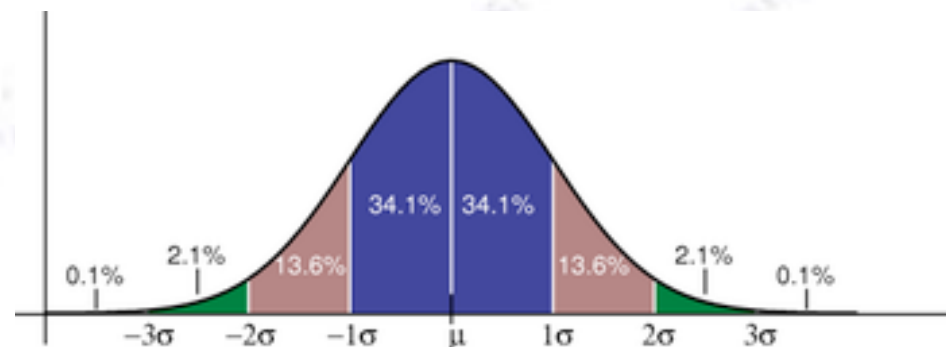


# Applied Statistics

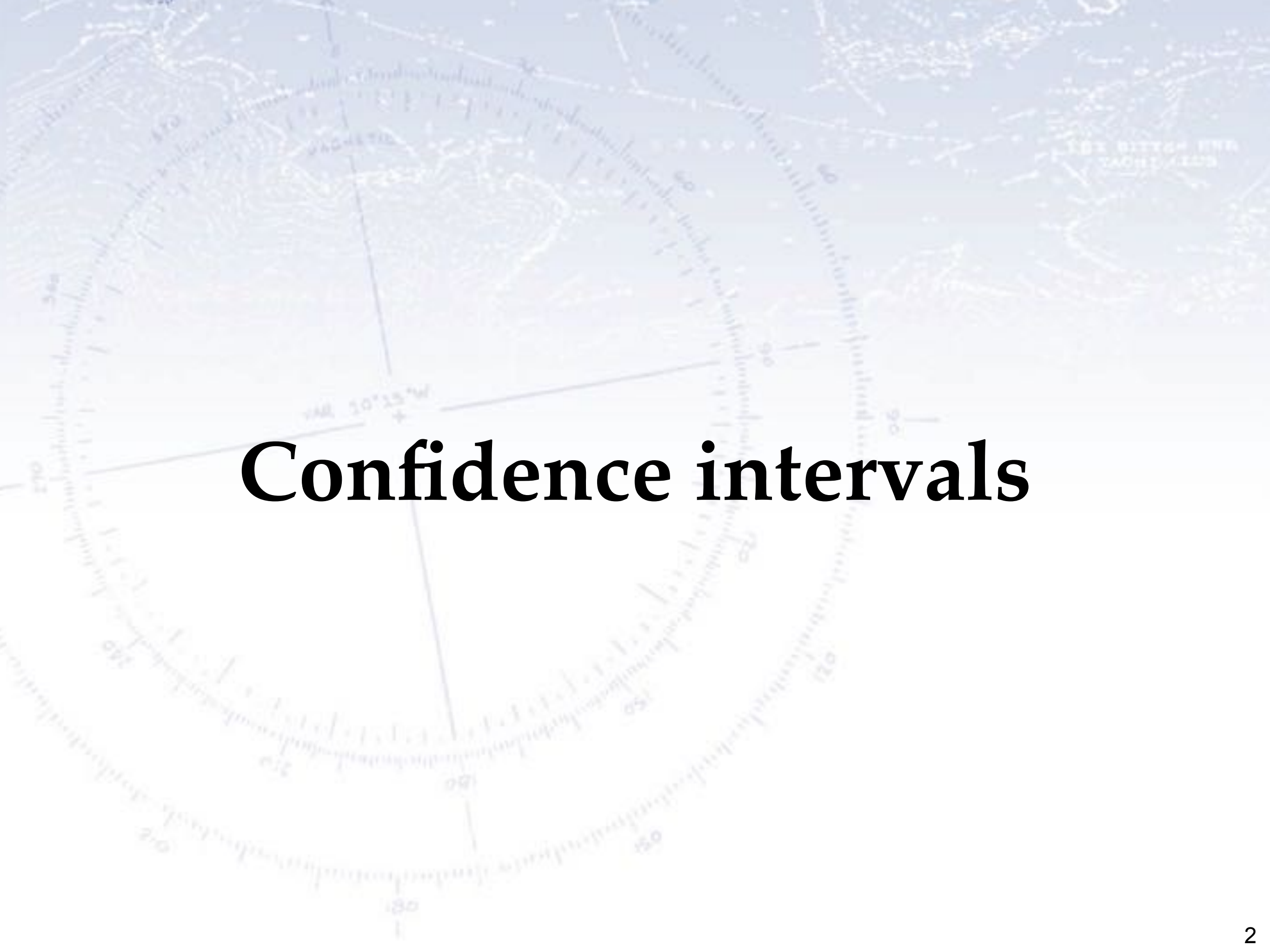
## Confidence intervals and Limits



Troels C. Petersen (NBI)



*"Statistics is merely a quantisation of common sense"*

The background is a map of the Pacific Ocean, showing bathymetry with depth contours (e.g., 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 1100, 1200, 1300, 1400, 1500, 1600, 1700, 1800, 1900, 2000, 2100, 2200, 2300, 2400, 2500, 2600, 2700, 2800, 2900, 3000, 3100, 3200, 3300, 3400, 3500, 3600, 3700, 3800, 3900, 4000, 4100, 4200, 4300, 4400, 4500, 4600, 4700, 4800, 4900, 5000, 5100, 5200, 5300, 5400, 5500, 5600, 5700, 5800, 5900, 6000, 6100, 6200, 6300, 6400, 6500, 6600, 6700, 6800, 6900, 7000, 7100, 7200, 7300, 7400, 7500, 7600, 7700, 7800, 7900, 8000, 8100, 8200, 8300, 8400, 8500, 8600, 8700, 8800, 8900, 9000, 9100, 9200, 9300, 9400, 9500, 9600, 9700, 9800, 9900, 10000). A confidence interval contour is shown as a dashed line, and a point is marked with a cross and labeled "10°15'W".

