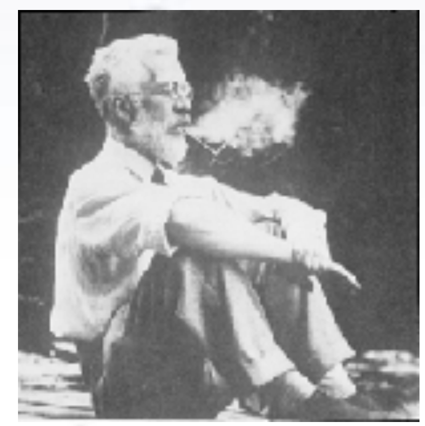
# Applied Statistics

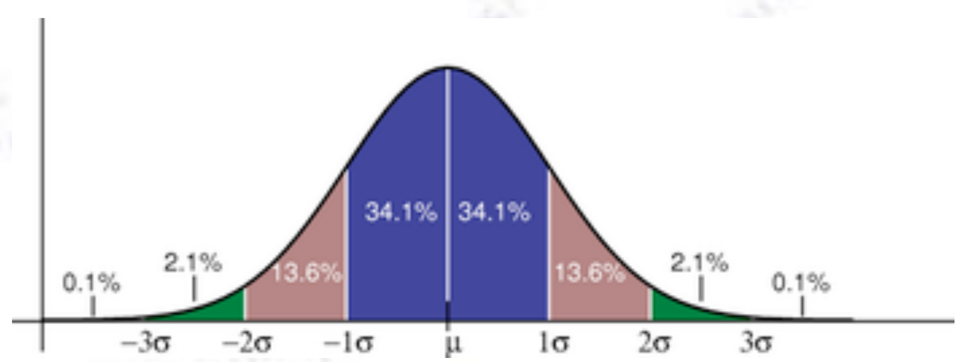
## Confidence intervals and Limits








Troels C. Petersen (NBI)




*"Statistics is merely a quantisation of common sense"*

# Confidence intervals

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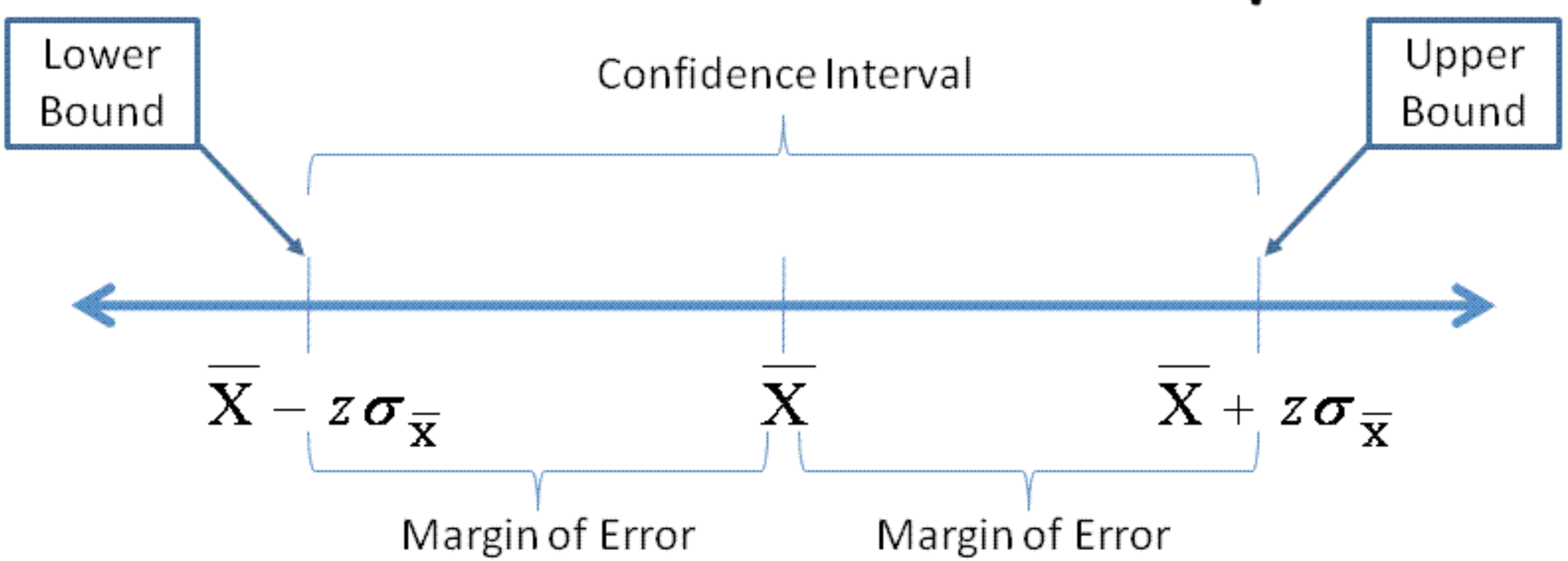
A very simple confidence interval for a Gaussian distribution can be constructed as:
(z denotes the number of sigmas wanted)

