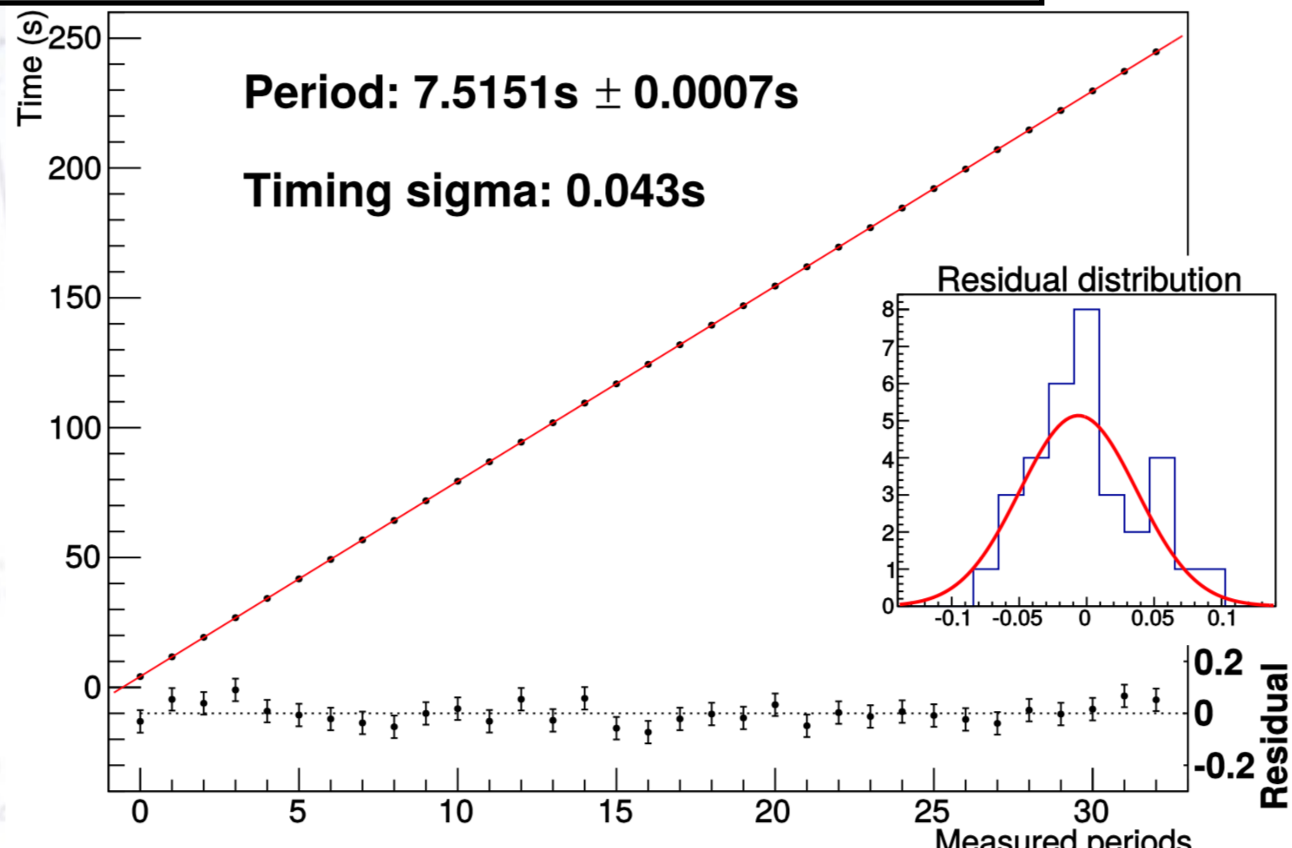

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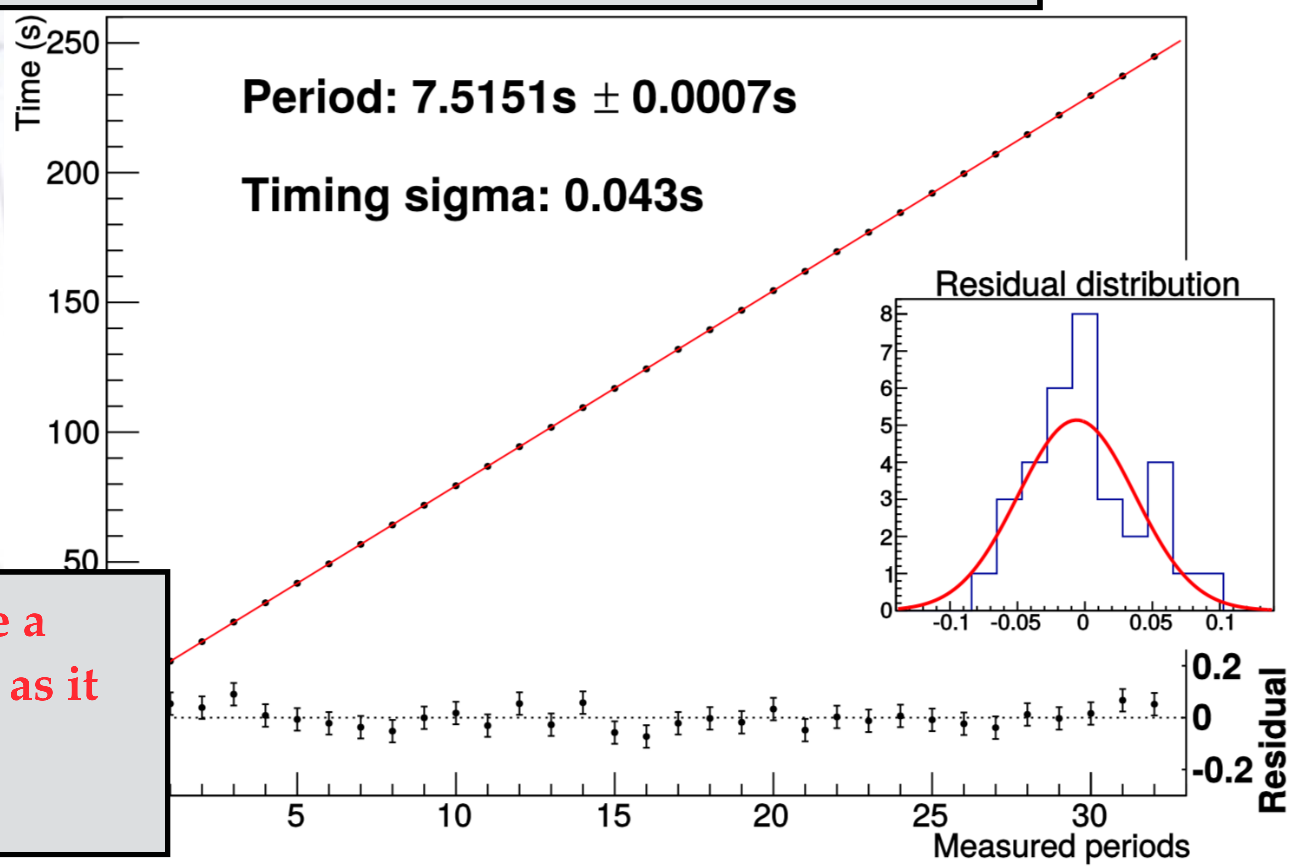

Note: This is a case, where a ChiSquare is not in place, as it will <u>by construction</u> yield around 50%.

Pendulum - comments

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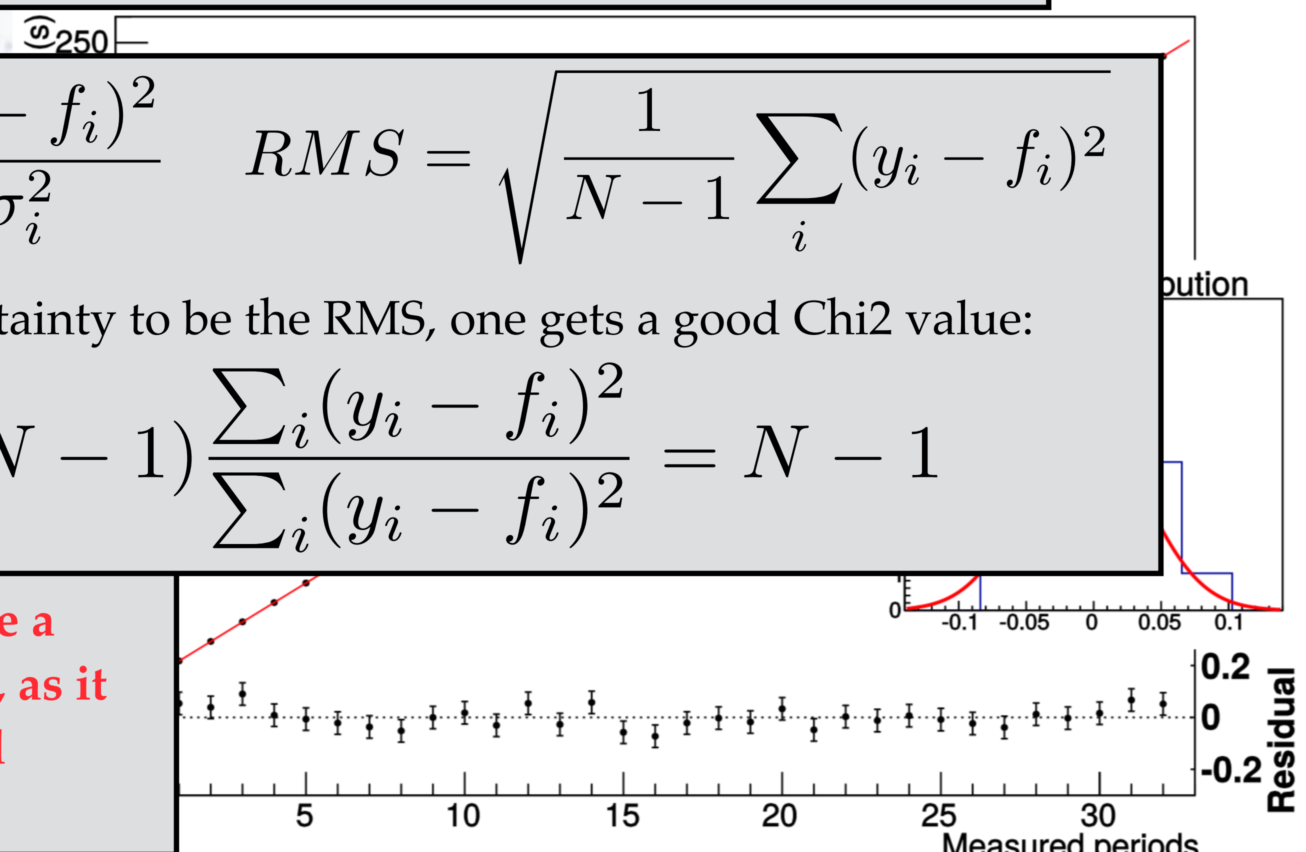
$$\chi^2 = \sum_i \frac{(y_i - f_i)^2}{\sigma_i^2} \qquad RMS = \sqrt{\frac{1}{N-1}\sum_i (y_i - f_i)^2}$$

Defining the uncertainty to be the RMS, one gets a good Chi2 value:

$$\chi^2 = (N-1)\frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - f_i)^2} = N - 1$$



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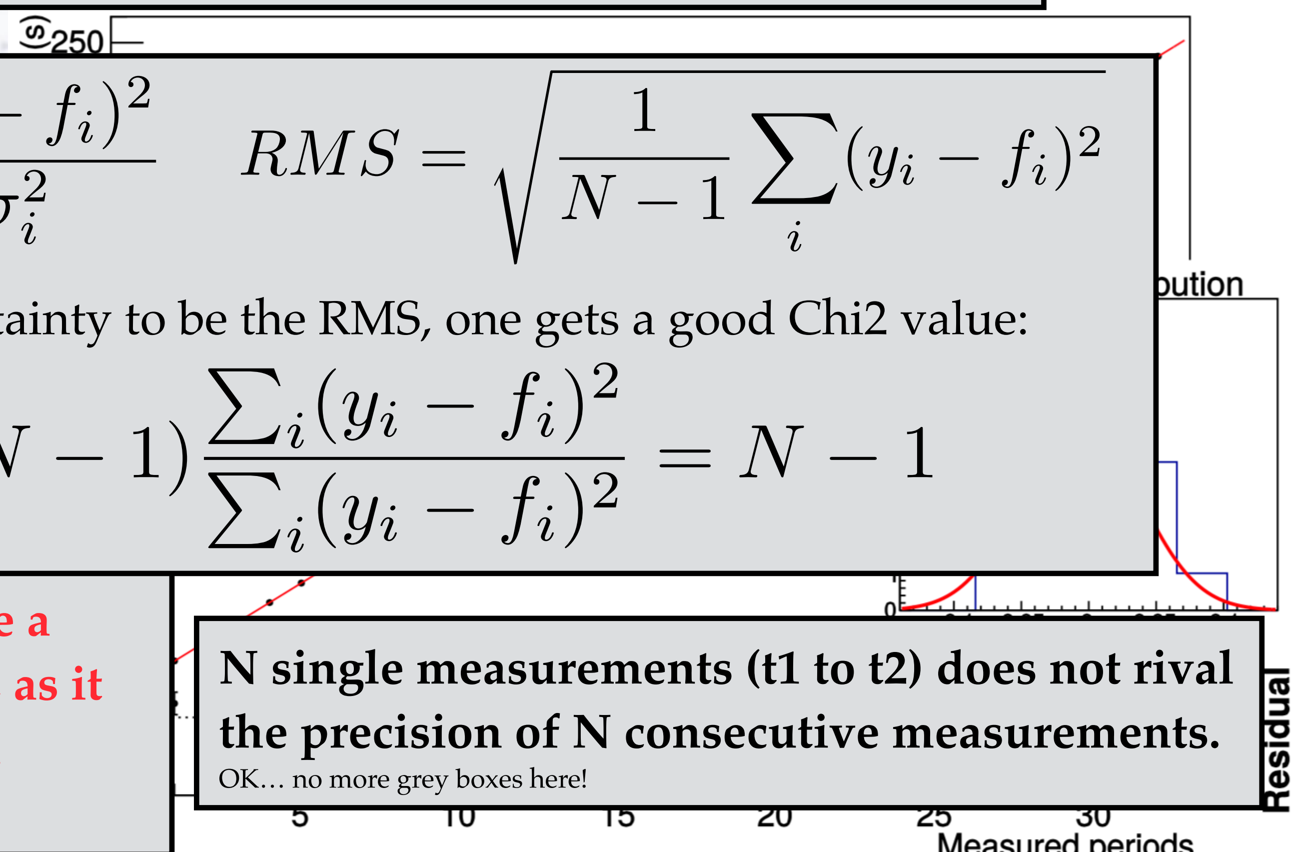

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Note: This is a case, where a ChiSquare is not in place, as it will by construction yield around 50%.

N single measurements (t1 to t2) does not rival the precision of N consecutive measurements.

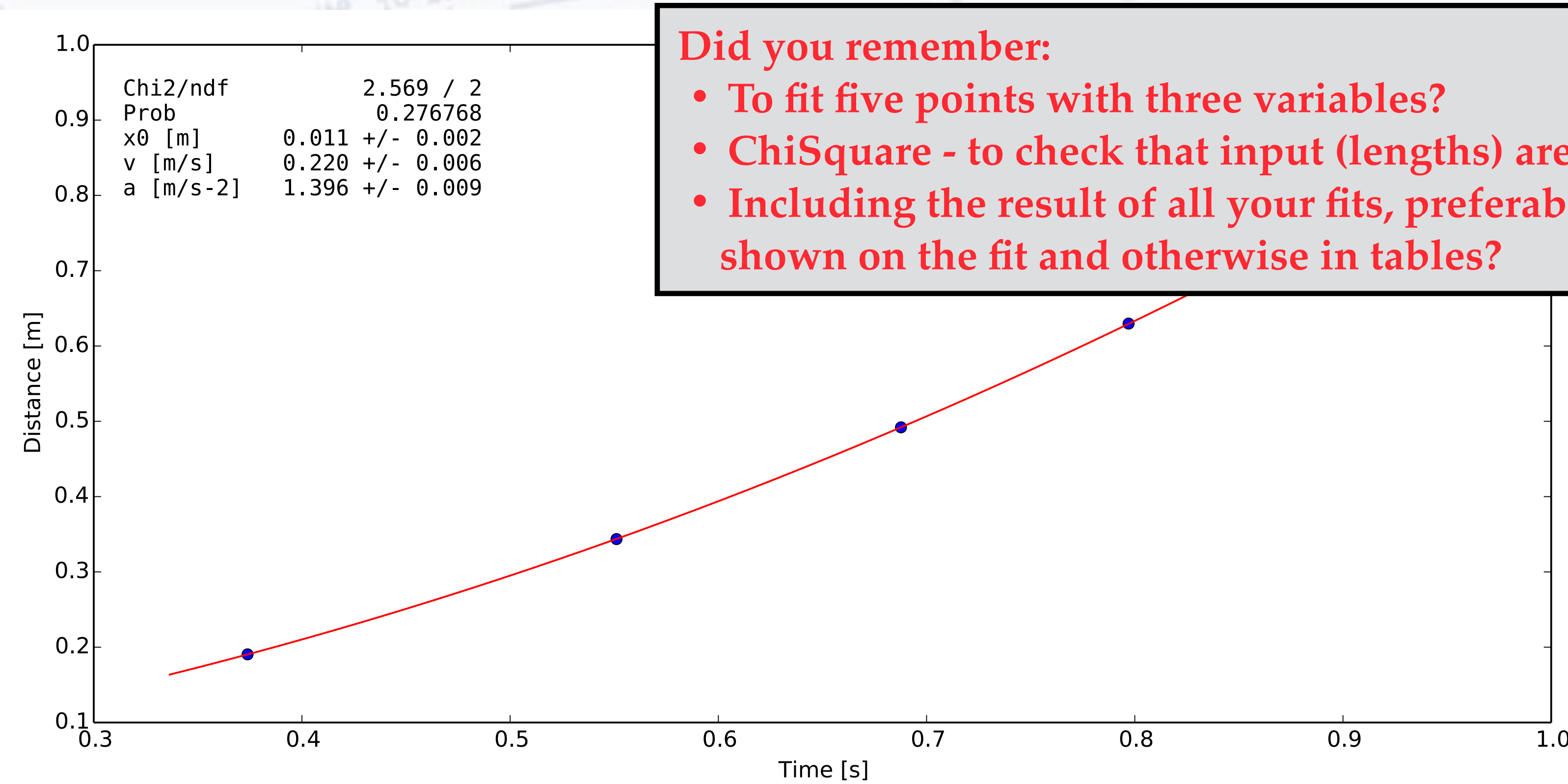
OK… no more grey boxes here!

Ball on incline - comments

Time measurement: Insignificant uncertainties, so repetition doesn't help!
However, it is helpful to detect unknown systematics.

Length measurements: Some uncertainties, some systematics, but OK errors.
Note that diode position is not central!

Angle measurement: This is the real challenge! And angle vs. lengths have very different systematics, so they are good to combine!