# Confidence intervals

# Confidence intervals

*“Confidence intervals consist of a range of values (interval) that act as good estimates of the unknown population parameter.”*

It is thus a way of giving a range where the true parameter value probably is.

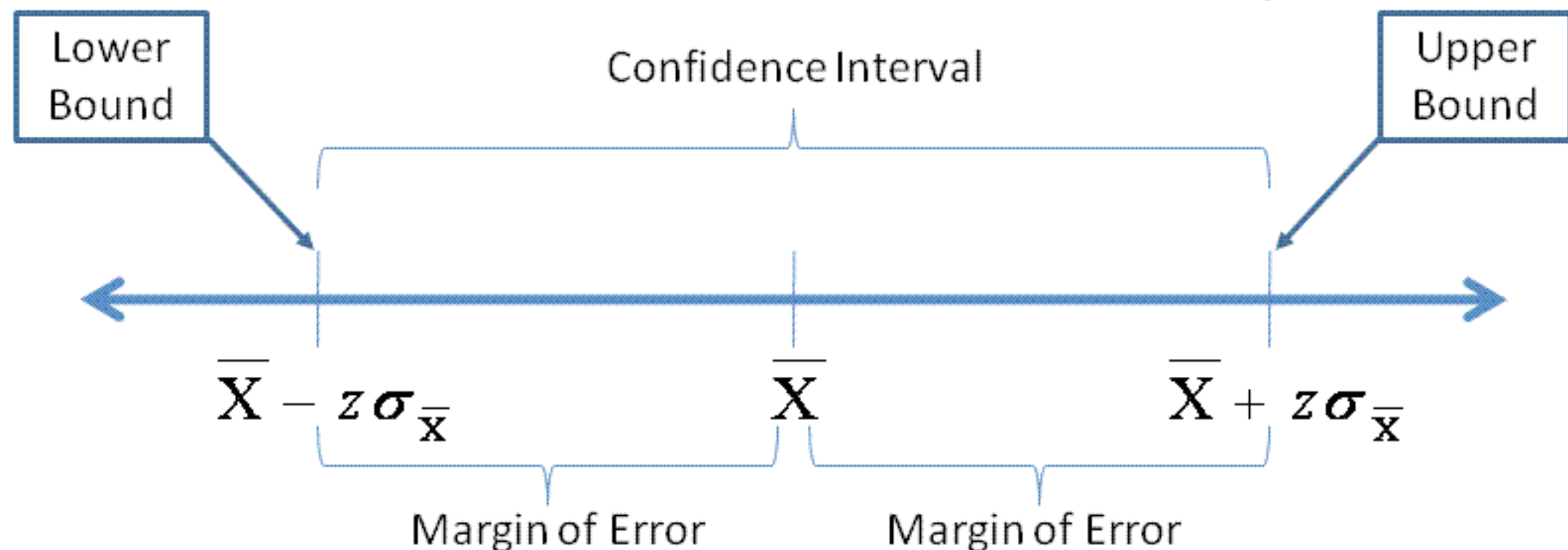
# Confidence intervals

*“Confidence intervals consist of a range of values (interval) that act as good estimates of the unknown population parameter.”*

It is thus a way of giving a range where the true parameter value probably is.

A very simple confidence interval for a Gaussian distribution can be constructed as:  
(z denotes the number of sigmas wanted)

$$\bar{x} \pm z \frac{s}{\sqrt{n}}$$



# Confidence intervals

Confidence intervals are constructed with a certain **confidence level C**, which is roughly speaking the fraction of times (for many experiments) to have the true parameter fall inside the interval:

$$Prob(x_- \leq x \leq x_+) = \int_{x_-}^{x_+} P(x) dx = C$$

Typically,  $C = 95\%$  (thus around  $2\sigma$ ), but  $68\%$ ,  $90\%$  and  $99\%$  are also used often.