$$\bar{x} \pm z \frac{s}{\sqrt{n}}$$



# Confidence intervals

Confidence intervals are constructed with a certain **confidence level C**, which is roughly speaking the fraction of times (for many experiments) to have the true parameter fall inside the interval:

$$Prob(x_- \leq x \leq x_+) = \int_{x_-}^{x_+} P(x)dx = C$$

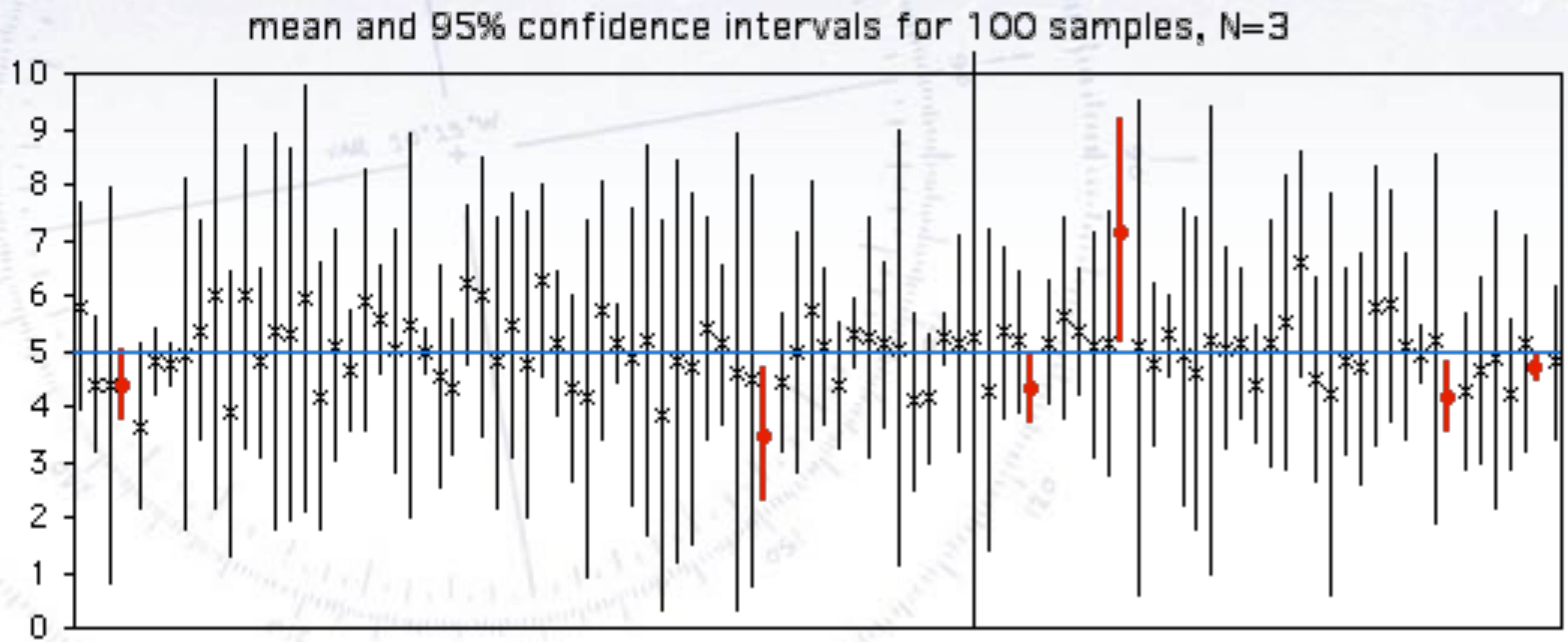
Typically, C = 95% (thus around 2σ), but 68%, 90% and 99% are also used often.