Did you remember:

- **To fit five points with three variables?**
- **ChiSquare - to check that input (lengths) are OK?**
- **Including the result of all your fits, preferably shown on the fit and otherwise in tables?**

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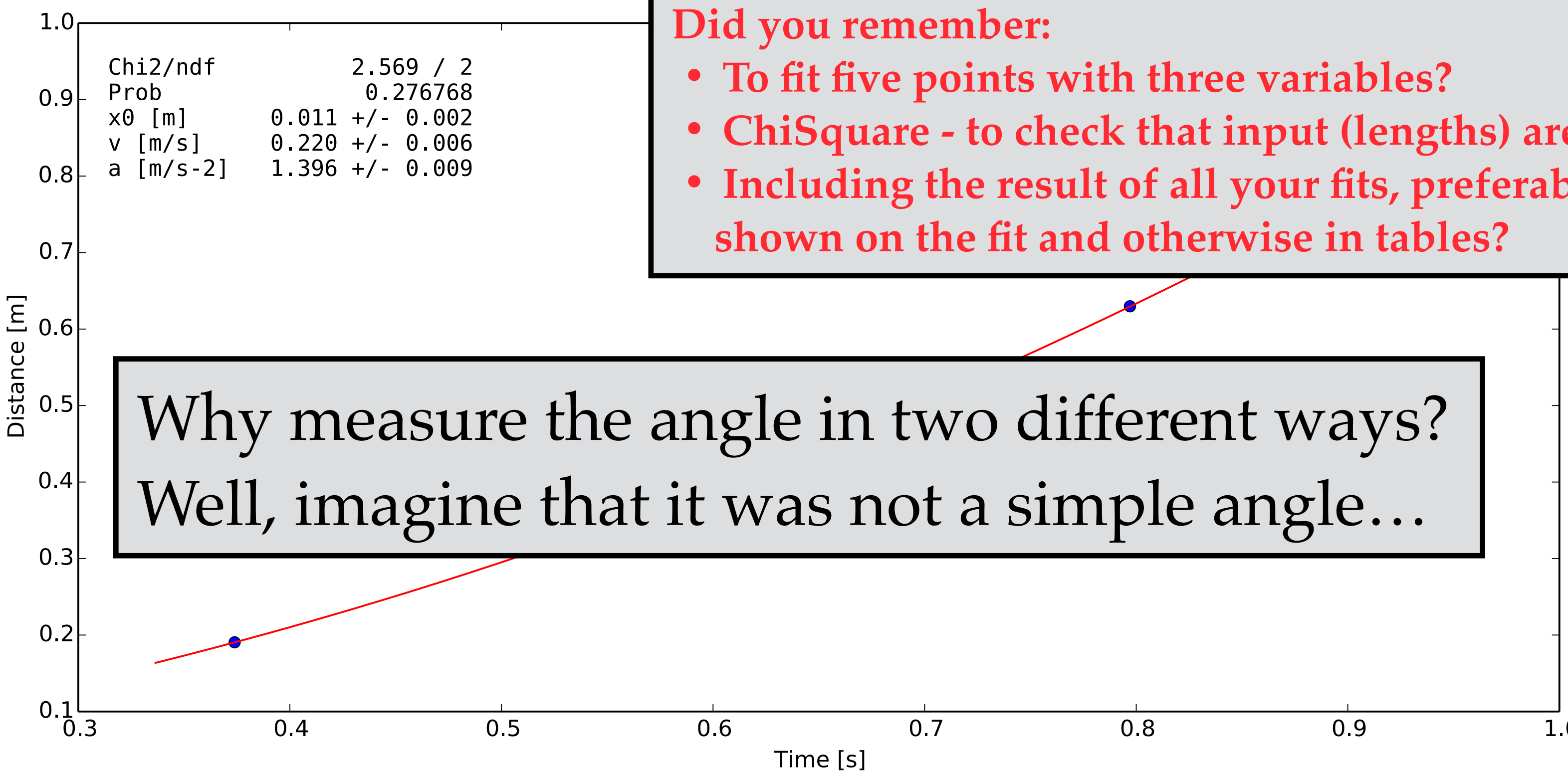

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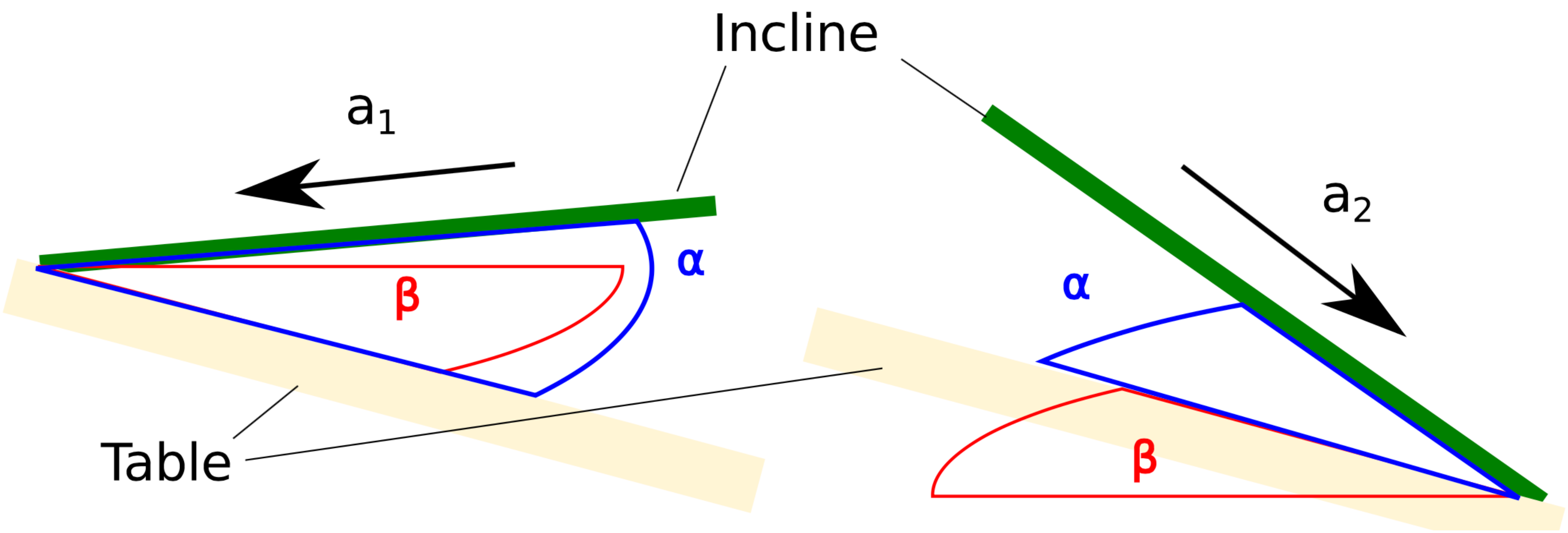
- **ChiSquare - to check that input (lengths) are OK?**
- **Including the result of all your fits, preferably shown on the fit and otherwise in tables?**

Why measure the angle in two different ways?
Well, imagine that it was not a simple angle…

Ball on incline - comments

After searching my soul (and trigonometric rule tables), I found that there is in fact an analytical solution for obtaining $\Delta\theta$. Both solution can be found on ERDA.

Calculate and utilize the table angle:



$$\frac{a_{left}}{a_{right}} = \frac{\sin(\theta + \Delta\theta)}{\sin(\theta - \Delta\theta)}. \qquad \Delta\theta = \frac{(a_{left} - a_{right})\sin(\theta)}{(a_{left} + a_{right})\cos(\theta)}.$$

General comments

Tests and cross checks:

- **For weighted average, calculate Chi2 and p-values (to test consistency).**
- Never combine inconsistent numbers (then you are SURE to make a mistake!).
- Comment (heavily) on inconsistent numbers.

Correlations:

- **Careful in not combining correlated numbers.**
- Careful not to repeat measurements that are small or systematically dominated.

Structure of report:

- **Plan your plots**. Here 3-4 is fitting. At least length vs. time for both experiments.
- **Plan your tables**. Make sure they are readable and complete.
- Put results and conclusions in abstract. Meant for saving readers time.
- Be short and precise. Label carefully and refer to these labels.
- Always remember correct number of significant digits.
- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Don’t put definition of mean, RMS, etc. in your text. Considered known to all.
- Be VERY detailed about removing data. Show explicitly why. Give p-values.

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> gravitational acceleration of 9.820 ± 0.0129 m/s^2 and $0.43\ \sigma$ for the pendulum and 9.661 ± 0.0672 m/s^2 and 2.29σ for the ball-on-incline. Both experiments were setup using standard hand-held

- Be VERY detailed about removing data. Show explicitly why. Give p-values.

Measurement situation

There are four possible situations in experimental measurements of a quantity:

One measurement, no error:

X = 3.14

Situation: You are f***ed!

You have no clue about uncertainty, and you can not obtain it!

Several measurements, no errors:

X1 = 3.14
X2 = 3.21
X3 = …

Situation: You are OK

You can combine the measurements, and from RMS get error on mean.

One measurement, with error:

X = 3.14 ± 0.13

Situation: You are OK

You have a number with error, which you can continue with.

Several measurements, with errors:

X1 = 3.14 ± 0.13
X2 = 3.21 ± 0.09
X3 = …

Situation: You are on top of things!

You can both combine to a weighted, average and check with a chi-square.

Specific comments

Pendulum experiment:

- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Pendulum line may be stretching. Requires times and lengths for individual g.
- Swinging pendulum at 90 degrees is a good check of impact of hook.
- Combine lengths (L) with normal mean, and get uncertainty from RMS.
- Combine periods (T) with weighted mean and use Chi2 to check consistency.
- Large swings (e.g. 7 degrees) result in 1/1000 violation of formula assumption!

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3\,072}\theta_0^4 + \frac{173}{737\,280}\theta_0^6 + \frac{22\,931}{1\,321\,205\,760}\theta_0^8 + \frac{1\,319\,183}{951\,268\,147\,200}\theta_0^{10} + \frac{233\,526\,463}{2\,009\,078\,326\,886\,400}\theta_0^{12} + \cdots\right),$$

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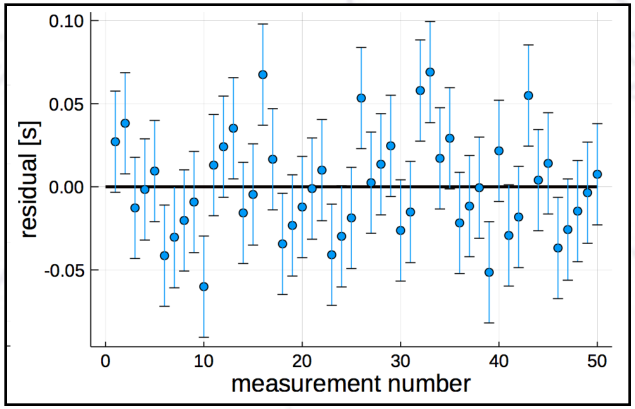
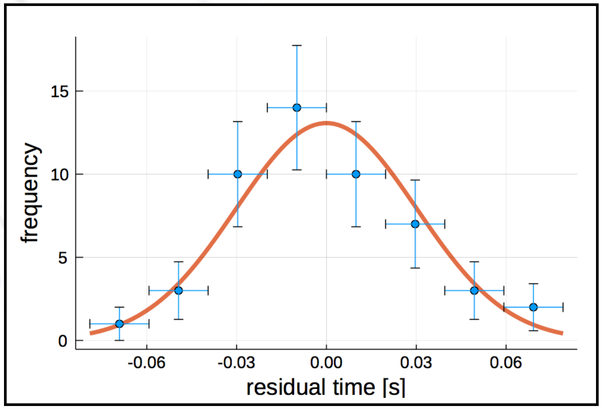
Note: Getting precise timing from a camera (16 f/s) is actually not that easy:





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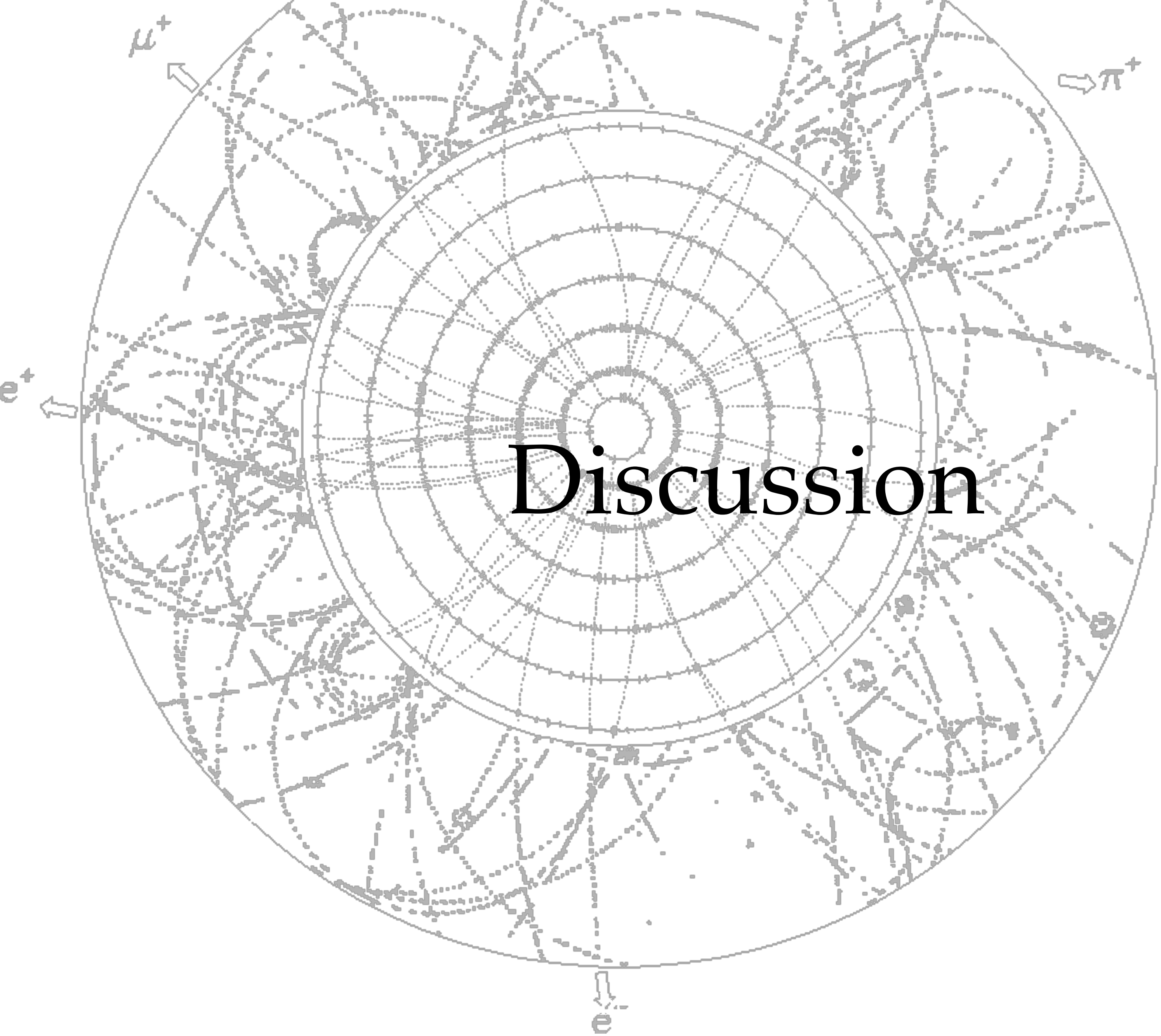
- Large swings (e.g. 7 degrees) result in 1/1000 violation of formula assumption!

Ball-on-incline experiment:

- Rerunning ball a few times to get σ(t_gate) only gives you σ(t_magnet release).
- Write IMPACT on g from each input variable. Each term in error prop. formula.
- Write fit function with constants in front, i.e. 0.5 * a * t^2. Gives correct errors!
- Different ball size give different results. 1-2% variations (10-15mm).
- No, you can't measure the angle with 0.001 degree precision!!!

Discussion



Correlated measurements

If measurements are independent, then they can be combined to decrease the uncertainty.

When are measurements correlated?

- When they are (partly) based on the same sub-measurement/input?
- When they are measured with the same instrument repeatedly?
- When they are from different methods for measuring the same quantity?
- When they involve a commonly extracted quantity?

Examples from the experiments:

- The gate positions in the Ball-on-Incline experiment, when they are measured
  a) as positions on one ruler placed in a fixed position?
  b) as distances between each adjacent gate?
- The pendulum times as measured by different people at the same time?
- The accelerations when repeating the balls roll?
- Two measurements of g based on turning setup around and measuring angle
  a) with a goniometer (i.e. angle measuring device)?
  b) using trigonometry?

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When are r

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- When th

Examples

- The gate measured
  a) as pos
  b) as dis

Note that correlations can be used to your advantage!

Imagine measuring the pendulum length with the laser. You have 1mm precision written on the instrument, but you actually don't know, and it is hard to claim anything better.

However, if you measure the pendulum length from the floor and up to the pendulum, and from the floor to the ceiling, then any bias will cancel, and repeated measurements can improve the precision beyond 1mm!

- The pendulum times as measured by different people at the same time?
- The accelerations when repeating the balls roll?
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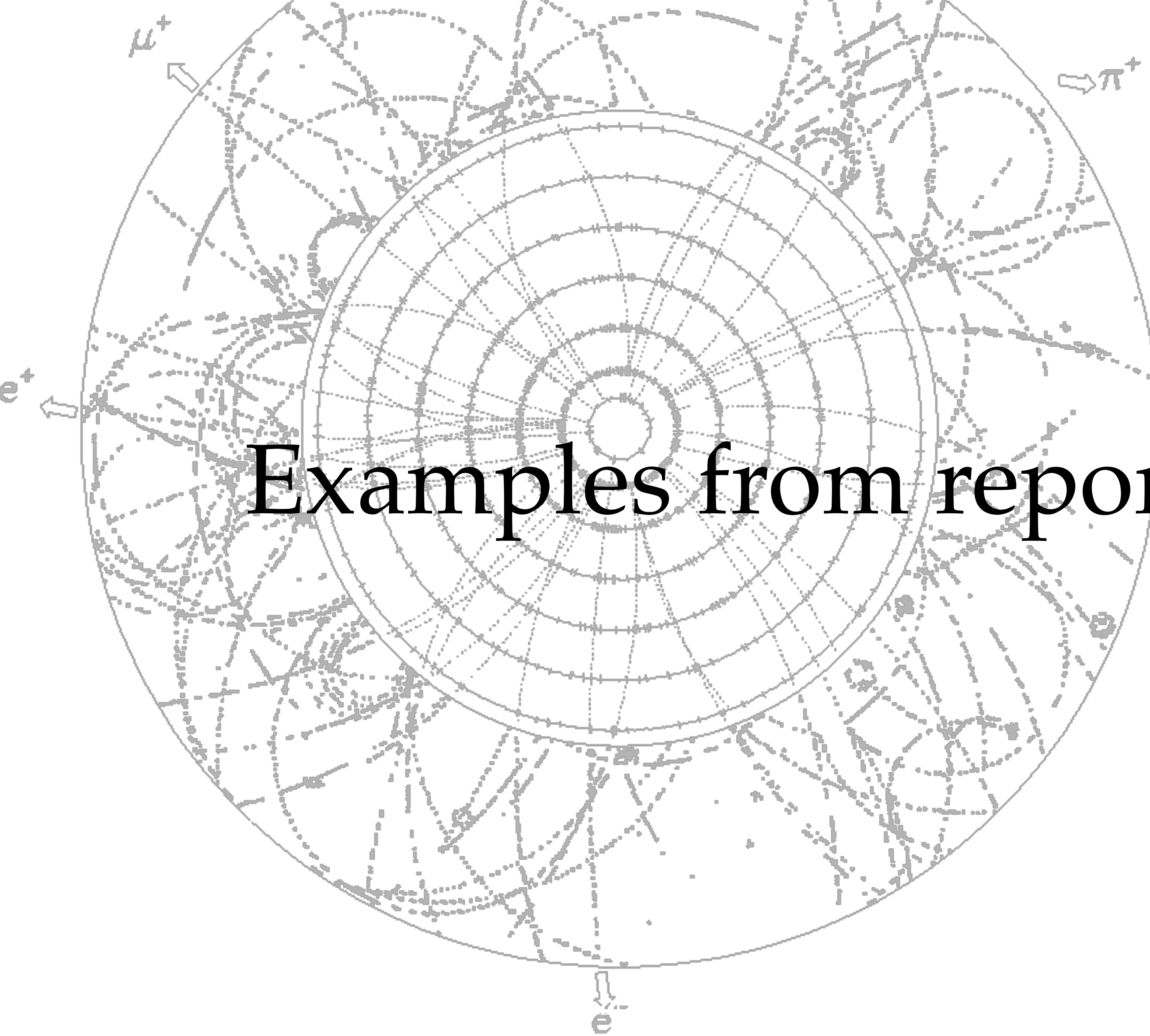
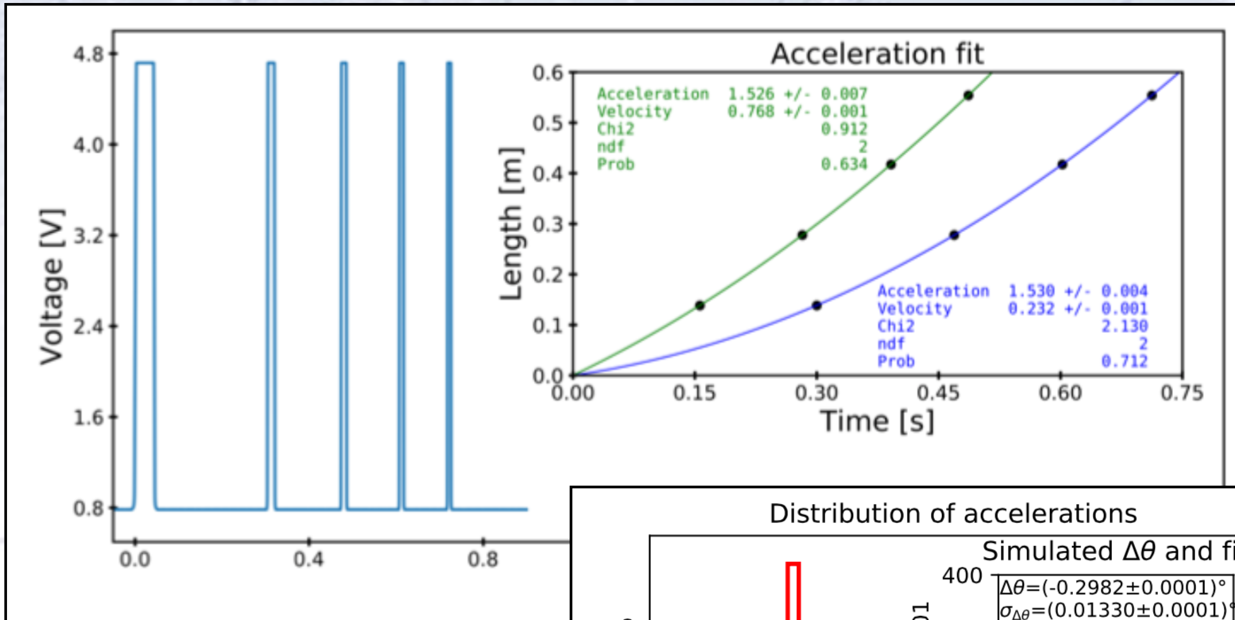
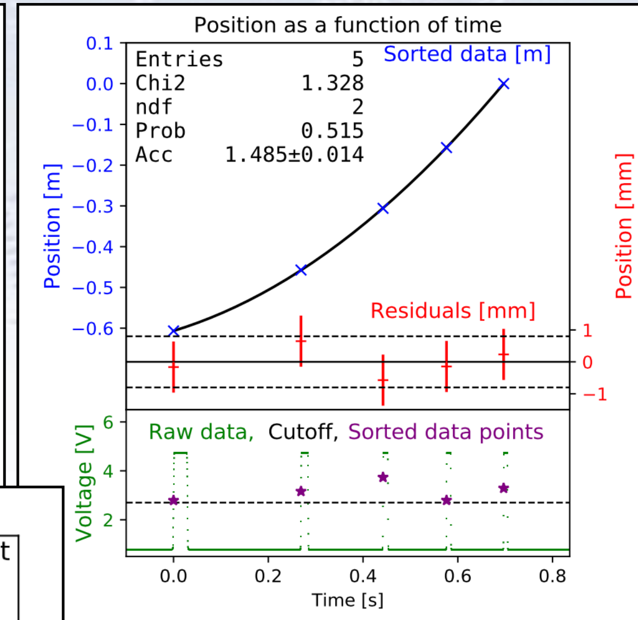
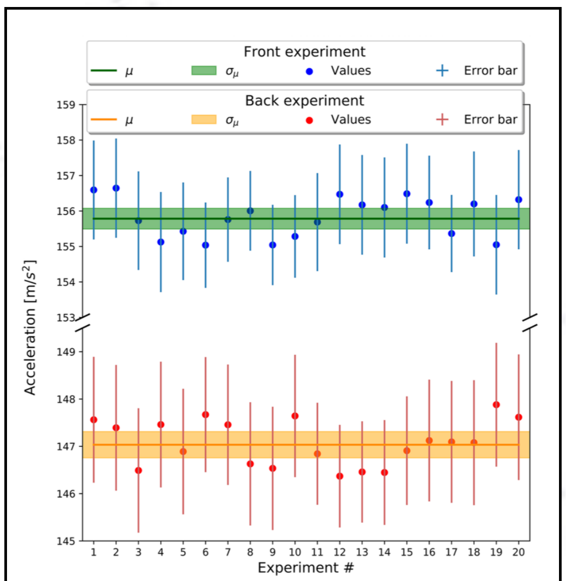