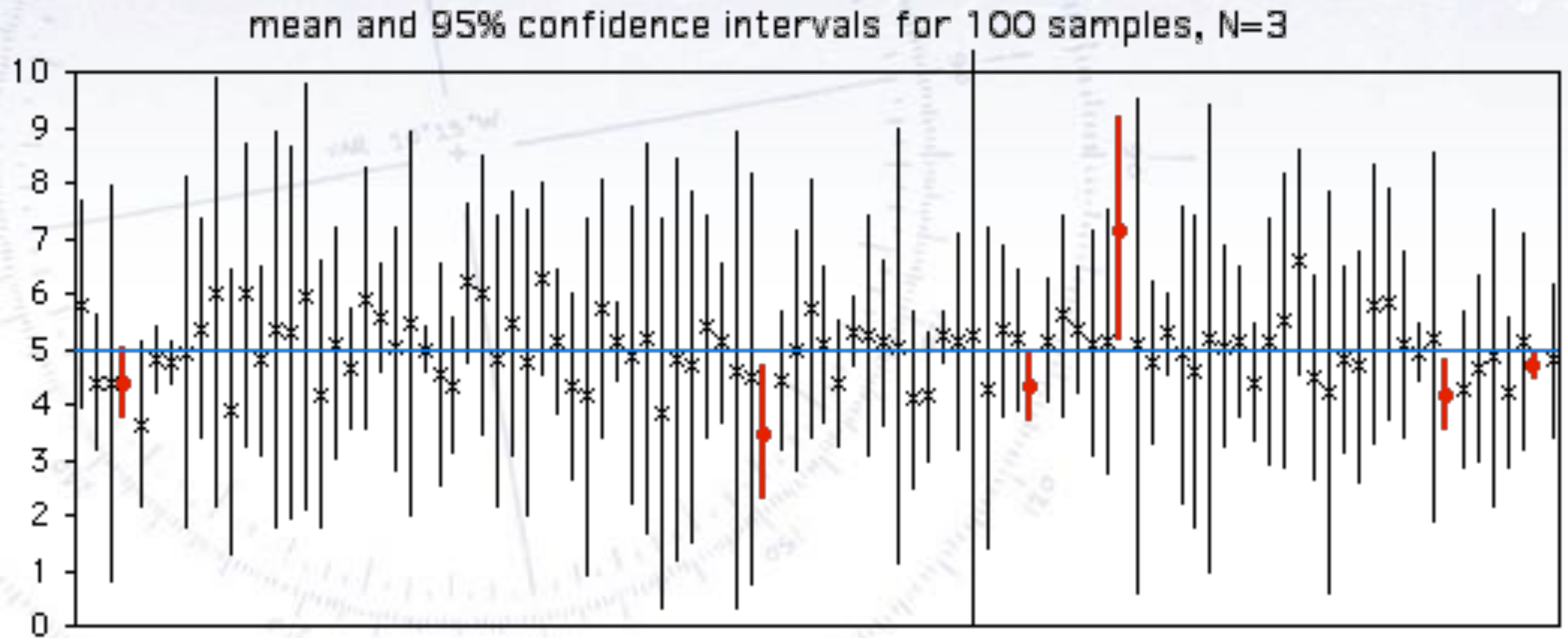
There is a choice as follows:

1. Require symmetric interval ( $x_+$  and  $x_-$  are equidistant from  $\mu$ ).
2. Require the shortest interval ( $x_+ - x_-$  is a minimum).
3. Require a central interval (integral from  $x_-$  to  $\mu$  is the same as from  $\mu$  to  $x_+$ ).

For the Gaussian, the three are equivalent!  
Otherwise, 3) is usually used.

# Confidence intervals

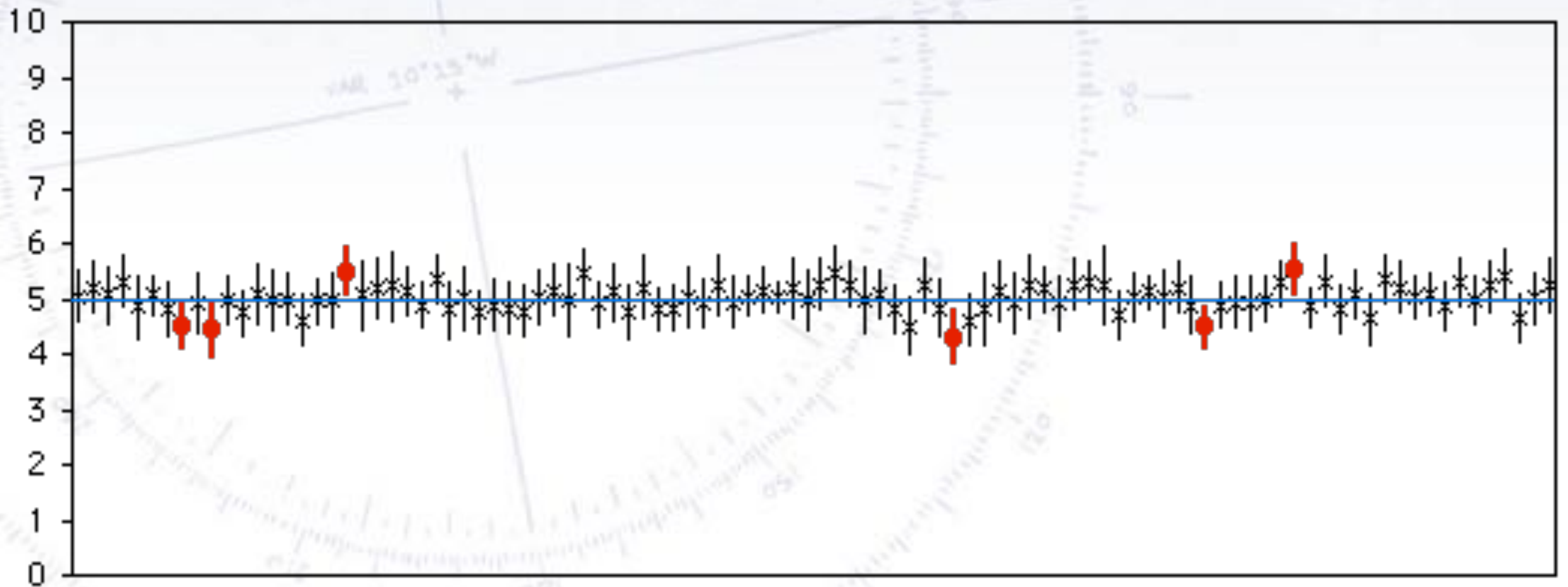
The confidence interval does not ALWAYS include the true value - only C fraction.



