Discussion of the angle θ

The angle θ, between the rail and the direction of gravity, can be measured in **two independent ways**, which allows for a vital cross check:

With the goniometer: $\theta = \theta_{\mathrm{gonio}}$

Using trigonometry and turning the experiment: $\theta = \theta_{\mathrm{trig}} + \Delta\theta_{\mathrm{turn}}$

You might think, that doing things in two independent ways is needless. Gut this is very important in experiments (which might be extremely complicated and rely on many assumptions!), as this ensures the correctness of the central value, and also tests of the uncertainties are realistic.

For this reason, the formula for g for the ball-on-incline experiment has two versions, depending on angular measurement, and with the above one has:

$$g = \frac{a}{\sin(\theta)}\left[1 + \frac{2}{5}\frac{D_{ball}^2}{D_{ball}^2 - d_{rail}^2}\right]$$