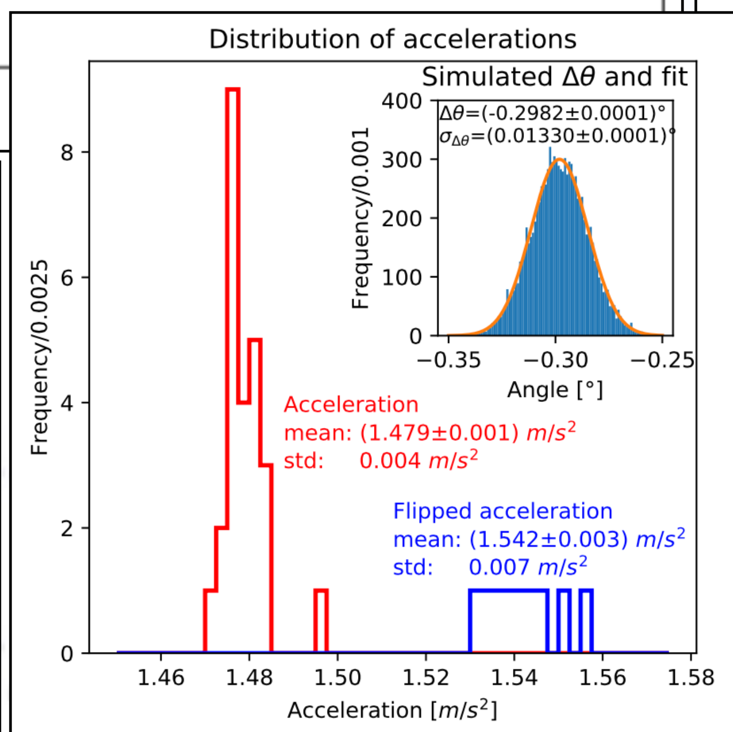
Examples from reports

Great figures



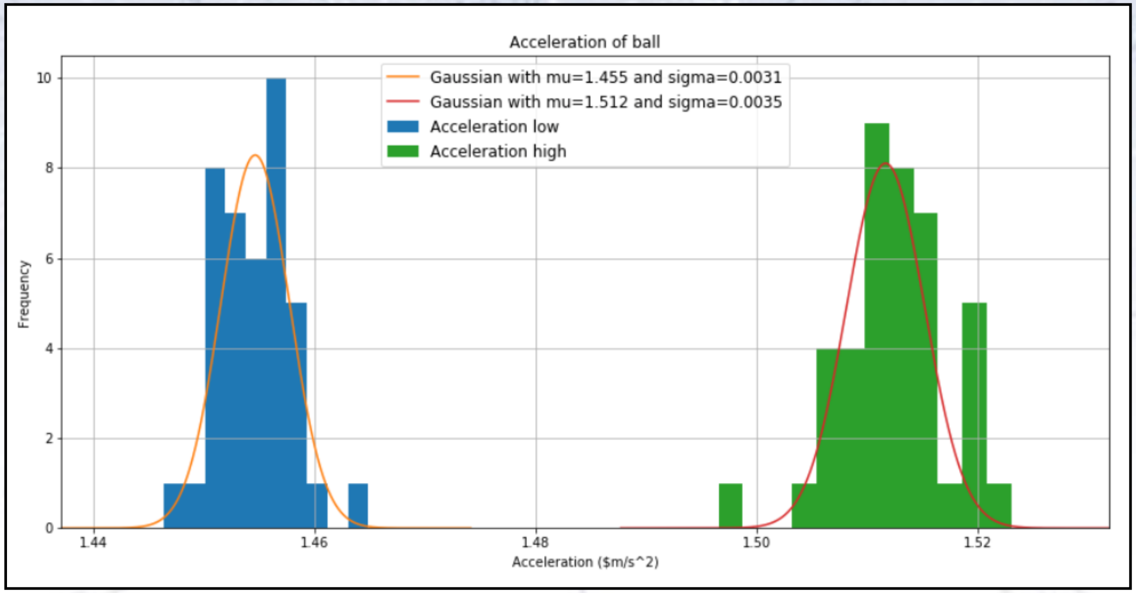


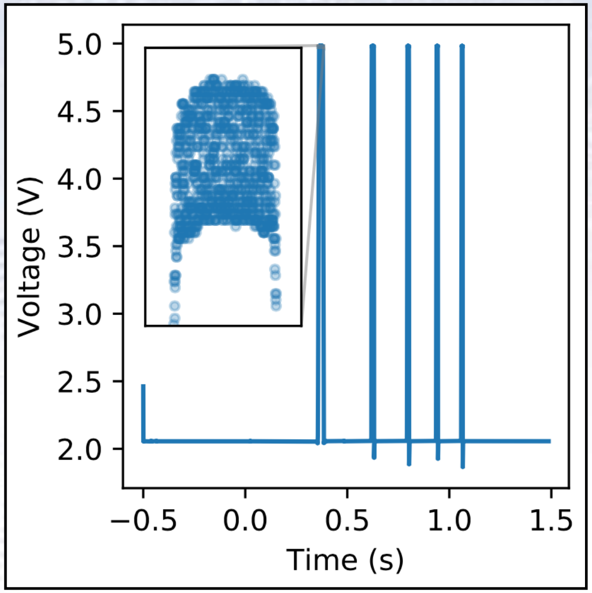


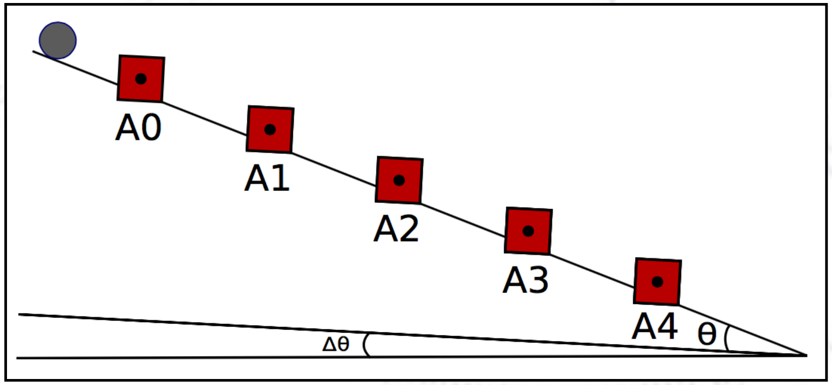


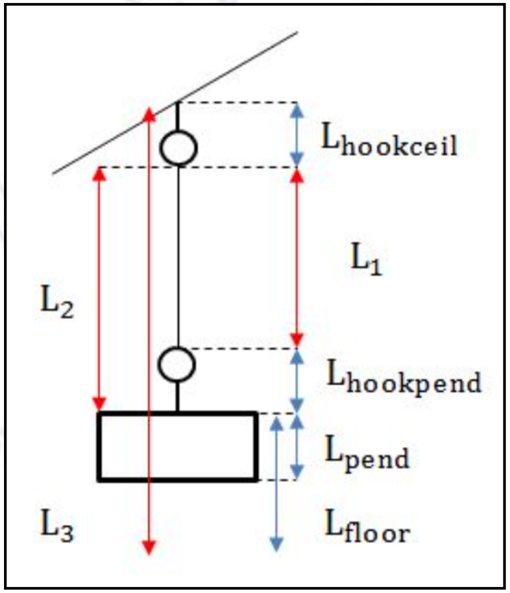
These figures are just a selection of the many great figures produced in your reports. Thanks a lot for that.

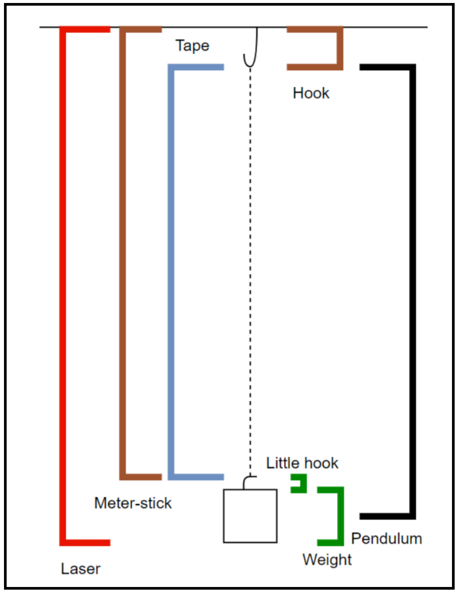
Great figures











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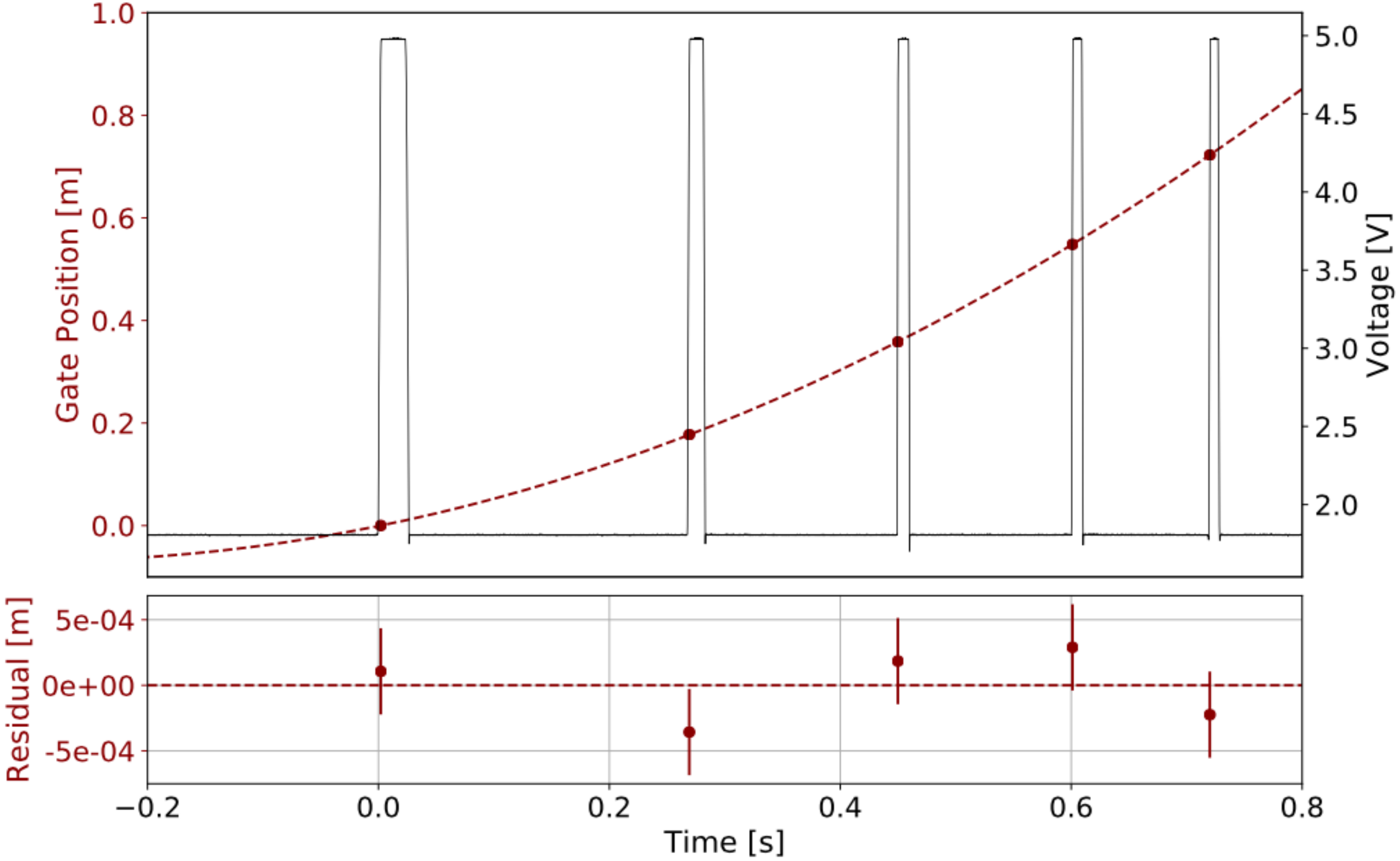

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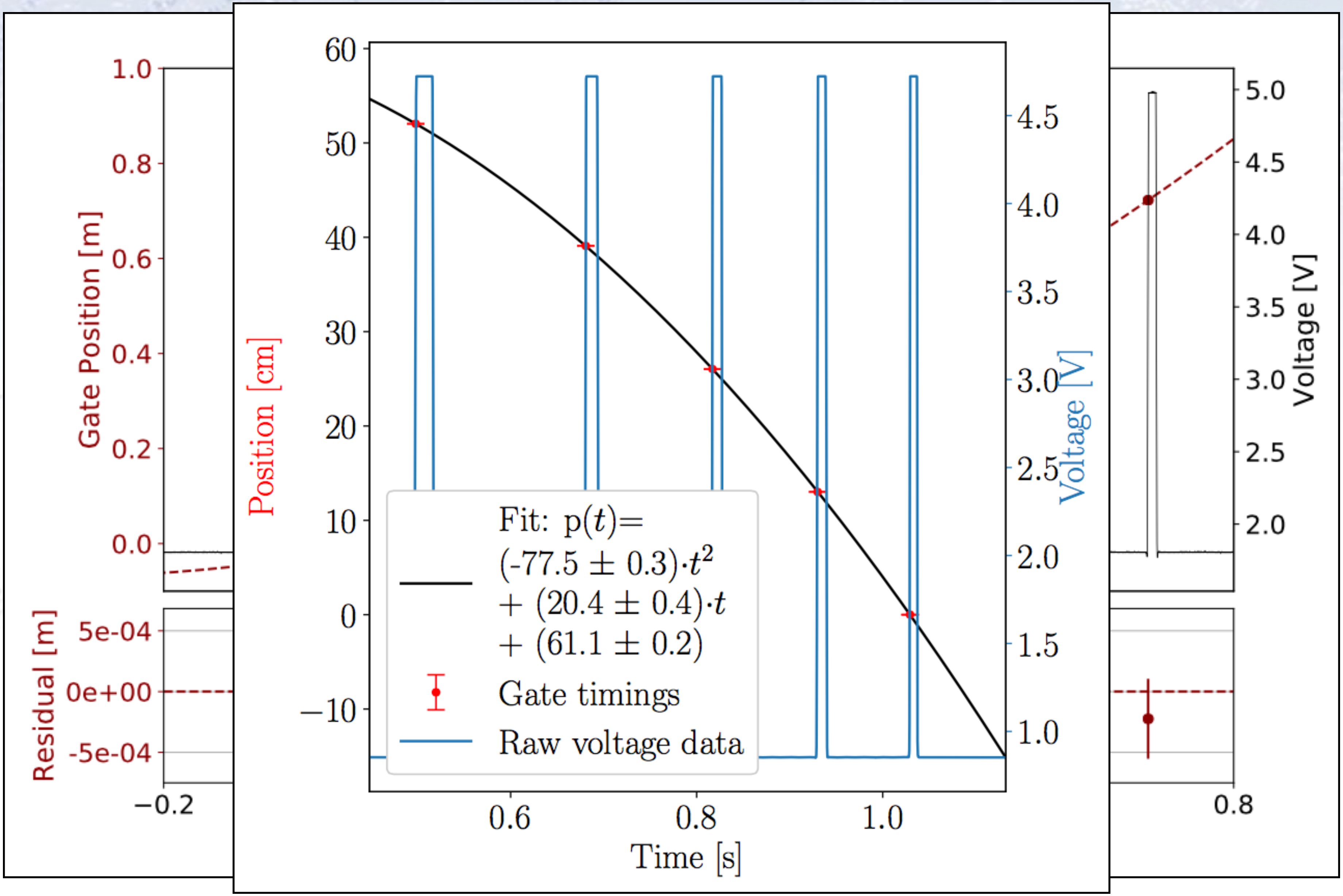
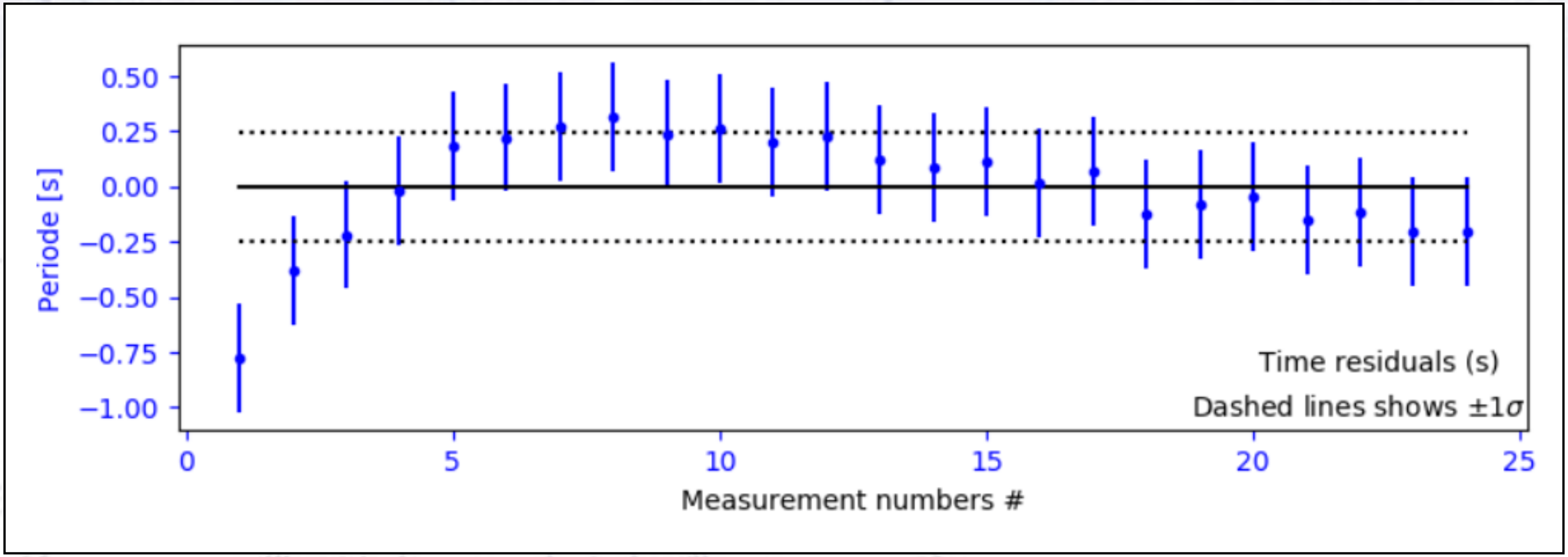