# Confidence intervals

The confidence interval does not ALWAYS include the true value - only C fraction.

mean and 95% confidence intervals for 100 samples, N=20

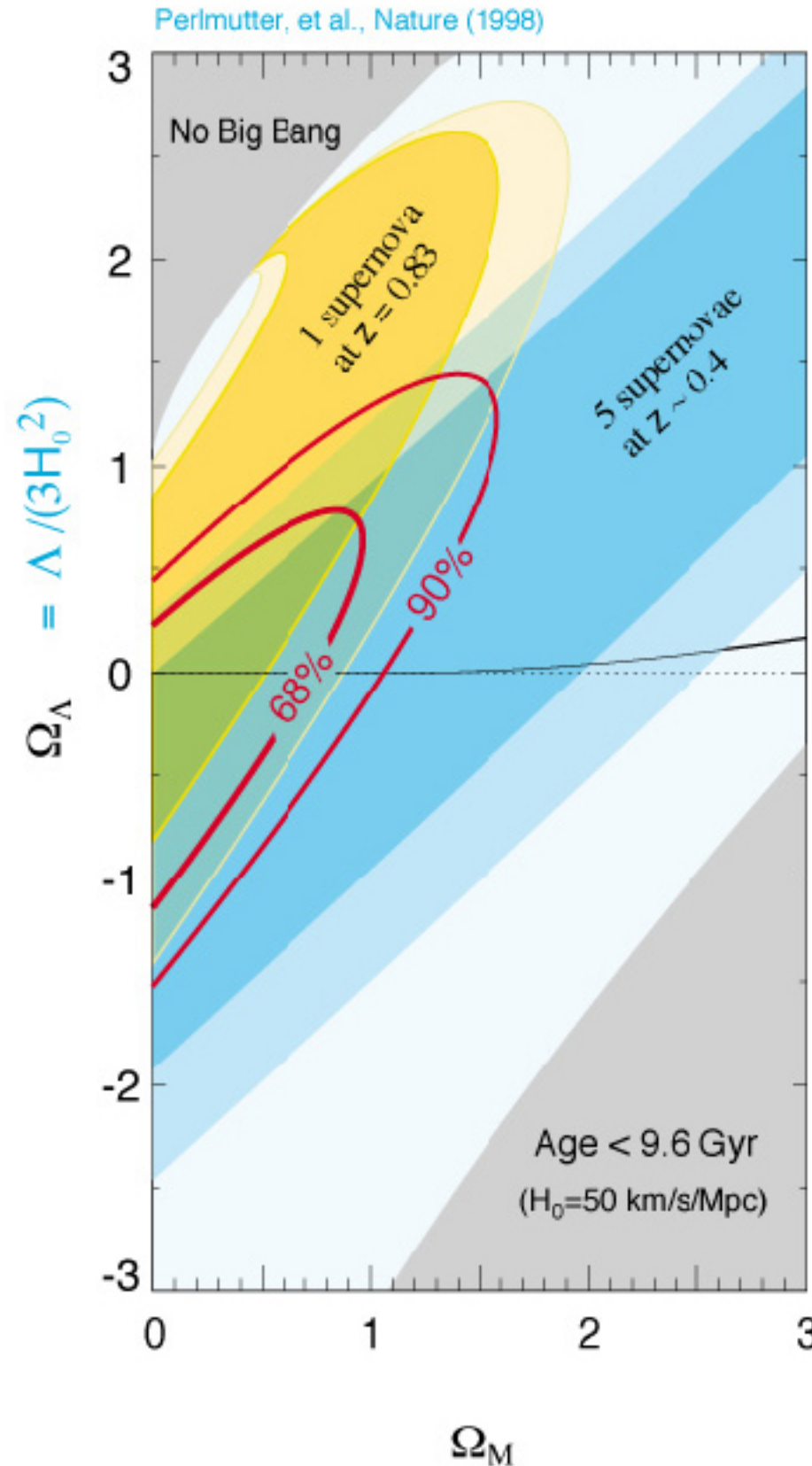


...and higher statistics does not help you!

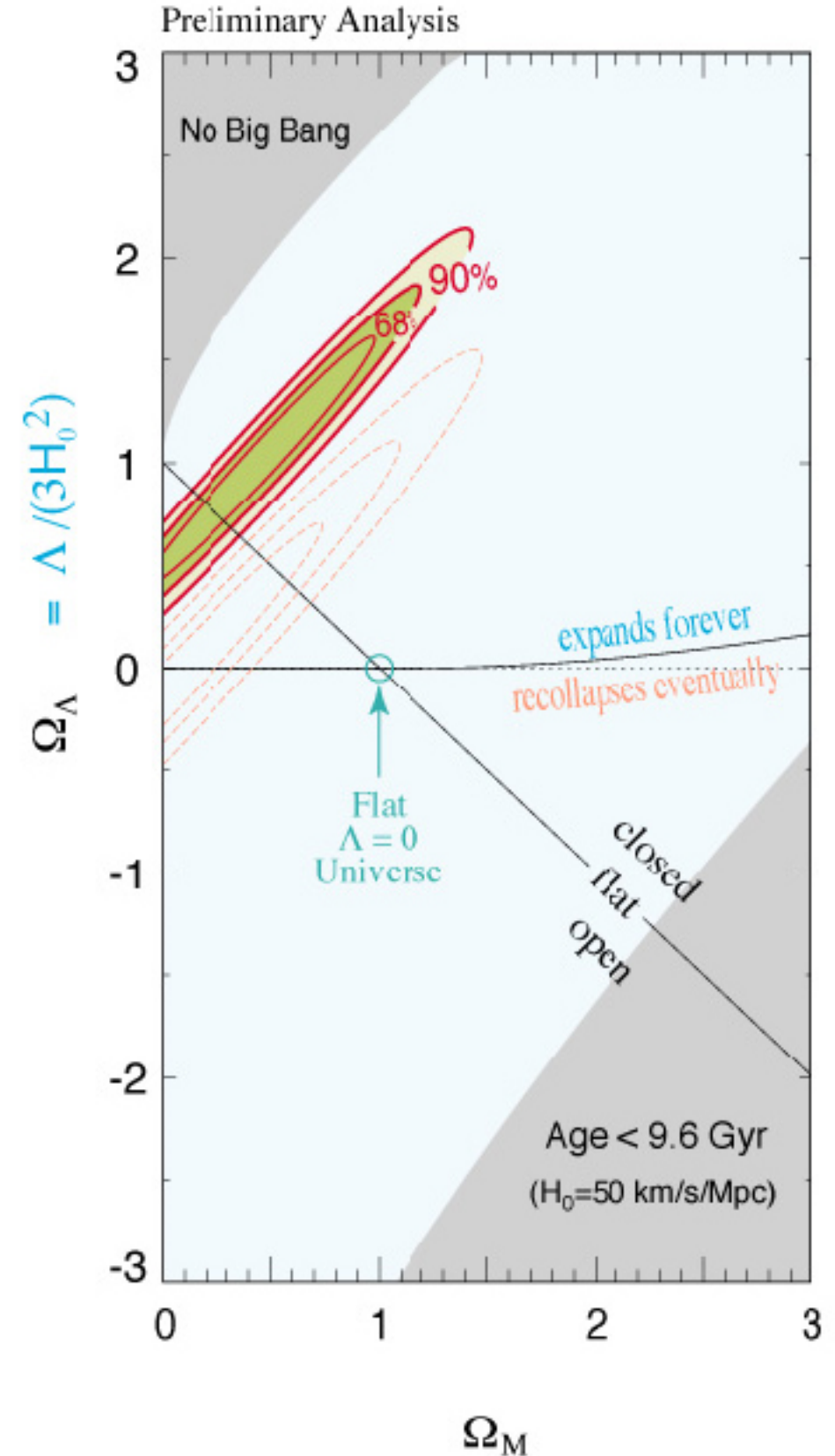
# Example

from cosmology

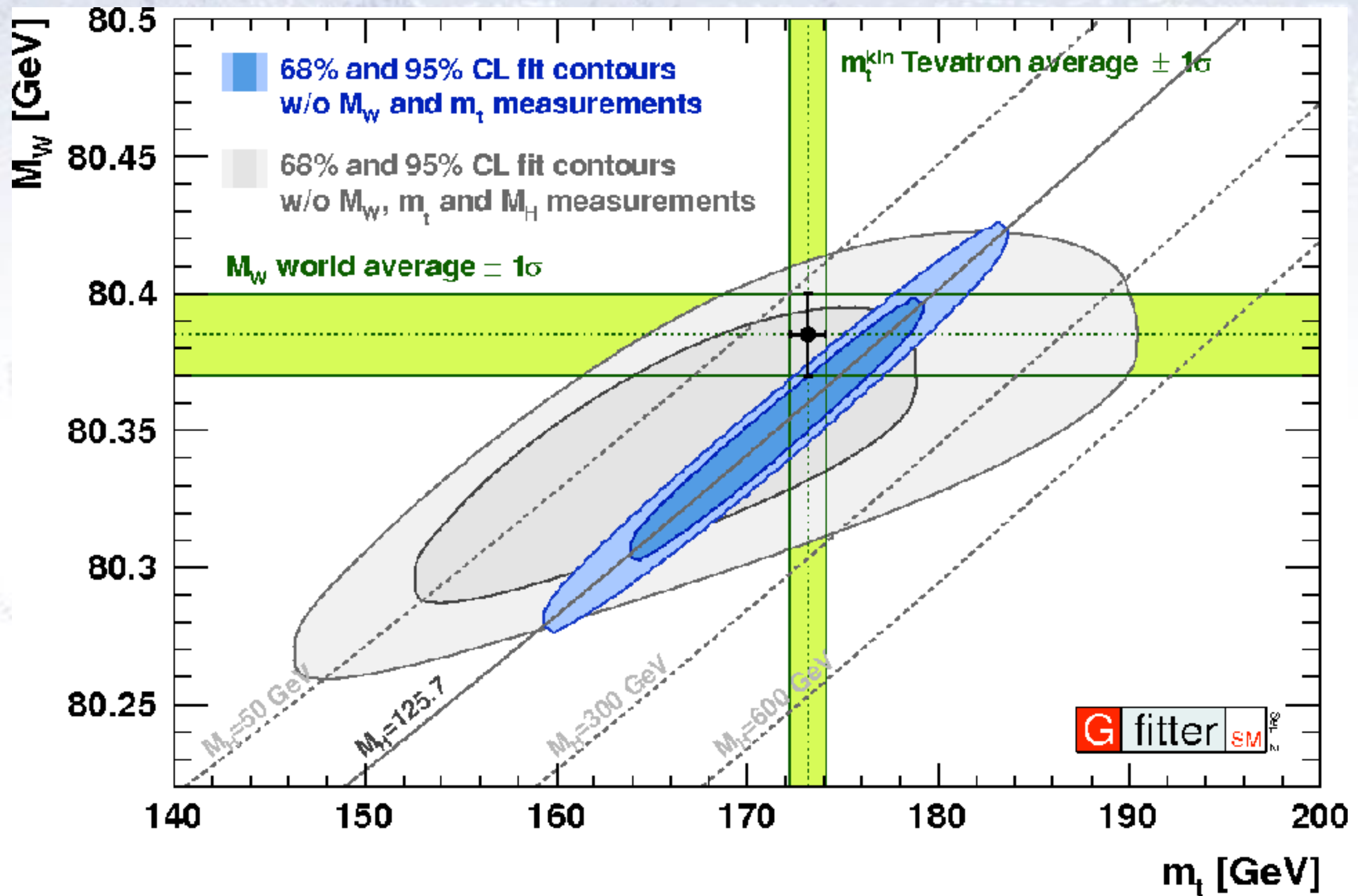
Results:  $\Omega$  vs  $\Lambda$   
from 6 supernovae