Great figures

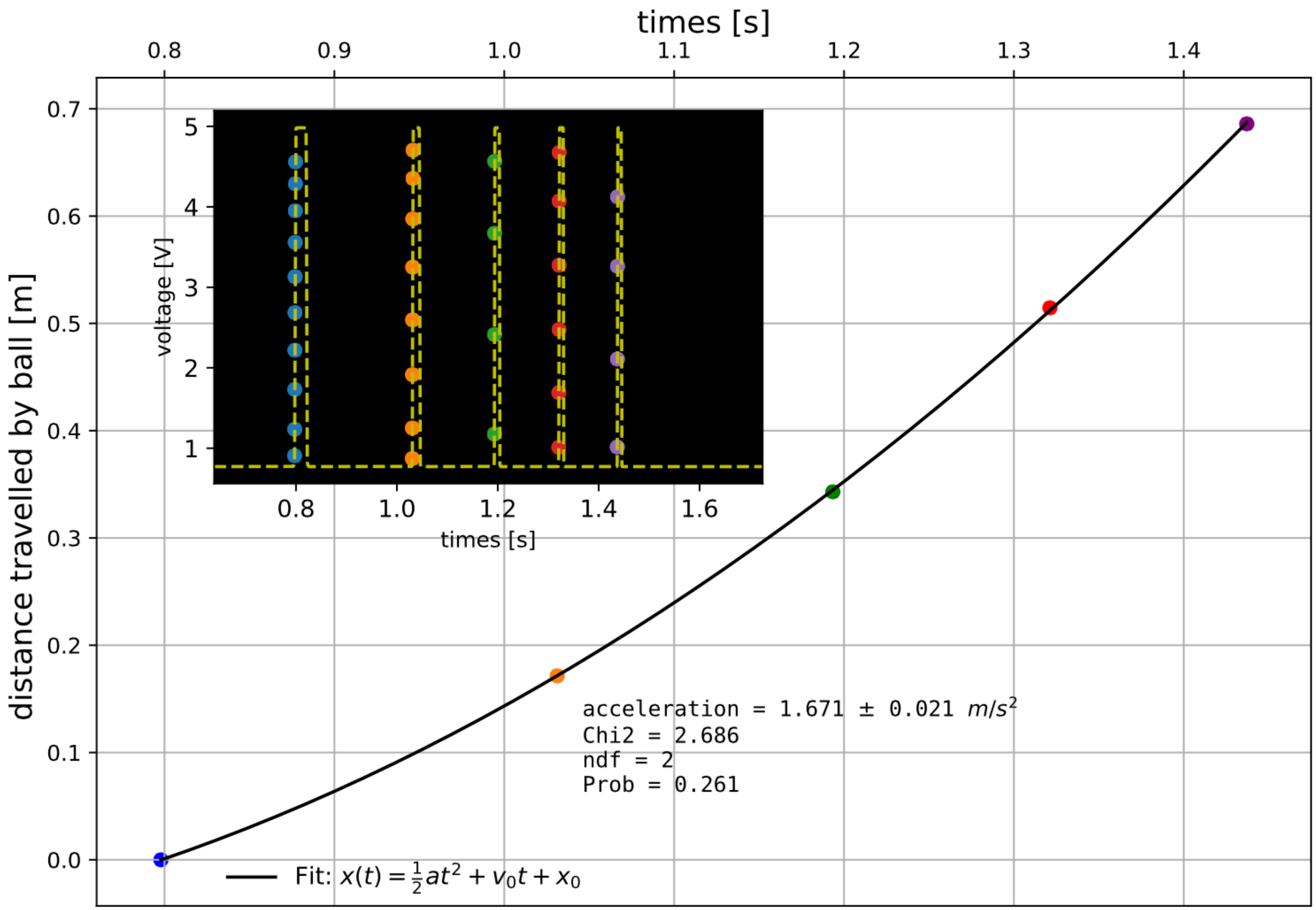
I like this figure….

It argues very strongly, why this data was discarded.



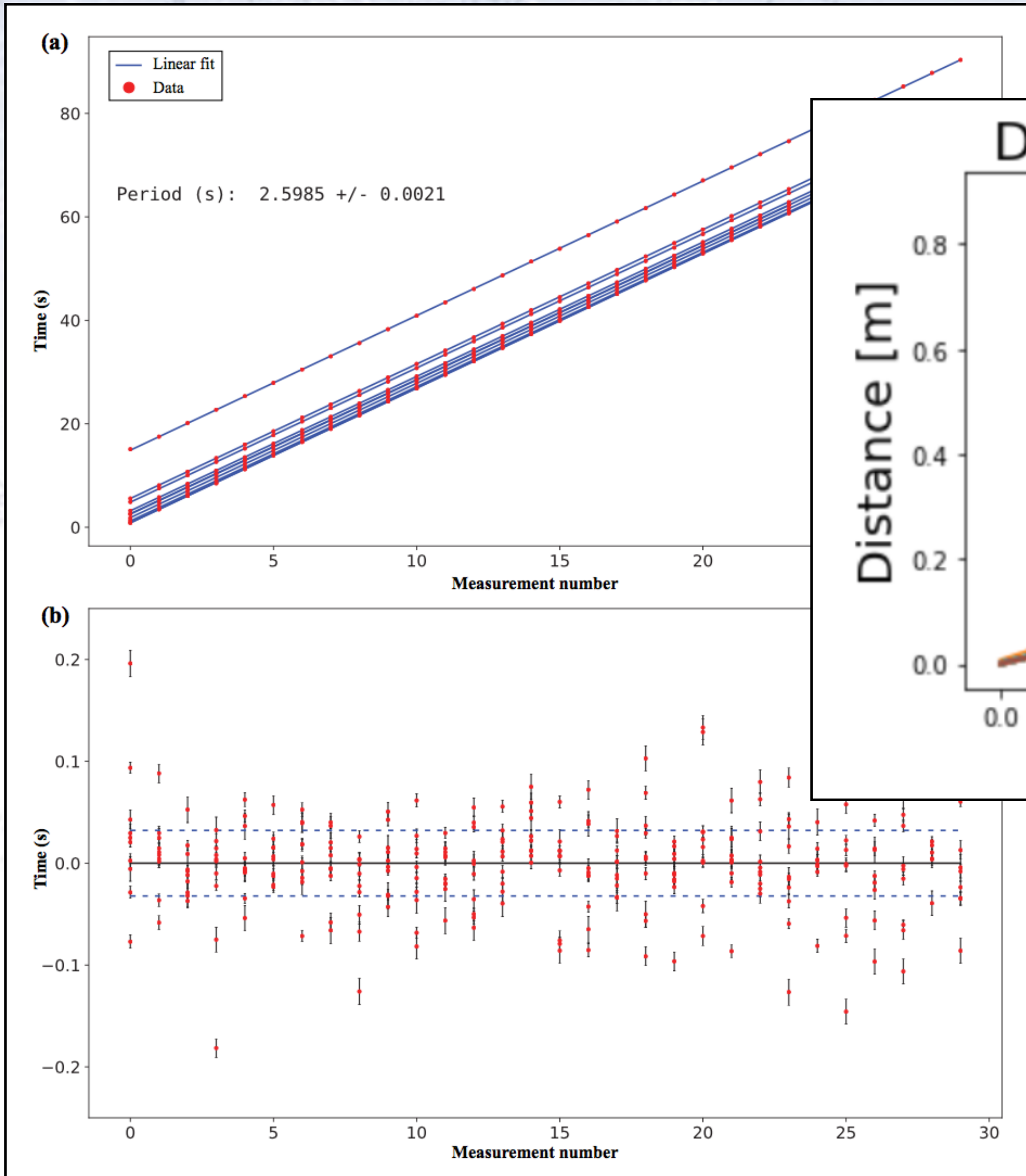
While such plots are not easy to put into publications, they server very well in the appendix of a thesis.

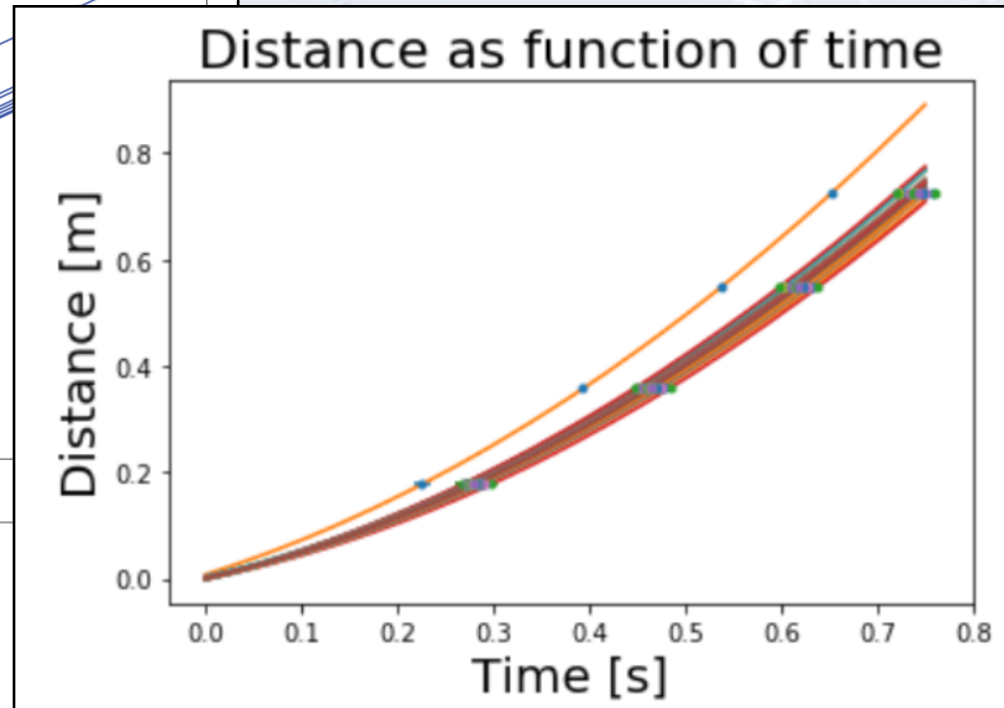
Great figures



	1.	2.	3.	4.	5.
Time error [s]	0.00372	0.00307	0.00211	0.0028	0.00241
Distance error [m]	0.002	0.002	0.002	0.002	0.002

Great figures, but…





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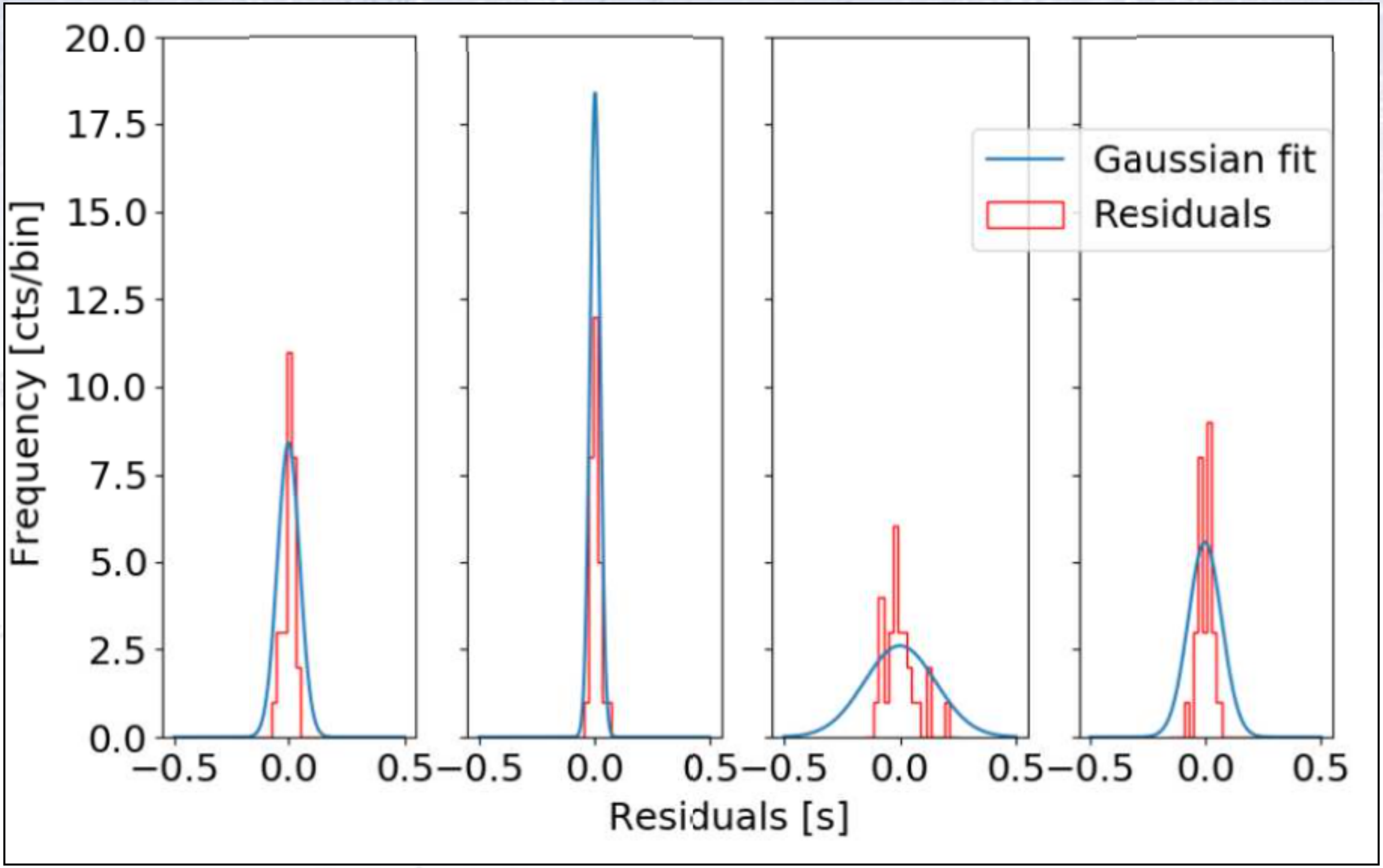

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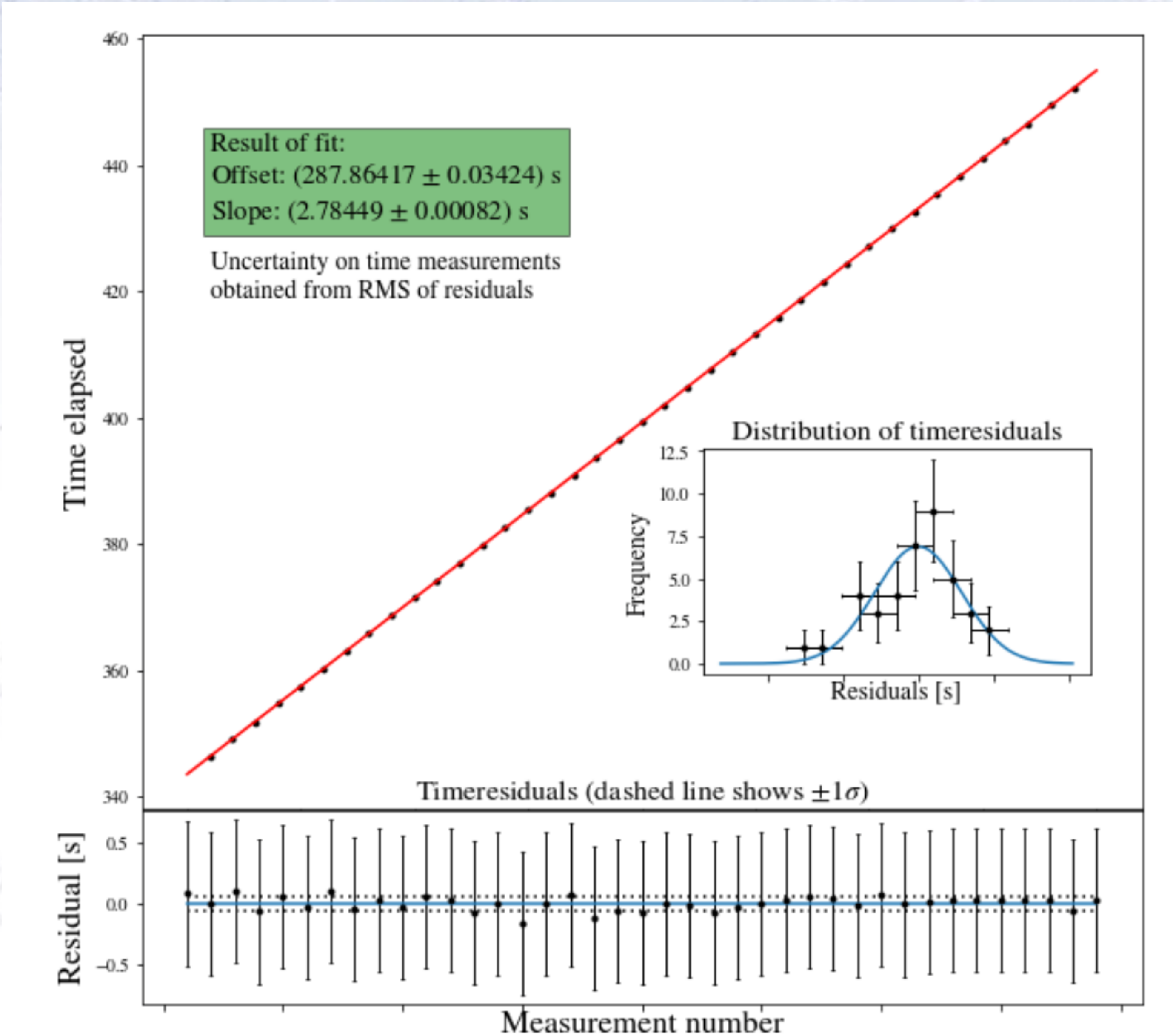


Great tables

Pendulum		
Variable	Value	Statistical error
Length L_1	2.6907 m	0.0017 m
L_2	2.6902 m	0.0017 m
L_3	2.6922 m	0.0017 m
L_4	2.6921 m	0.0017 m
Period T_1	3.2879 s	0.0012 s
T_2	3.2881 s	0.0012 s
T_3	3.2872 s	0.0004 s
T_4	3.2900 s	0.0011 s

Variable	Mean	Statistical error	RMS	χ^2	χ^2_{prob}
T	3.2876s	0.0004s	0.0006s	5.85	0.12
L	2.6913m	0.0009m	0.0005m	1.02	0.80

Great tables

Pendulum

Variable	Mean	Statistical error	RMS	χ^2	χ^2_{prob}
T	3.2876s	0.0004s	0.0006s	5.85	0.12
L	2.6913m	0.0009m	0.0005m	1.02	0.80

g - Pendulum

Variable	Value	Statistical error
g	9.830m/s^2	0.004m/s^2
Err. cont. L	0.003m/s^2	
Err. cont. T	0.002m/s^2	

Great table, but…

The two bottom values are probably correlated, and thus not fit for a weighted mean.

	Value	Con. σ_g	Con. σ_g, flipped
a [m/s^2]	1.479 ± 0.001	$0.008\ [m/s^2]$	-
a_F [m/s^2]	1.542 ± 0.003	-	$0.015\ [m/s^2]$
θ [deg]	13.99 ± 0.02	$0.015\ [m/s^2]$	$0.014\ [m/s^2]$
$\Delta\theta$ [deg]	-0.298 ± 0.013	$0.008\ [m/s^2]$	$0.008\ [m/s^2]$
r [mm]	6.374 ± 0.005	$0.001\ [m/s^2]$	$0.001\ [m/s^2]$
d [mm]	6.025 ± 0.005	$0.002\ [m/s^2]$	$0.002\ [m/s^2]$

Result	Value	Value, flipped	Weighted mean
g [m/s^2]	9.470 ± 0.019	9.47 ± 0.02	9.470 ± 0.014

Great table, but…

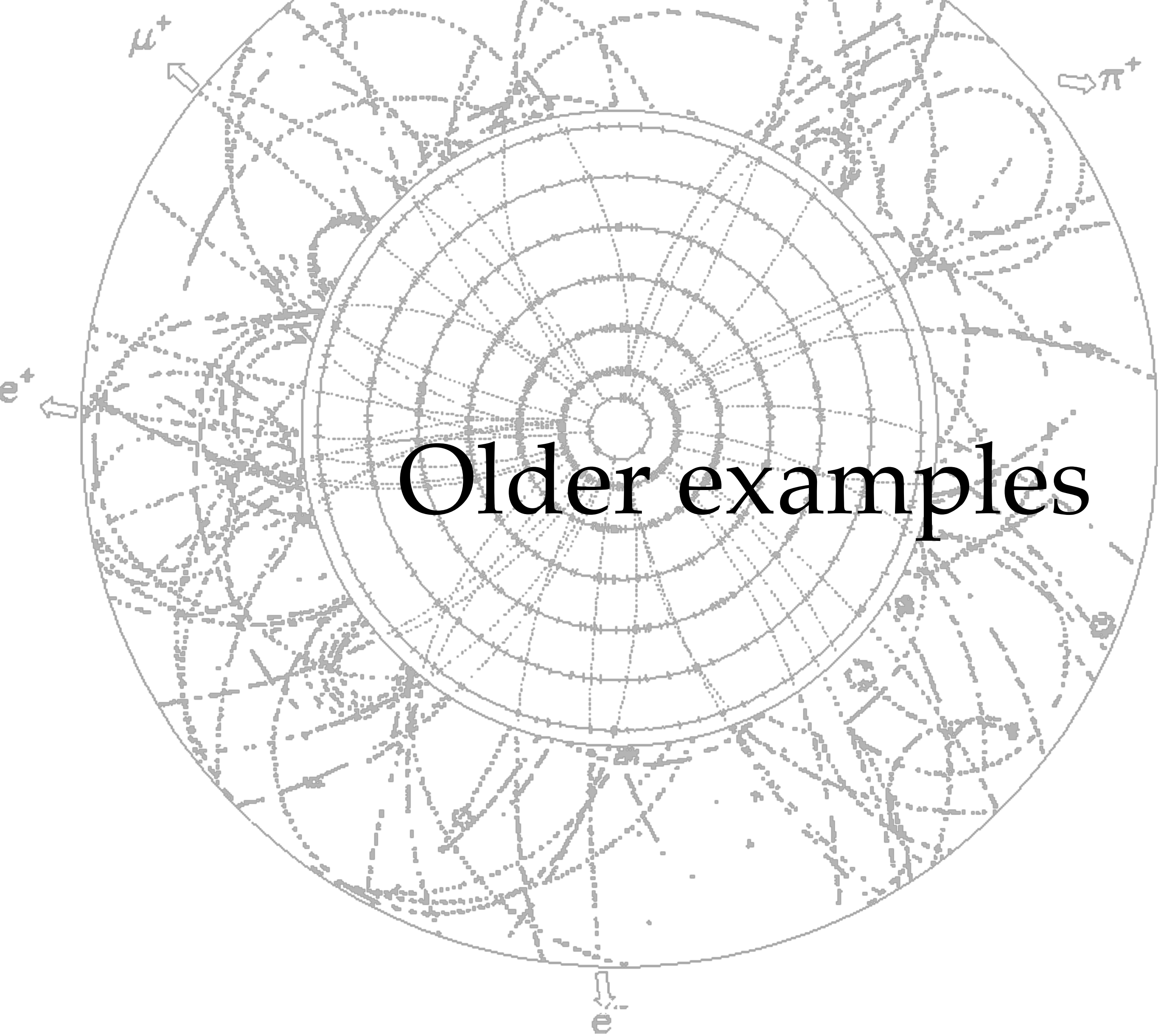
I'm quite convinced that the three Ball-on-Incline experiments are very correlated, and hence their combination is not that straight forward:

Final g Results		
Pendulum g (laser)	$g = (9.87 \pm 0.07)$	m/s^2
Pendulum g (tape measure)	$g = (9.872 \pm 0.007)$	m/s^2
Ball on incline g ($\rho = 0.0$)	$g = (9.8 \pm 0.9)$	m/s^2
Ball on incline g ($\rho = 0.5$)	$g = (9.8 \pm 0.9)$	m/s^2
Ball on incline g ($\rho = 1.0$)	$g = (9.8 \pm 0.8)$	m/s^2
Weighted mean g pendulum	$g = (9.872 \pm 0.007)$	m/s^2
Weighted mean g ball on incline	$g = (9.8 \pm 0.5)$	m/s^2
Weighted mean g total	$g = (9.872 \pm 0.007)$	m/s^2

Even great equations!

$$
\begin{aligned}
\sigma_g^2 = \sigma_{\Delta\theta}^2 & \frac{\left(\frac{8\,R^2}{4\,R^2-d^2}+5\right)^2 a^2 \cos(\theta+\Delta\theta)^2}{25\,\sin(\theta+\Delta\theta)^4} \Bigg\} I_{\Delta\theta}^2 \\
& + \sigma_\theta^2 \frac{\left(\frac{8\,R^2}{4\,R^2-d^2}+5\right)^2 a^2 \cos(\theta+\Delta\theta)^2}{25\,\sin(\theta+\Delta\theta)^4} \Bigg\} I_\theta^2 \\
& + \sigma_R^2 \frac{256\left(\frac{4\,R^3}{(4\,R^2-d^2)^2}-\frac{R}{4\,R^2-d^2}\right)^2 a^2}{25\,\sin(\theta+\Delta\theta)^2} \Bigg\} I_R^2 \\
& + \sigma_d^2 \frac{256\,R^4 a^2 d^2}{25\,(4\,R^2-d^2)^4 \sin(\theta+\Delta\theta)^2} \Bigg\} I_d^2 \\
& + \sigma_a^2 \frac{\left(\frac{8\,R^2}{4\,R^2-d^2}+5\right)^2}{25\,\sin(\theta+\Delta\theta)^2} \Bigg\} I_a^2
\end{aligned}
$$

Older examples



Examples from other reports

Using different ball sizes gave variations in the result (10, 12.7, and 15 mm).

$$g_{Big} = (9.865 \pm 0.040_{stat} \pm 0.001_{sys}) \;\; \mathrm{m/s}^2,$$

$$g_{Medium} = (9.822 \pm 0.042_{stat} \pm 0.001_{sys}) \;\; \mathrm{m/s}^2,$$

$$g_{Small} = (9.741 \pm 0.055_{stat} \pm 0.001_{sys}) \;\; \mathrm{m/s}^2,$$

Table of (changing) impact is GREAT:

Parameter	Big Ball	Medium Ball	Small Ball
a	0.024	0.024	0.022
θ	0.030	0.030	0.030
$\Delta\theta$	0.0014	0.0014	0.0013
D	0.003	0.007	0.019
d	0.010	0.016	0.035
Total	0.040	0.042	0.055

Examples from other reports

The table shows a great example of reporting the inputs for the measurement of g, along with their impact on the final precision.

The figure shows the result of 10 period measurements and their weighted average, where the ChiSquare reveals, that one measurement seems flawed.

Variable	Value	Stat. error	Impact on g
Pendulum			
Period T	8.5373 s	0.0005 s	0.0011 $\frac{m}{s^2}$
Length L	18.1363 m	0.0002 m	0.0001 $\frac{m}{s^2}$
Gravity g	9.8319 $\frac{m}{s^2}$	0.0011 $\frac{m}{s^2}$	
Ball on Incline			
Acc. a_{LR}	0.7193 $\frac{m}{s^2}$	0.0018 $\frac{m}{s^2}$	0.02 $\frac{m}{s^2}$
Acc. a_{RL}	0.7851 $\frac{m}{s^2}$	0.0019 $\frac{m}{s^2}$	0.02 $\frac{m}{s^2}$
Angle θ	7.303°	0.002°	*
$\Delta\theta$	0.321°	0.013°	*
Radius R	0.0071 m	0.0002 m	0.04 $\frac{m}{s^2}$
Width d	0.0064 m	0.0002 m	0.05 $\frac{m}{s^2}$
Gravity g	8.89 $\frac{m}{s^2}$	0.06 $\frac{m}{s^2}$	
Resulting g	**9.8316**	**$\pm$0.0011 $\frac{m}{s^2}$**	

Table III: Final results from both experiments. *θ and $\Delta\theta$ have a combined impact on g, which is calculated as 0.02°.

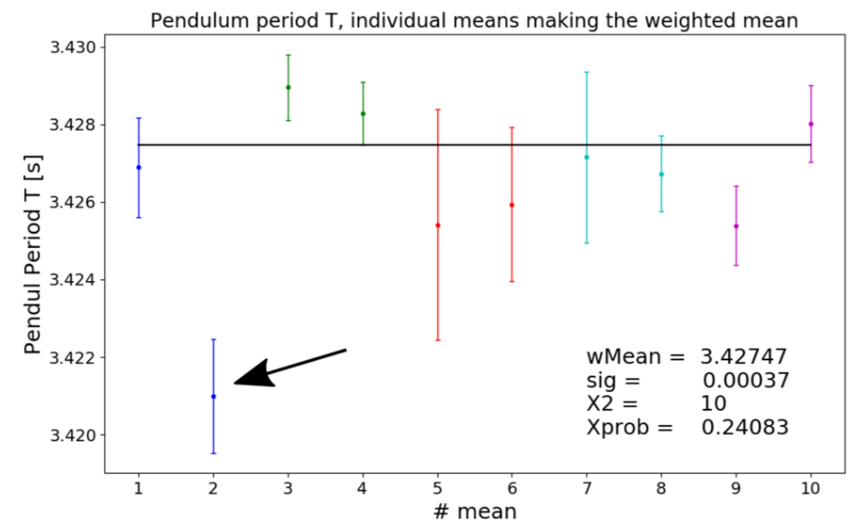


Figure 3: Period T, colours indicate different peoples individual mean, and the black line is the weighted mean of T=3.4275 $\pm$ 0.0004. The highlighted data point is 4.4σ away from the mean, we excluded this from our data analysis. We note the χ^2 is within our 9 Degrees of Freedom. The χ^2prob. is the probability of getting a worse fit, 24% is somewhat low but not low enough to raise any alarms

Examples from other reports

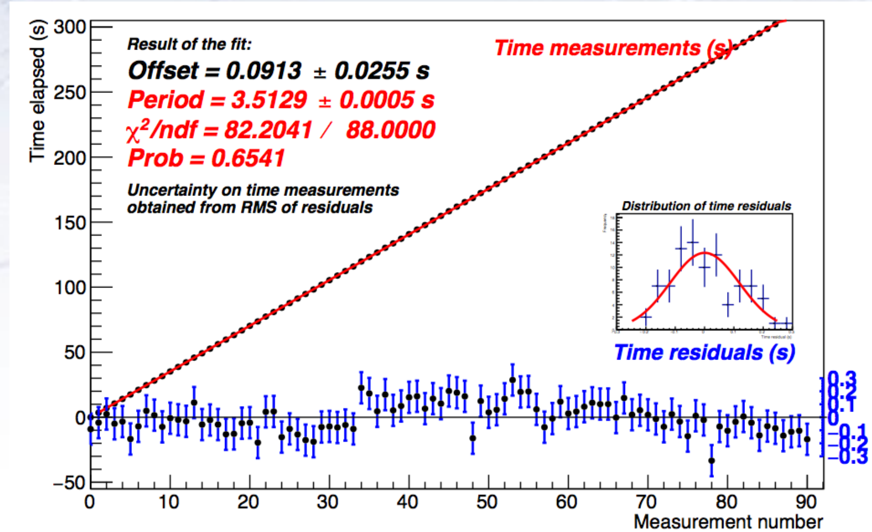
Here are some very nice tables and figures:

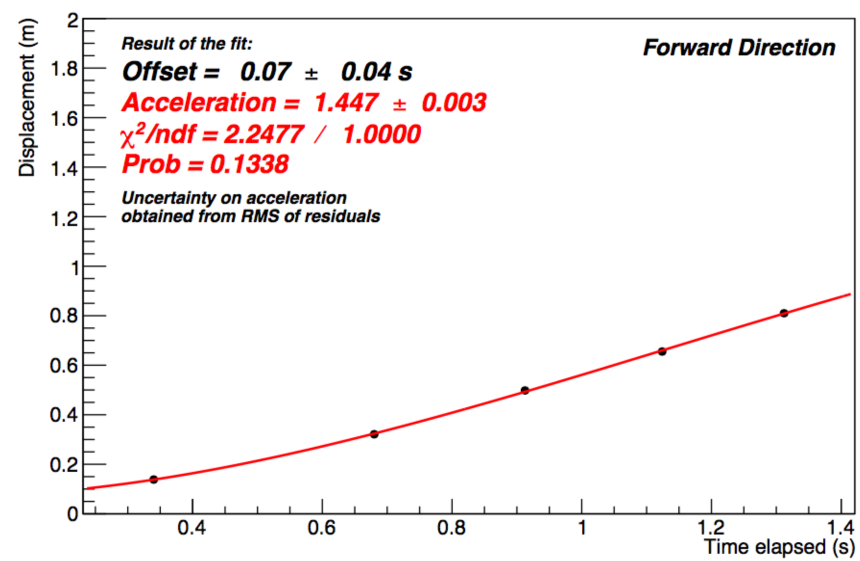
Variable	Value & error	Impact on g	Comment
Pendulum			
Period	$3.513 \pm 0.001 \pm 0.006$ s	0.034 m/s^2	good χ^2
Length	$3.048 \pm 0.005 \pm 0.01$ m	0.036 m/s^2	only laser
Gravity	9.75 ± 0.05 m/s^2		fair result

Table I: Different variables and their uncertainty from the pendulum experiment used to determine g. See Fig. 2 for χ^2 and more goodness of fit comments

Variable	Value & error	Impact on g	Comment
Ball on Incline			
Acceleration a	1.447 ± 0.003 m/s^2	0.023 m/s^2	forward
Incliend plane θ	11 ± 1 °	1.085 m/s^2	impact of
Table $\Delta\theta$	0.42 ± 0.04 °	m/s^2	total angle
Ball radius R	15 ± 1 mm	0.0018 m/s^2	
track width d	6 ± 1 mm	0.0046 m/s^2	
Gravity	11.16 ± 1.09 m/s^2	m/s^2	poor result

Table II: Different variables and their uncertainty from the ball on incline experiment used to determine g. For this experiment i struggled to find quantitative estimates on the systematic errors, so the table only lists the total error. See Fig. 3 for χ^2 and more goodness of fit comments





Examples from reports

Which method is best for obtaining good precision on the pendulum timing:

- When the pendulum is **at speed in "the middle" (resting position)**?
- When the pendulum is **still on "the side" (outer position)**?

Examples from reports

Which method is best for obtaining good precision on the pendulum timing:

- When the pendulum is **at speed in "the middle" (resting position)**?
- When the pendulum is **still on "the side" (outer position)**?

That can be measured:

Variable	Value	Stat. error
First approach: Resting pos.		
μ_T [s]	3.4306	0.0019
g [m/s^2]	9.8098	0.0035
Second approach: Outer pos.		
μ_T [s]	3.4345	0.0071
g [m/s^2]	9.7826	0.0113

Examples from reports

Careful of combining (unknowingly) correlated measurements:

Experiments	a m/s^2	Chi2	Probability
1 (Regular)	0.756 ± 0.048	1.1689	0.5574
2 (Regular)	0.737 ± 0.061	1.1244	0.5699
3 (Regular)	0.741 ± 0.056	1.1459	0.5639
4 (Regular)	0.754 ± 0.048	1.1987	0.5492
5 (Inverted)	0.667 ± 0.042	1.1781	0.5548
6 (Inverted)	0.664 ± 0.043	1.1987	0.5492
7 (Inverted)	0.651 ± 0.052	1.1210	0.5709
Weighted mean of a in normal direction 0.749 ± 0.026 m/s^2			
Weighted mean of a in inverted direction 0.662 ± 0.026 m/s^2			

The main uncertainty in the accelerations are the lengths, which are common!!!

On weighted means and Chi2

Measurement nr.	Value	Error
Length of line before experiment		
Length 1	195.45 cm	0.05 cm
Length 2	195.60 cm	0.05 cm
Length 3	195.45 cm	0.05 cm
Length 4	195.55 cm	0.05 cm
Length 5	195.50 cm	0.05 cm
Resulting length	**195.51 cm**	**0.06 cm**
Length of line after experiment		
Length 1	195.32 cm	0.05 cm
Length 2	195.40 cm	0.05 cm
Length 3	195.51 cm	0.05 cm
Length 4	195.50 cm	0.05 cm
Length 5	195.25 cm	0.05 cm
Resulting length	**195.40 cm**	**0.10 cm**

These values are so well measured, and agree well.

But this is not tested, nor are they combined with a weighted mean.

The second set agrees less well.

Fortunately, this was at least commented on.

Examples from reports

Remember to only put 1-2 significant digits on the error, and then the SAME number of digits on the result.

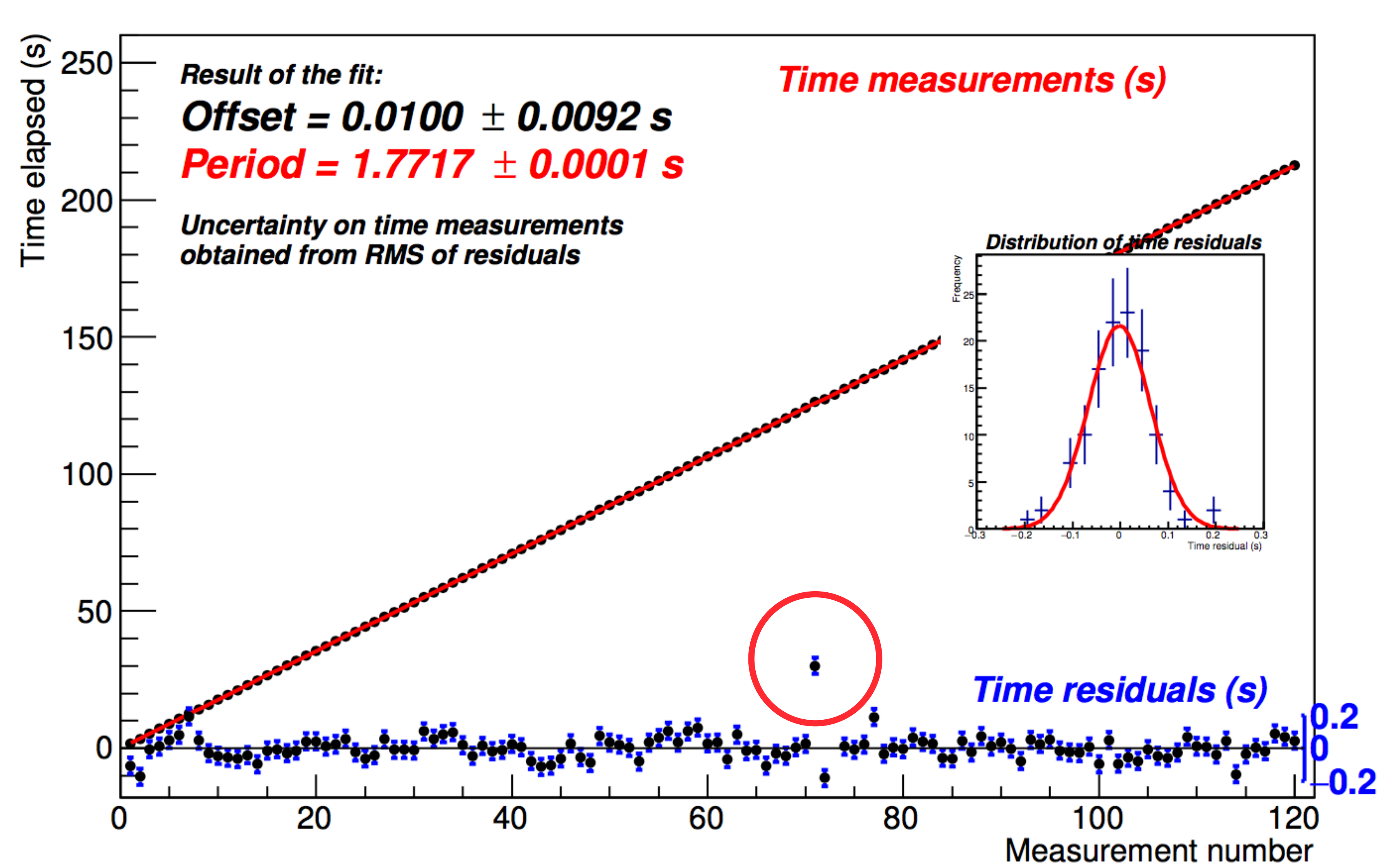
Exp.	a_h	σ_{ah}	a_v	σ_{av}
1	1.541 m/s^2	0.01170 m/s^2	1.473 m/s^2	0.008515 m/s^2
2	1.543 m/s^2	0.01176 m/s^2	1.472 m/s^2	0.008517 m/s^2
3	1.543 m/s^2	0.01173 m/s^2	1.475 m/s^2	0.008550 m/s^2
4	1.541 m/s^2	0.01172 m/s^2	1.474 m/s^2	0.008533 m/s^2
5	1.539 m/s^2	0.01175 m/s^2	1.474 m/s^2	0.008544 m/s^2
6	1.545 m/s^2	0.01177 m/s^2	1.473 m/s^2	0.008521 m/s^2
7	1.543 m/s^2	0.01173 m/s^2	1.471 m/s^2	0.008541 m/s^2
8	1.540 m/s^2	0.01171 m/s^2	1.474 m/s^2	0.008513 m/s^2
9	1.543 m/s^2	0.01175 m/s^2	1.471 m/s^2	0.008498 m/s^2
10	discarded	discarded	1.475 m/s^2	0.008529 m/s^2
Result	**1.54** m/s^2	**0.0039** m/s^2	**1.47** m/s^2	**0.0027** m/s^2
χ^2/Ndf	0.20/8		0.27/9	
Prob.	1		1	

Tool	Height	Statistical error	Systematic error
Tomme	313.29 cm	0.1331 cm	0.05 cm
Lineal	311.88 cm	0.1664 cm	0.05 cm
Vinkel	311.80 cm	0.1464 cm	0.05 cm
Result	**312.46 cm**	**0.085 cm**	0.0162 cm
χ^2/Ndf	66.19/2		
Prob.	$4.234 \cdot 10^{-15}$		

Furthermore, remember to COMMENT on your Chi2 probabilities!
This is very important, because a test is worthless, if not acted upon.