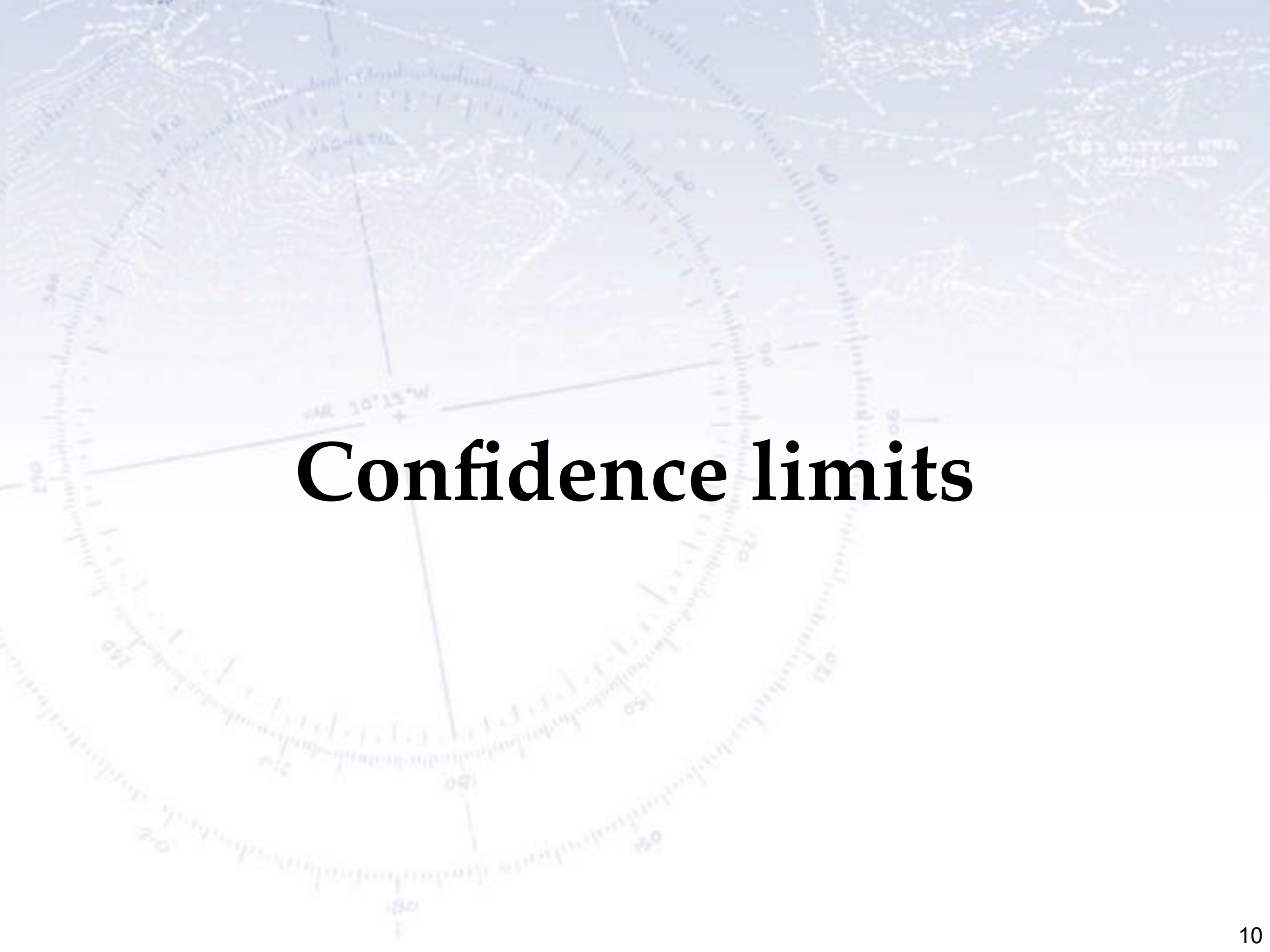


Results:  $\Omega$  vs  $\Lambda$   
from 40 supernovae



# Example from particle physics



The background is a grayscale bathymetric map of the Pacific Ocean. It features contour lines representing ocean depths, with labels such as 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 1100, 1200, 1300, 1400, 1500, 1600, 1700, 1800, 1900, 2000, 2100, 2200, 2300, 2400, 2500, 2600, 2700, 2800, 2900, 3000, 3100, 3200, 3300, 3400, 3500, 3600, 3700, 3800, 3900, 4000, 4100, 4200, 4300, 4400, 4500, 4600, 4700, 4800, 4900, 5000, 5100, 5200, 5300, 5400, 5500, 5600, 5700, 5800, 5900, 6000, 6100, 6200, 6300, 6400, 6500, 6600, 6700, 6800, 6900, 7000, 7100, 7200, 7300, 7400, 7500, 7600, 7700, 7800, 7900, 8000, 8100, 8200, 8300, 8400, 8500, 8600, 8700, 8800, 8900, 9000, 9100, 9200, 9300, 9400, 9500, 9600, 9700, 9800, 9900, 10000. A prominent contour line is labeled '1000'. Another contour line is labeled '2000'. A third contour line is labeled '3000'. A fourth contour line is labeled '4000'. A fifth contour line is labeled '5000'. A sixth contour line is labeled '6000'. A seventh contour line is labeled '7000'. An eighth contour line is labeled '8000'. A