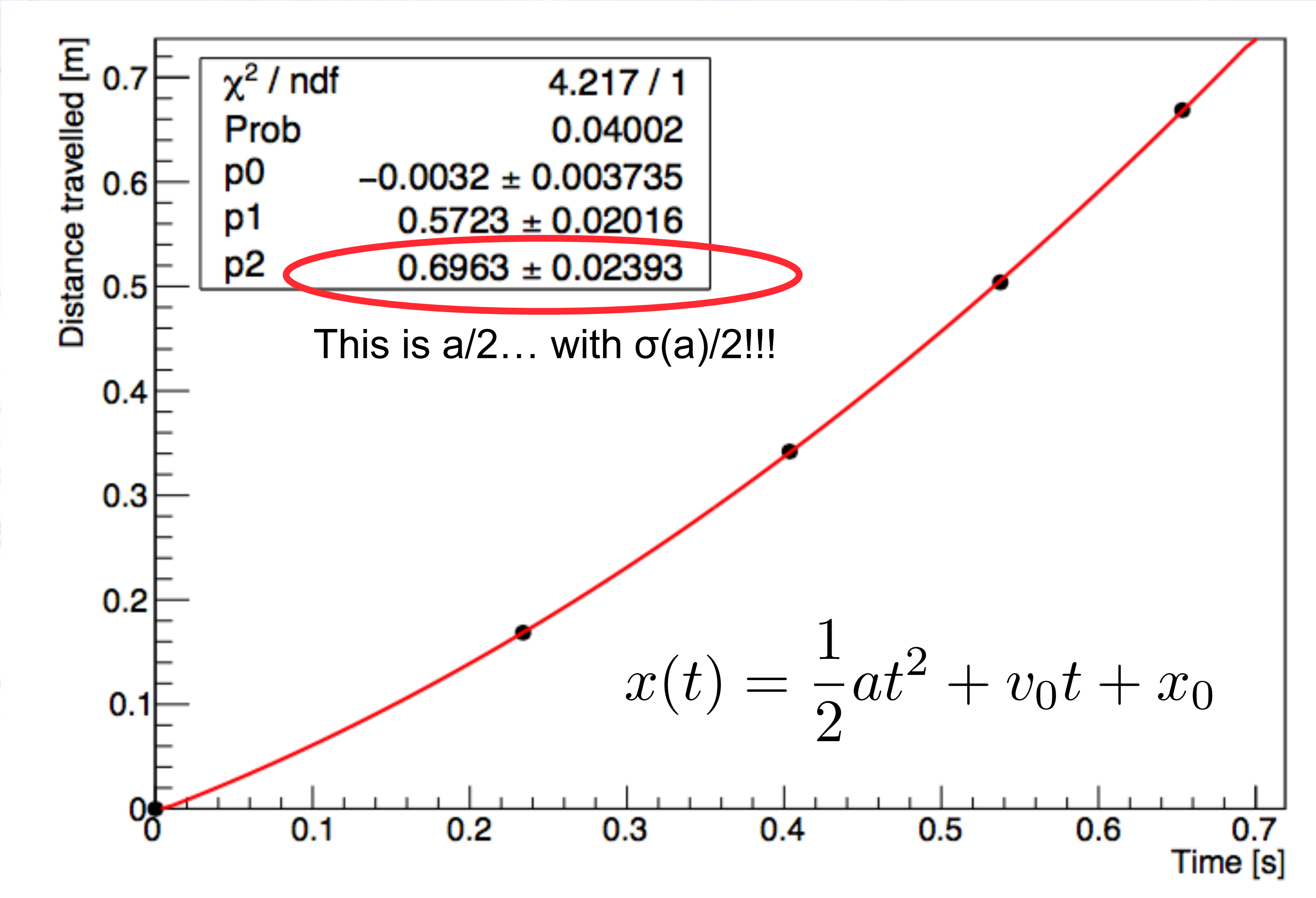
Examples from reports

Sometimes, single measurement are clearly off… consider/comment on these!

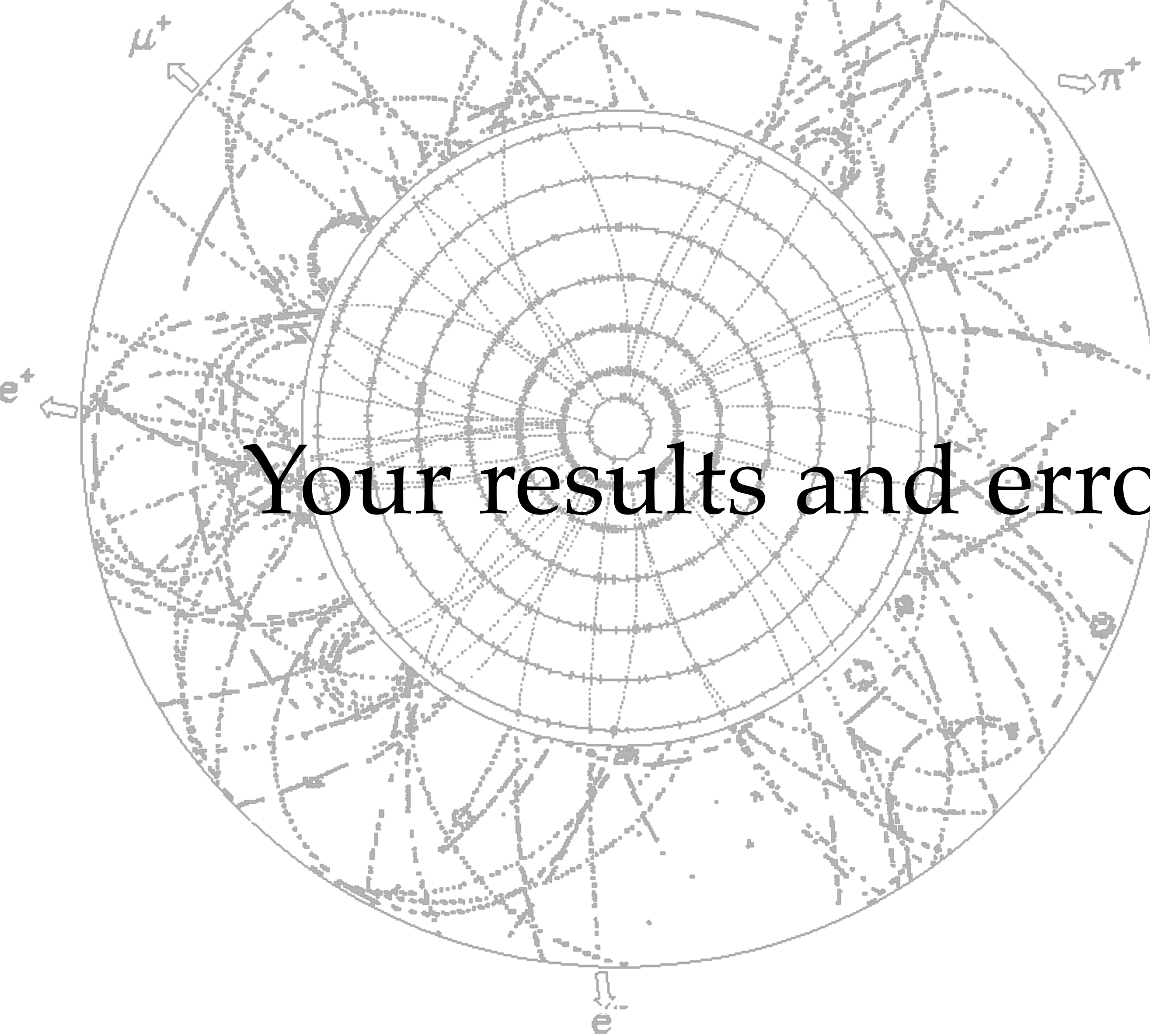


Examples from reports

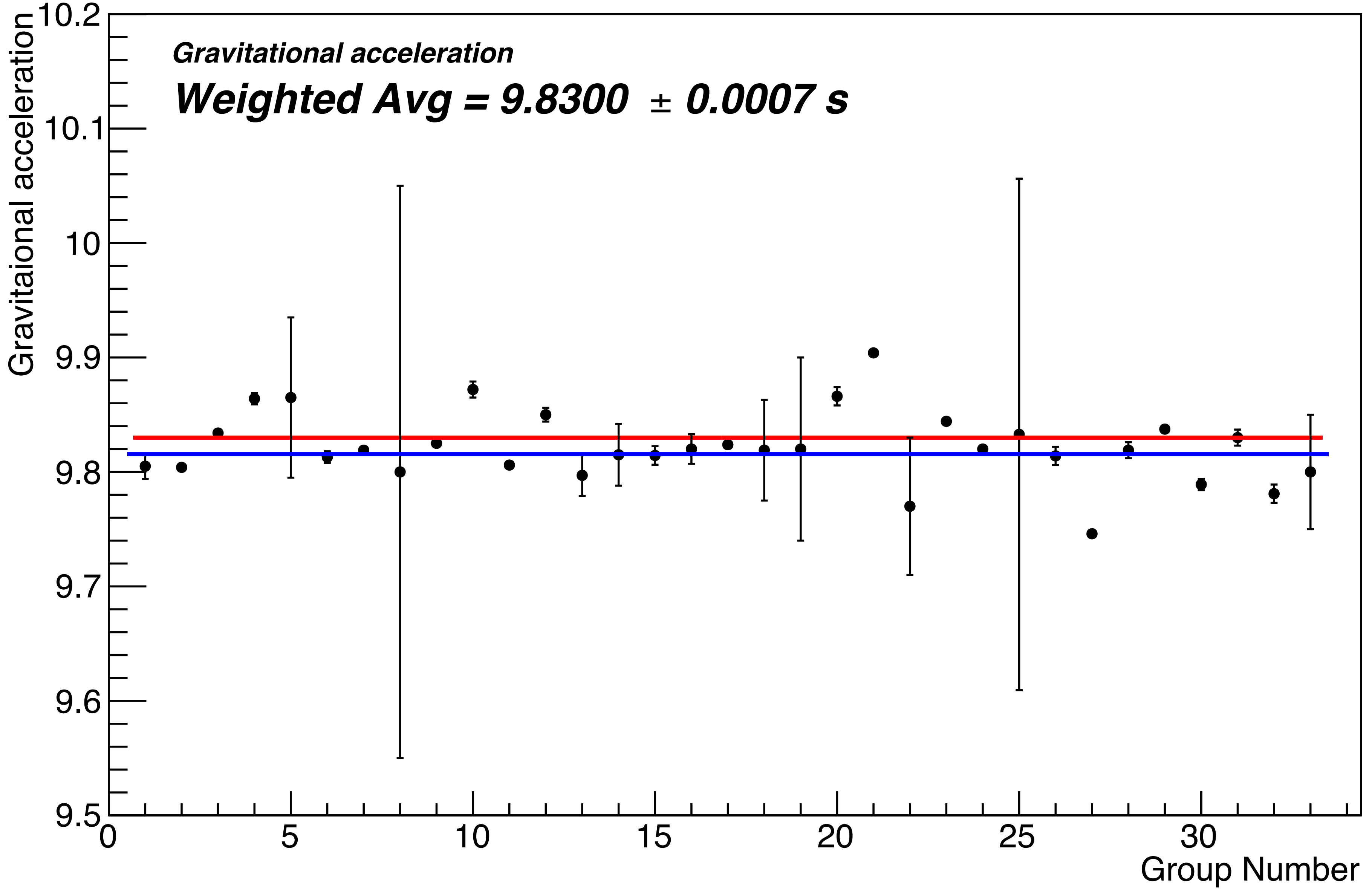
Think (a lot) about constraining points in the fit (i.e. by giving 0.0 in error).
Also, it is a good idea to fit with all constants in place in formula, i.e. factor 1/2.



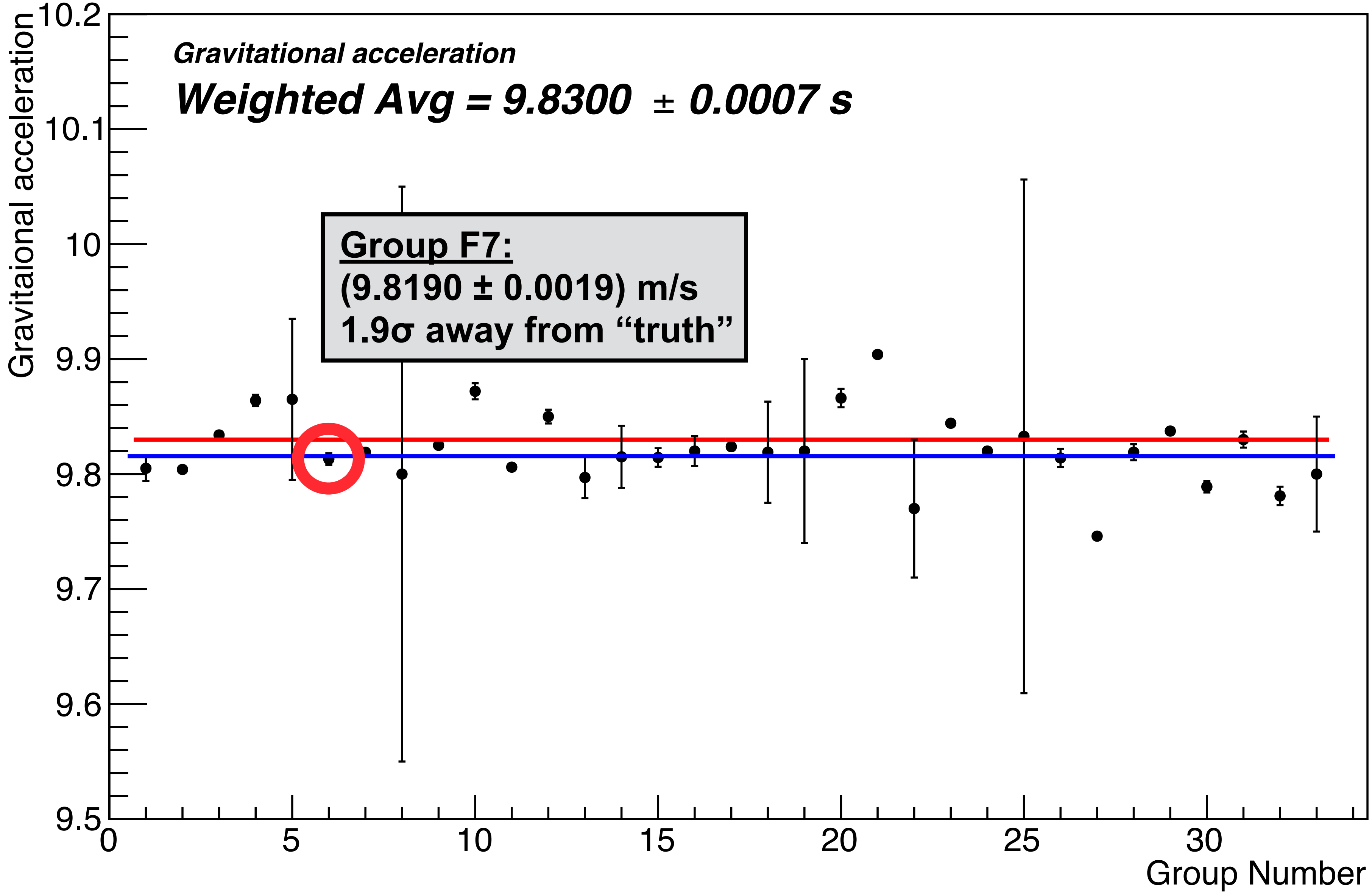
Your results and errors



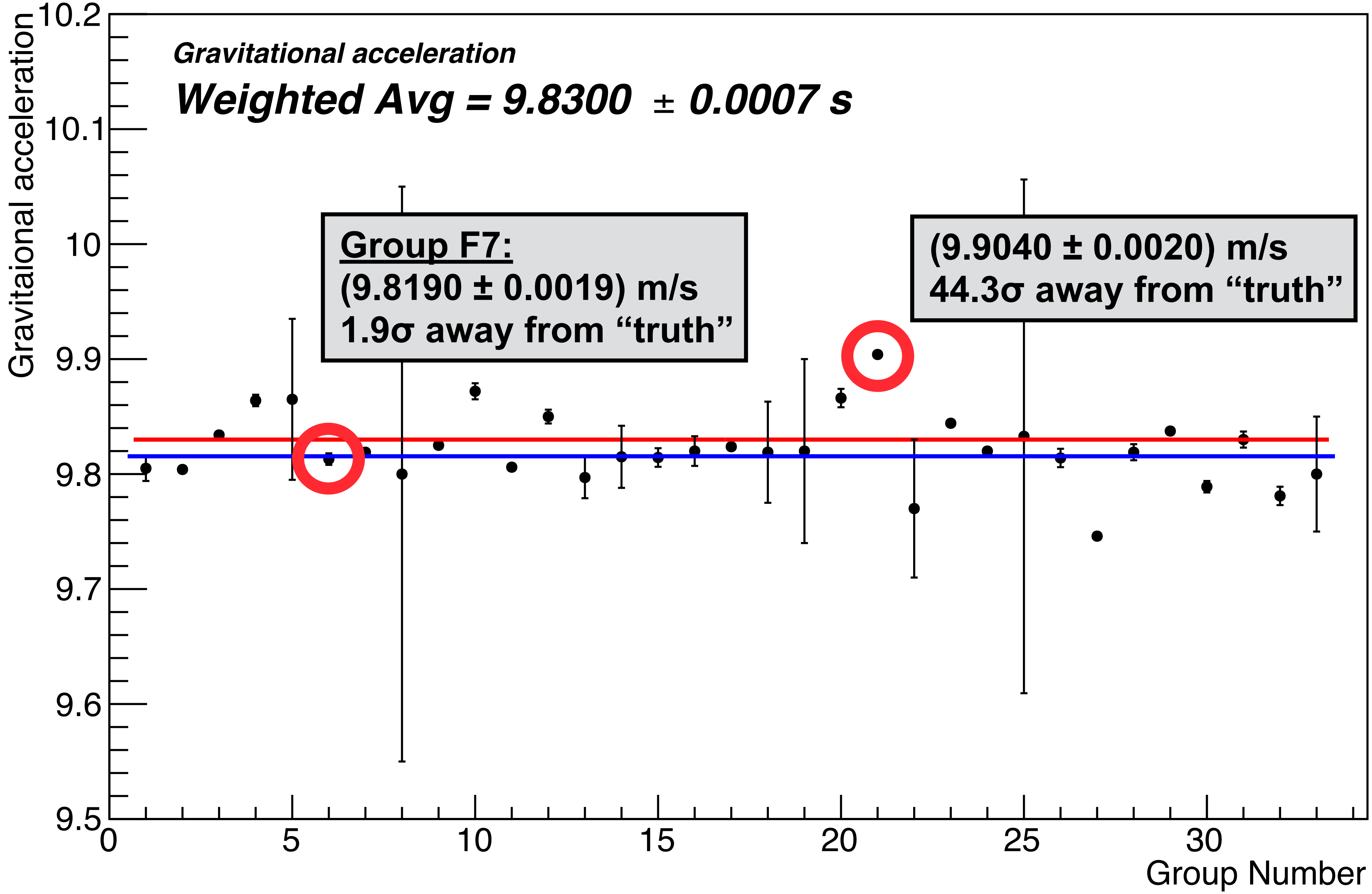
Your results - pendulum



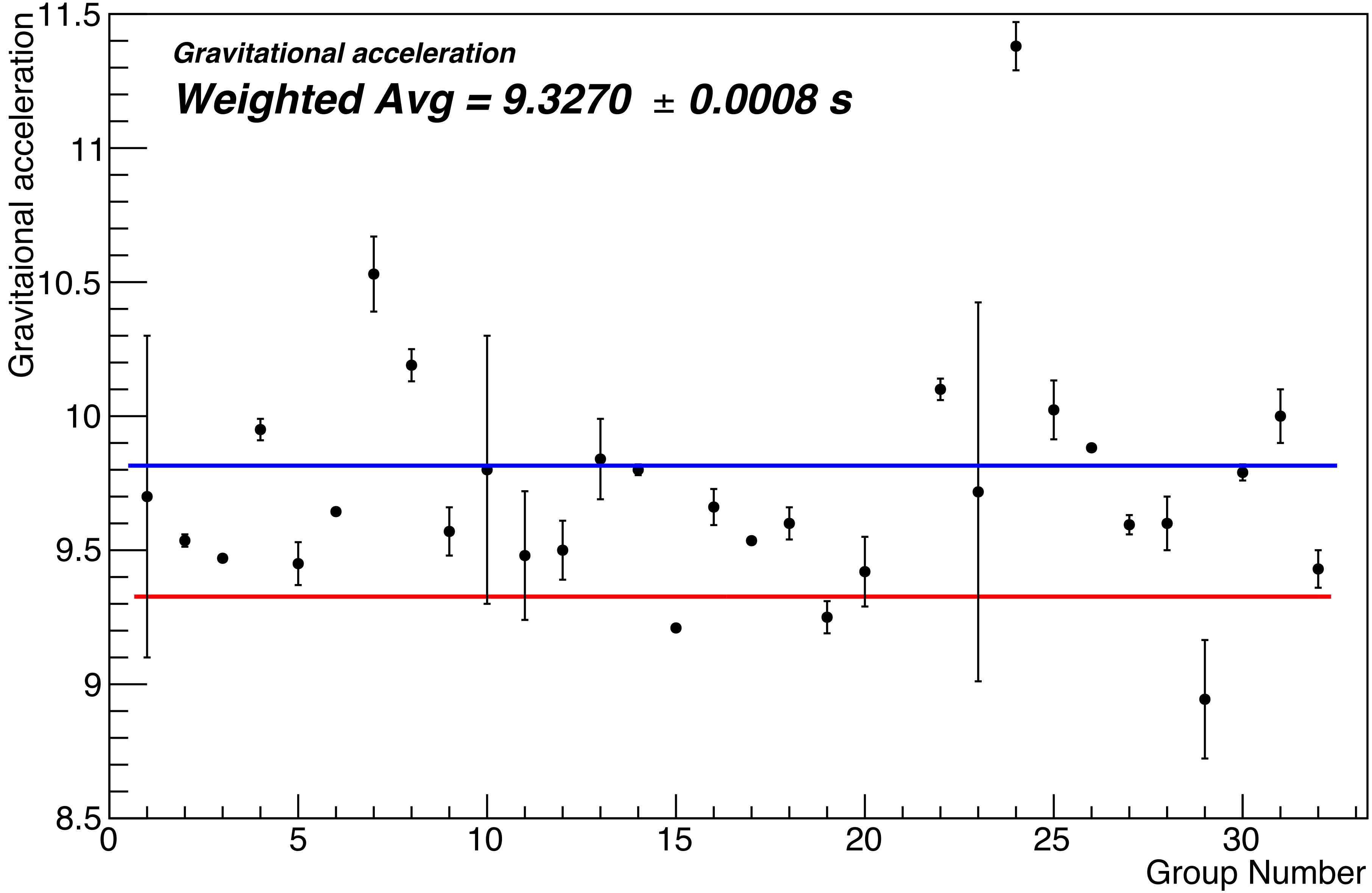
Your results - pendulum



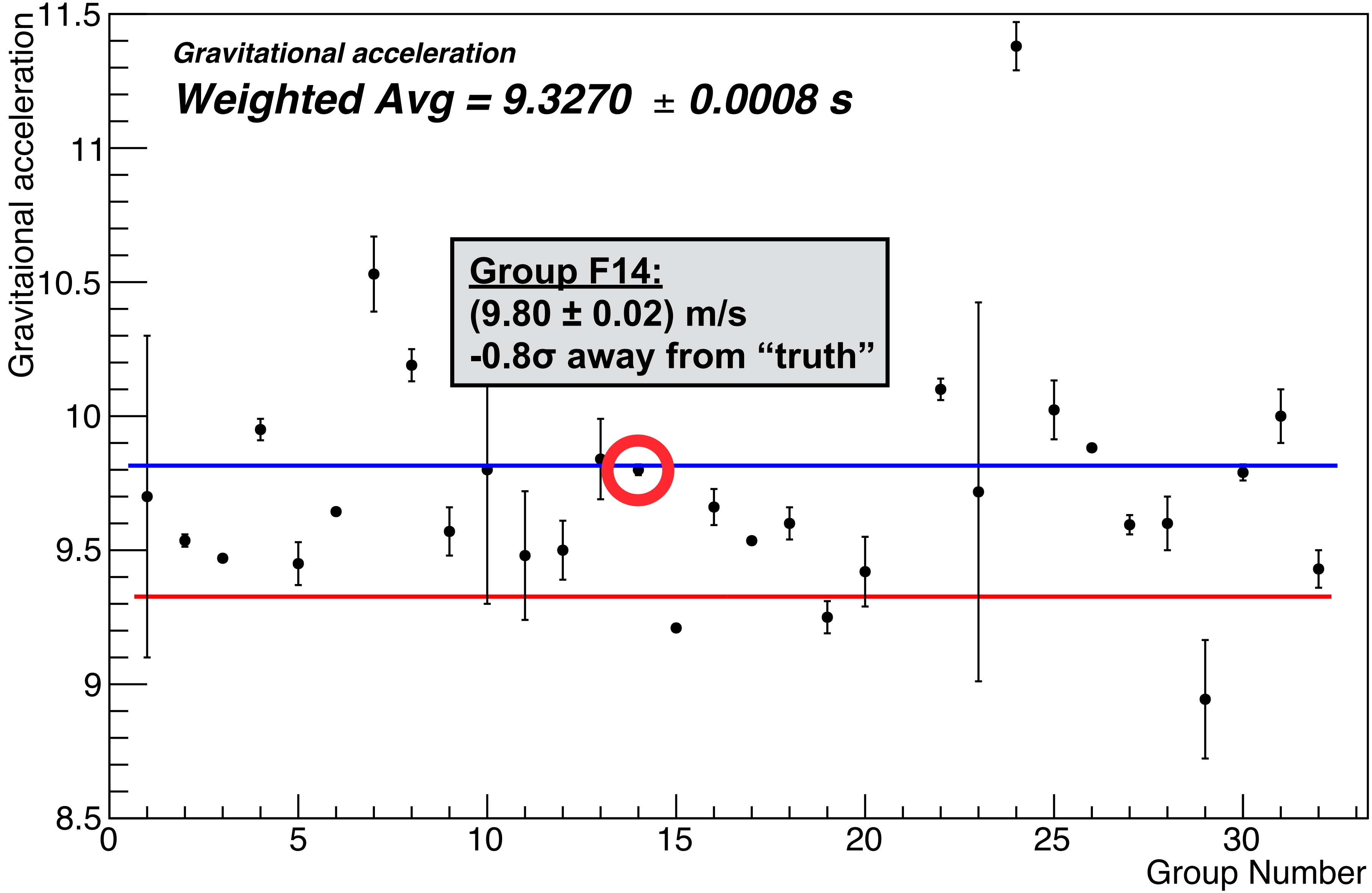
Your results - pendulum



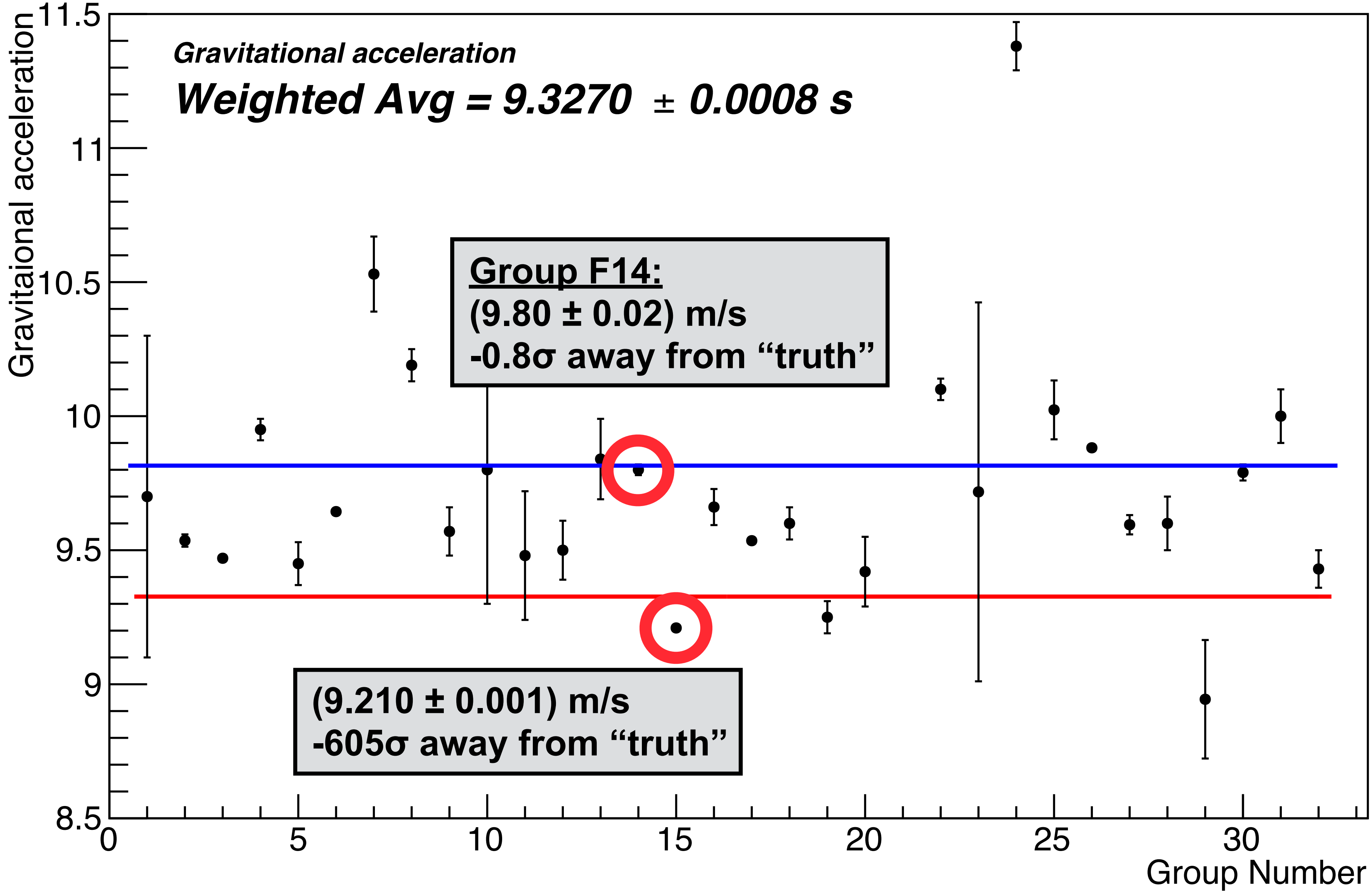
Your results - Ball on Incline



Your results - Ball on Incline



Your results - Ball on Incline



Various uncertainties

Group:	sT (s)	sL (m)	g_pend	sAcc.	sTheta	sDtheta	g_ball	Points
1:	0.007	0.0009	9.84+-0.04	0.0009	0.05	-1	7.10+-0.04	60
2:	0.0005	0.00002	9.8319+-0.0011	0.0019	0.10	0.013	8.89+-0.06	90
3:	0.0002	0.0015	9.815+-0.005	0.001	0.10	-1	6.55+-0.13	70
4:	0.0007	0.0011	9.759+-0.006	0.0006	0.40	0.40	10.6 +-0.80	85
5:	0.02	0.003	9.8+-0.1	0.0017	0.5	-1	9.7+-0.3	80
6:	0.0000003	0.001	10.31+-0.0018	0.03	-1	0.36	3.9+-0.2	60
7:	0.003	0.0004	9.824+-0.003	0.003	0.5	-1	8.7+-0.5	100
8:	0.004	0.0008	9.831+-0.003	0.015	0.1	-1	9.4+-0.1	95
9:	0.0003	0.004	9.8324+-0.0002	0.004	0.1	-1	9.31+-0.10	95
10:	0.0008	0.007	9.8252+-0.0018	0.016	0.06	0.06	10.14+-0.05	85
11:	0.0003	0.0009	9.827+-0.003	0.09	0.06	0.06	9.48+-0.04	100
12:	0.005	0.018	9.79+-0.07	0.02	-	-1	9.59+-0.06	70
13:	0.004	0.0001	9.8169+-0.0014	0.0014	0.2	-1	6.8+-0.2	75
14:	0.001	0.0008	9.811+-0.003	0.001	0.03	-1	9.25+-0.03	95
15:	0.003	0.004	9.825+-0.003	0.03	0.13	0.08	8.9+-0.2	80
16:	0.0001	0.001	9.820+-0.002	-1	0.08	-1	9.809+-0.009	75
MJ:	0.0009	0.0014	9.821+-0.003	0.012	0.09	-1	8.84+-0.009	90
GH:	0.001	0.005	9.75+-0.05	0.003	1.0	0.04	11.16+-1.09	95

My best estimates of a "minimum" uncertainty:

2016:	0.0002s	0.0005m	0.002	0.005	0.20	0.02	0.05
2017:	0.00015s	0.0005m	0.0015	0.002	0.02 (trig) 0.10 (gonio)	0.02	0.05

g(Pend)	sigma_g(Pend)	sigma(T)	sigma(L)	g(Bol)	sigma_g(Bol)	sigma(a)	sigma(theta)	sigma(dtheta)
9.805	0.011	na	na	9.7	0.6	na	na	na
9.804	0.003	na	na	9.536	0.023	0.002	0.03	na
9.834	0.003	0.003	0.001	9.47	0.014	0.011	0.015	0.008
9.864	0.005	0.0005	0.005	9.95	0.04	0.007	0.04	na
9.865	0.07	0.07	0.003	9.45	0.08	0.02	0.08	na
9.813	0.005	na	na	9.644	0.002	na	na	na
9.819	0.0019	0.0005	0.0018	10.53	0.14	0.135	0.05	na
9.8	0.25	na	na	10.19	0.06	0.03	0.04	0.04
9.825	0.004	0.0004	0.0009	9.57	0.09	0.004	0.12	0.03
9.872	0.007	na	na	9.8	0.5	na	na	na
9.806	0.0017	0.0396	0.0005	9.48	0.24	0.0007	0.022	0.014
9.85	0.006	0.0008	0.0006	9.5	0.11	0.006	0.17	ng
9.797	0.018	0.0038	0.0012	9.84	0.15	0.0006	0.0042	0.0042
9.815	0.027	0.00043	0.18	9.8	0.02	0.002	0.018	0.012
9.8144	0.0081	0.0009	0.002	9.21	0.001	na	0.05	ng
9.82	0.0129	0.0021	0.0016	9.661	0.0673	0.0015	0.05	0.0029
9.8238	0.0028	0.0003	0.005	9.5355	0.0029	0.0114	0.08	ng
9.819	0.044	0.007	0.001	9.6	0.06	0.005	0.25	0.25
9.82	0.08	0.04	0.0004	9.25	0.06	0.005	0.14	ng
9.8661	0.008	0.0004	0.0021	9.42	0.13	0.00008	0.13	0.18
9.904	0.002	na	na	7.48	0.06	0.001	0.16	0.02
9.77	0.06	0.01	0.0008	10.1	0.04	0.00006	0.5	na
9.8442	0.0041	0.0004	0.0005	9.7176	0.7067	na	na	na
9.82	0.004	0.0006	0.00005	11.38	0.09	0.005	0.08	0.5
9.8328	0.2234	0.0376	0.0008	10.0234	0.1098	0.0018	0.2067	0.0007
9.814	0.008	0.0004	0.0013	9.882	0.008	0.0006	0.17	na
9.746	0.004	na	na	9.595	0.036	0.004	0.06	0.05
9.819	0.007	0.003	0.0007	9.6	0.1	0.001	0.3	na
9.8374	0.0018	0.0008	0.0007	8.944	0.221	0.034	0.035	na
9.789	0.005	0.0007	0.0004	9.79	0.03	0.3	0.0573	0.0573
9.83	0.007	0.001	0.0003	10	0.1	0.0003	0.1	NA
9.781	0.008	0.0005	0.13	9.43	0.07	0.008	0.11	0.03

Group:	sT (s)	l	Points
1:	0.007	-0.04	60
2:	0.0005	-0.06	90
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6:	0.0000	0.2	60
7:	0.003	0.5	100
8:	0.004	0.1	95
9:	0.0003	-0.10	95
10:	0.0008	+-0.05	85
11:	0.0003	-0.04	100
12:	0.005	-0.06	70
13:	0.004	0.2	75
14:	0.001	-0.03	95
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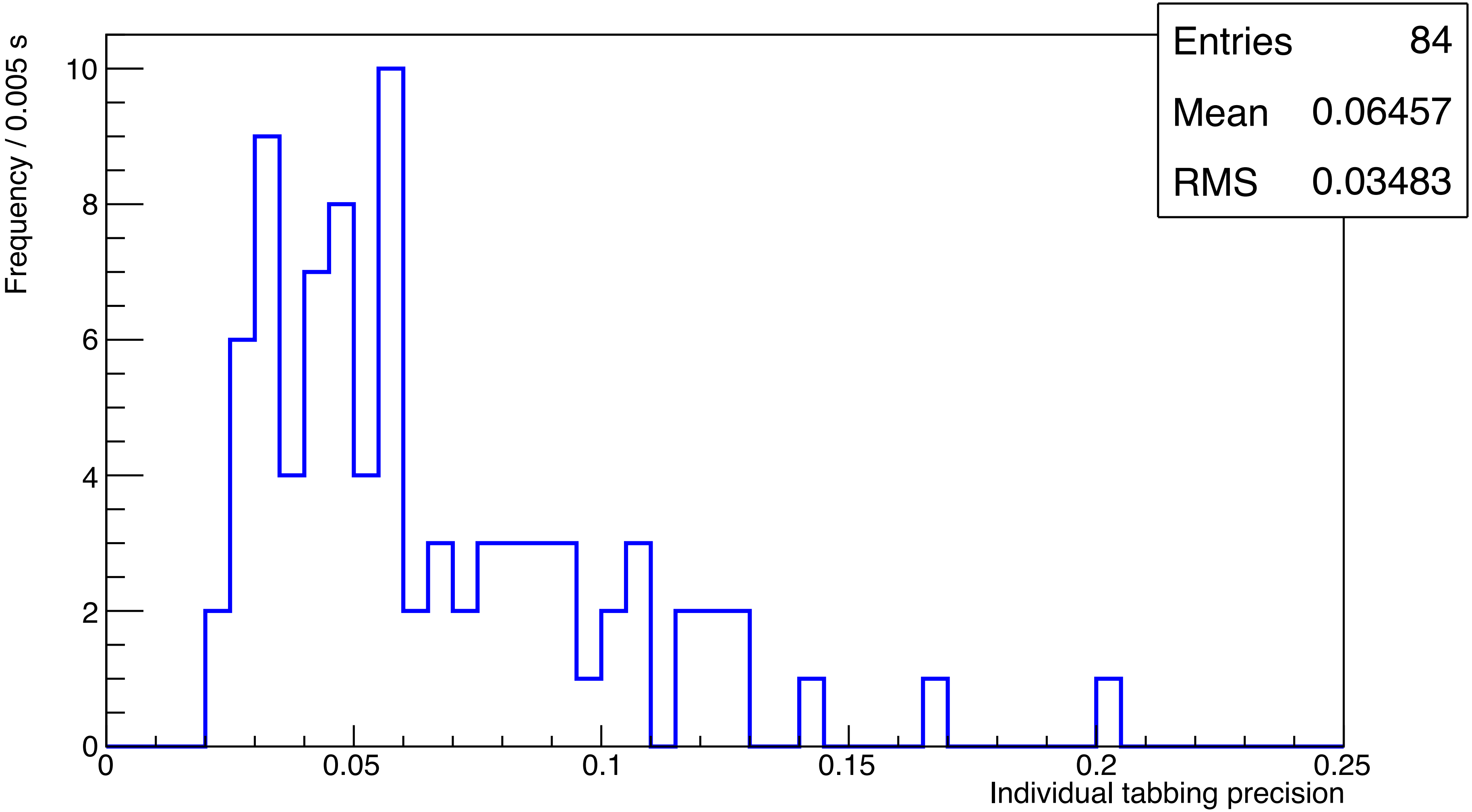
My best estimates of a "minimum" uncertainty:

2016:	0.0002s	0.0005m	0.002	0.005	0.20	0.02	0.05
2017:	0.00015s	0.0005m	0.0015	0.002	0.02 (trig) 0.10 (gonio)	0.02	0.05

Individual tabbing precisions

The individual precision varies quite a lot… who is then "Captain Accurate"?