ninth contour line is labeled '9000'. A tenth contour line is labeled '10000'. A specific point is marked with a cross and labeled '10°15'N'. A confidence limit contour is shown as a dashed line. The text 'CONFIDENCE LIMIT' is visible in the upper right corner. The text '10°15'N' is visible near the marked point. The text '1000' is visible on a contour line. The text '2000' is visible on a contour line. The text '3000' is visible on a contour line. The text '4000' is visible on a contour line. The text '5000' is visible on a contour line. The text '6000' is visible on a contour line. The text '7000' is visible on a contour line. The text '8000' is visible on a contour line. The text '9000' is visible on a contour line. The text '10000' is visible on a contour line.

# Confidence limits

# Confidence limits

Imagine that you do an experiment to search for an unknown but predicted phenomenon (aether, planet Vulcan, dark matter, medical effect, etc.), and that find  
...nothing!

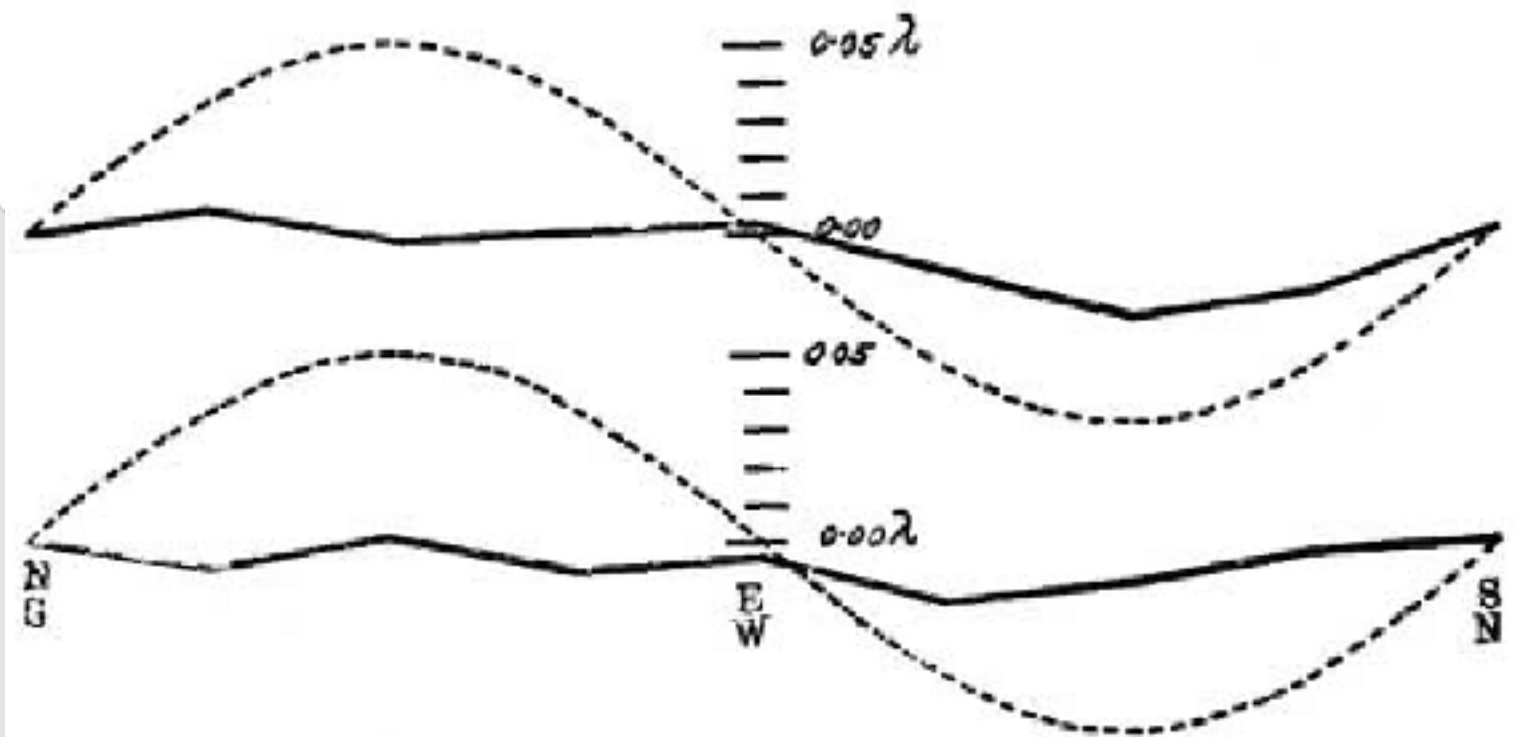
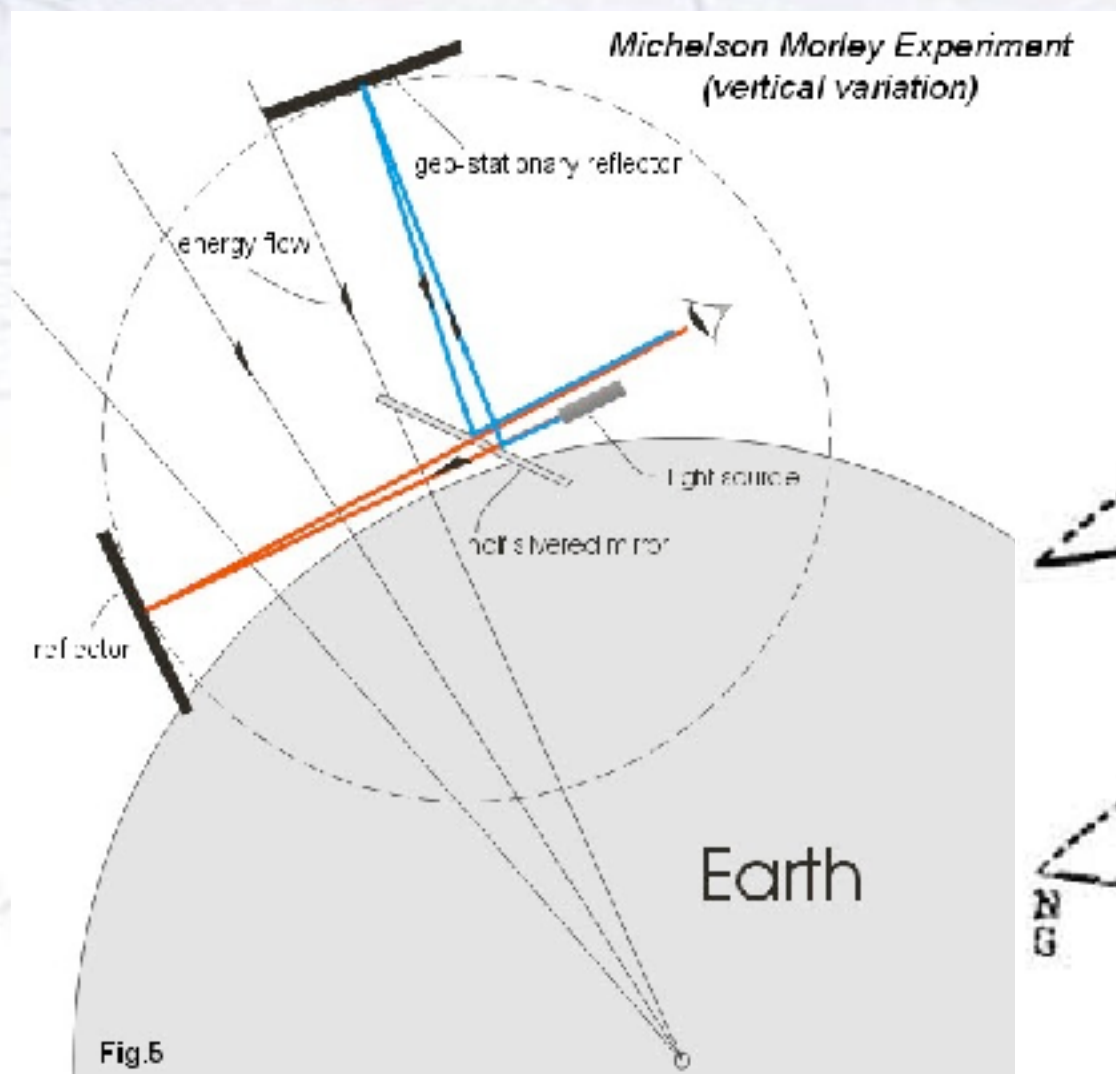
Reporting this result, you wish to state *what you would have discovered, if it had been there*, i.e. something along the lines:

*“If the aether had affected the speed of light by  $X\%$ , we would have seen the effect with 95% confidence”.*

This is a **confidence limit** (much like a one-sided confidence interval).

# Confidence limits example

In the case of Michelson-Morley, a limit could be set on the “degree of dragging” of the aether (though they didn’t do this, as statistics was still in its infancy!).



# Confidence limits - Poisson

Poisson statistics is a neat special case, perhaps best explained by numbers:

## Example:

If you in a day observe 0 red cars on Blegdamsvej, you can with 95% confidence say that there are less than 3.00 pr. day, and with 90% confidence say that there are less than 2.30 pr. day.

If you in a day observe 2 red cars, you can say at 95% CL that there are more than 0.355 and less than 6.30 red cars.

| $n$ | $1 - \alpha = 90\%$ |            | $1 - \alpha = 95\%$ |            |
|-----|---------------------|------------|---------------------|------------|
|     | $\mu_{lo}$          | $\mu_{up}$ | $\mu_{lo}$          | $\mu_{up}$ |
| 0   | —                   | 2.30       | —                   | 3.00       |
| 1   | 0.105               | 3.89       | 0.051               | 4.74       |
| 2   | 0.532               | 5.32       | 0.355               | 6.30       |
| 3   | 1.10                | 6.68       | 0.818               | 7.75       |
| 4   | 1.74                | 7.99       | 1.37                | 9.15       |
| 5   | 2.43                | 9.27       | 1.97                | 10.51      |
| 6   | 3.15                | 10.53      | 2.61                | 11.84      |
| 7   | 3.89                | 11.77      | 3.29                | 13.15      |
| 8   | 4.66                | 12.99      | 3.98                | 14.43      |
| 9   | 5.43                | 14.21      | 4.70                | 15.71      |
| 10  | 6.22                | 15.41      | 5.43                | 16.96      |