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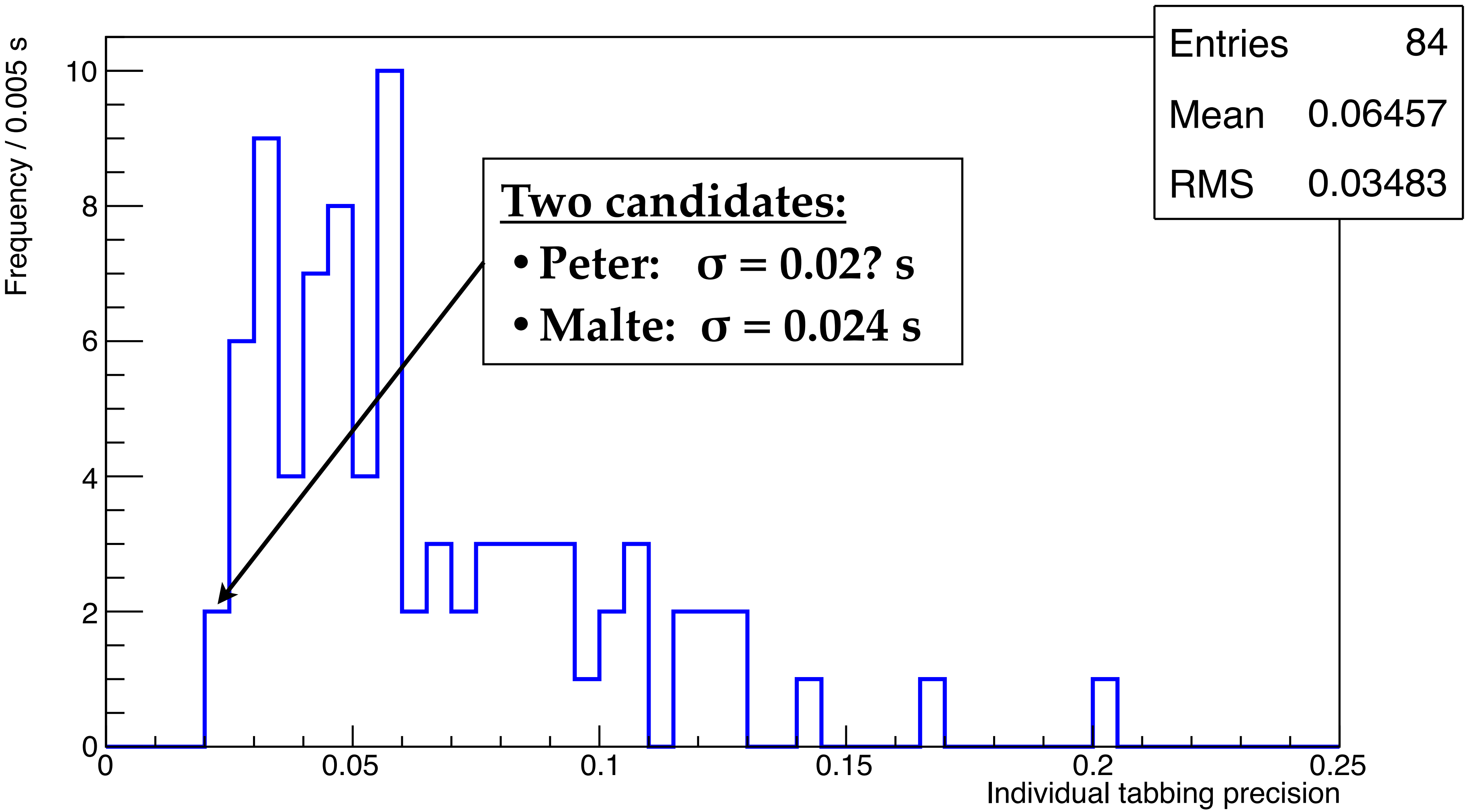


General considerations

Be careful with error estimation and propagation:

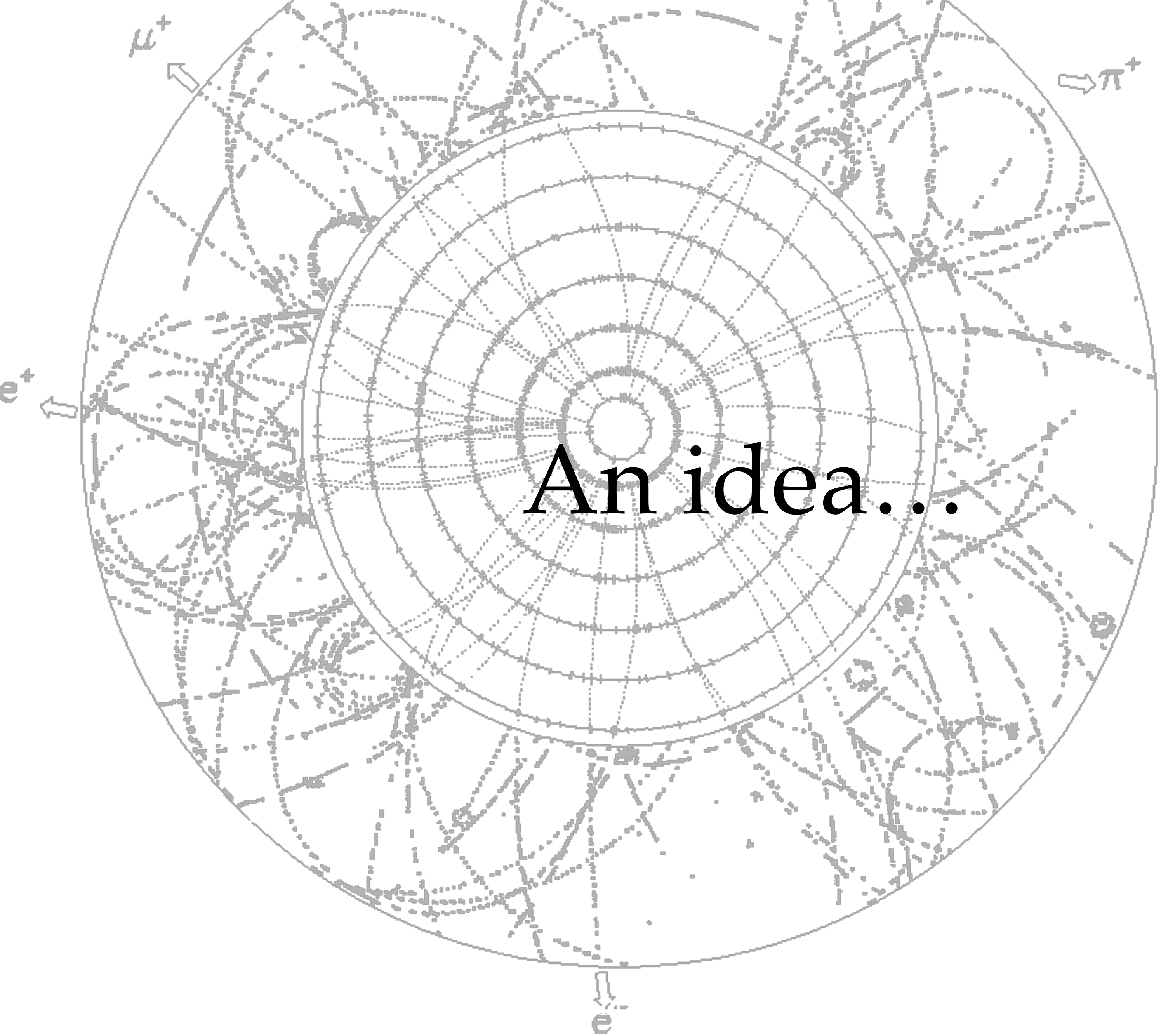
- Systematic errors do not decrease with repetition!
- **Correlated quantities** can not simply be averaged over.
- Propagation of error needs consideration - not always in quadrature.
- Be conservative, if you are uncertain about your errors.

Always show/check what you're doing:

- Make good plots.
- Give measurement tables.
- Compare the difference between results and methods.
- Combine results with uncertainties with a **weighted mean and Chi2.**

Also:

- Put the result in the abstract.
- Explain your symbols in formulae.



An idea…

Additional ideas

It is possible to “leave” the pendulum swinging between to sets of measurements. This maximises the period over which you measure, without requiring your activity all the time…

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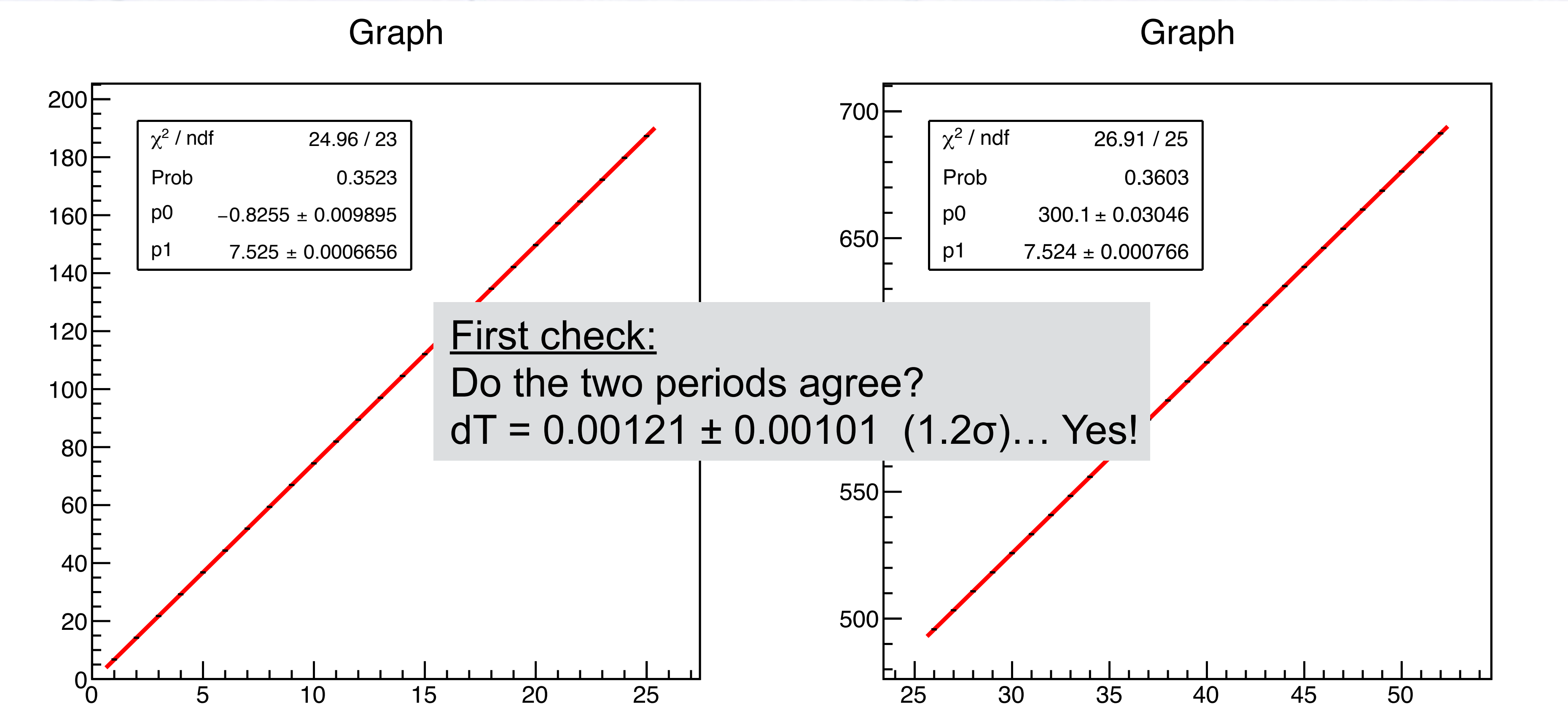


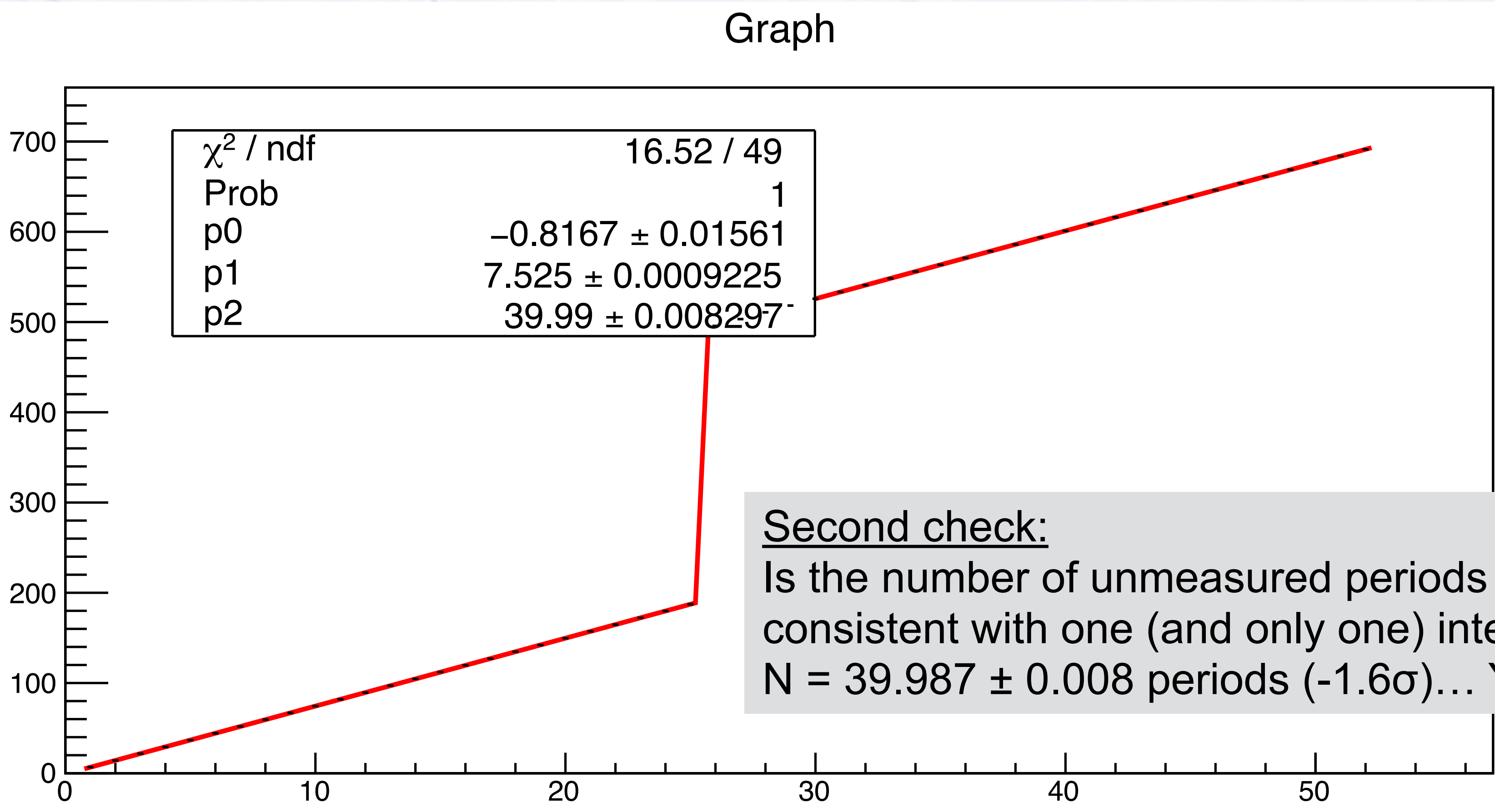
<u>First check:</u>

Do the two periods agree?

dT = 0.00121 ± 0.00101 (1.2σ)… Yes!

Additional ideas

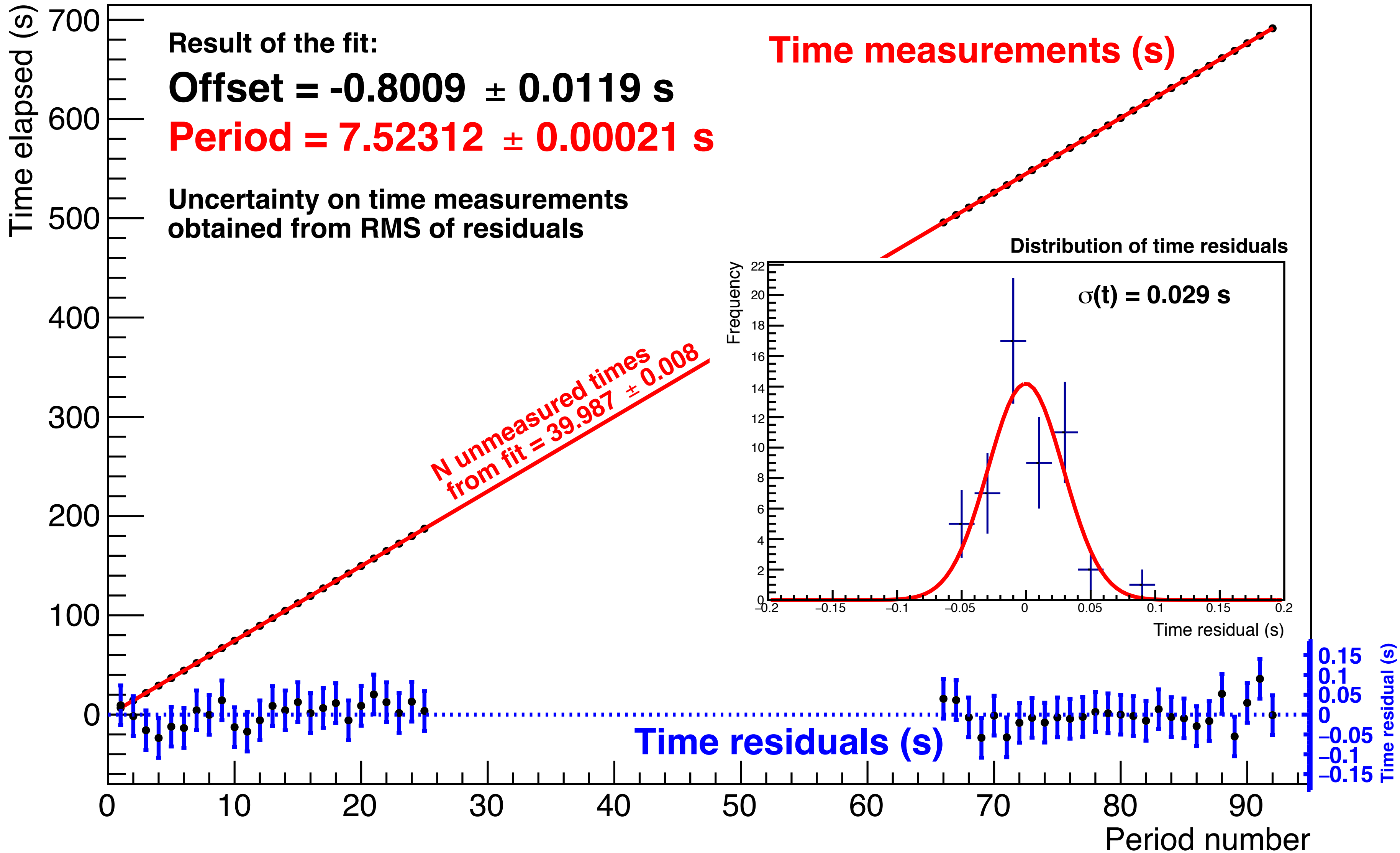
It is possible to "leave" the pendulum swinging between to sets of measurements. This maximises the period over which you measure, without requiring your activity all the time…



Second check:
Is the number of unmeasured periods consistent with one (and only one) integer?
N = 39.987 ± 0.008 periods (-1.6σ)… Yes!

Additional ideas

After checks, fit the entire time span to get "insanely great" precision.



Bonus slides