<u>There is a choice as follows:</u>
1. Require symmetric interval (x+ and x- are equidistant from μ).
2. Require the shortest interval (x+ - x- is a minimum).
3. Require a central interval (integral from x- to μ is the same as from μ to x+).

For the Gaussian, the three are equivalent!
Otherwise, 3) is usually used.

# Confidence intervals

The confidence interval does not ALWAYS include the true value - only C fraction.



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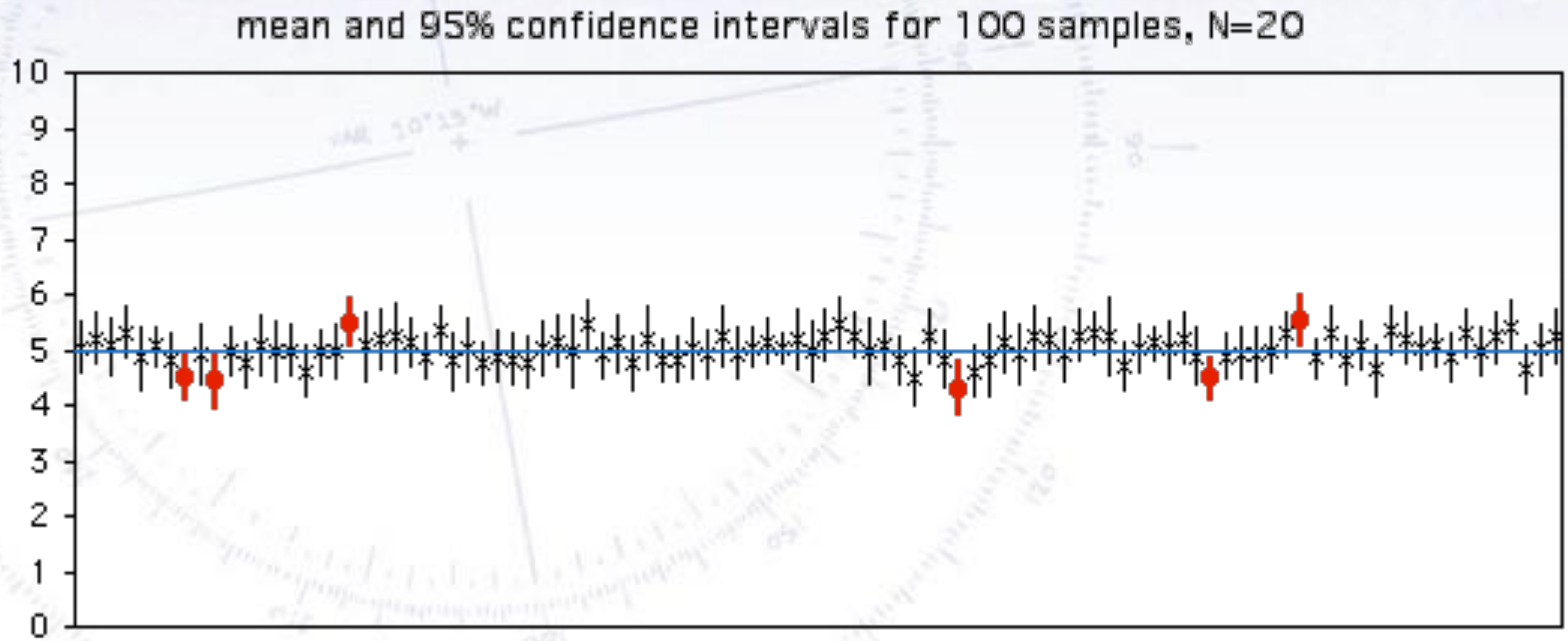
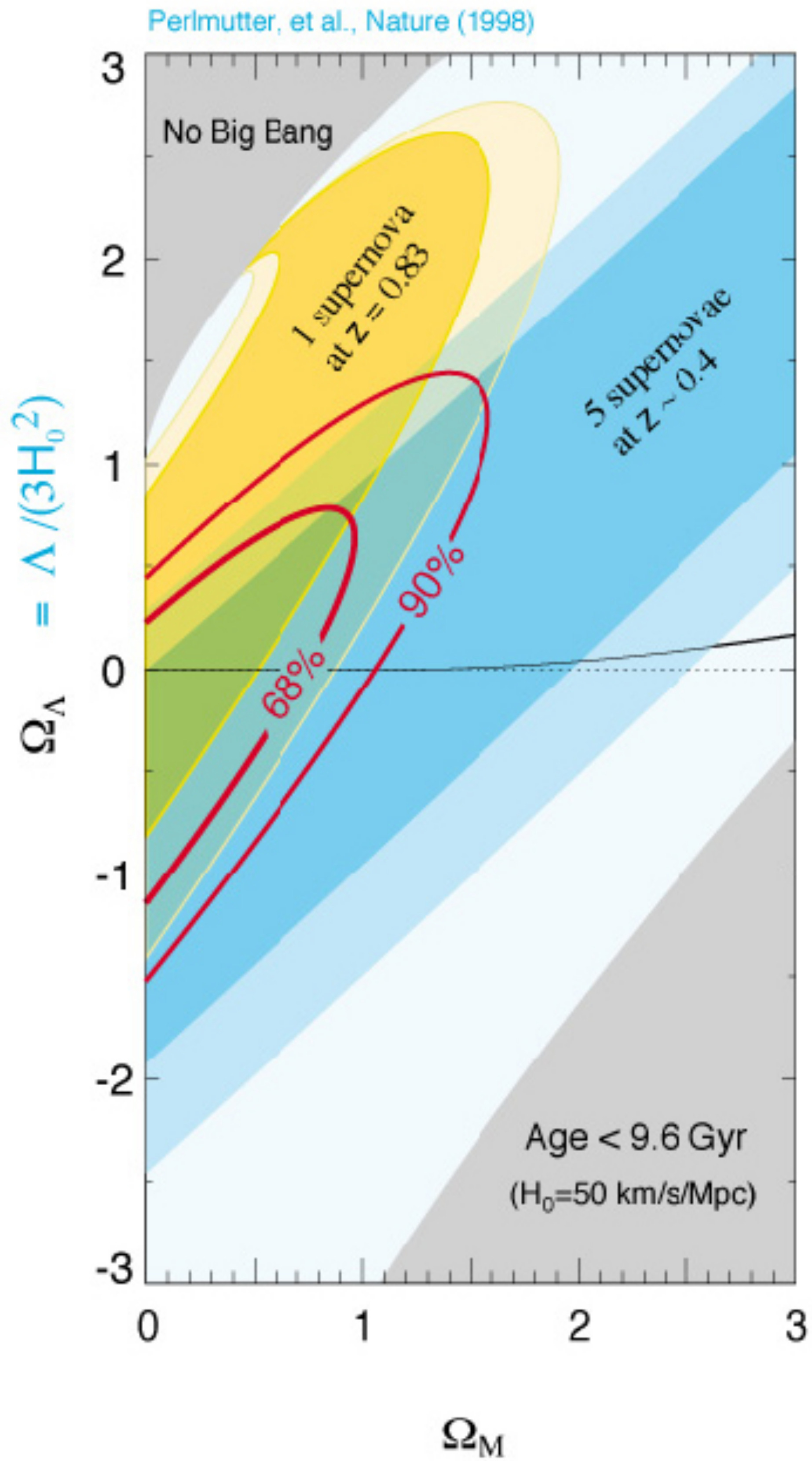



...and higher statistics does not help you!

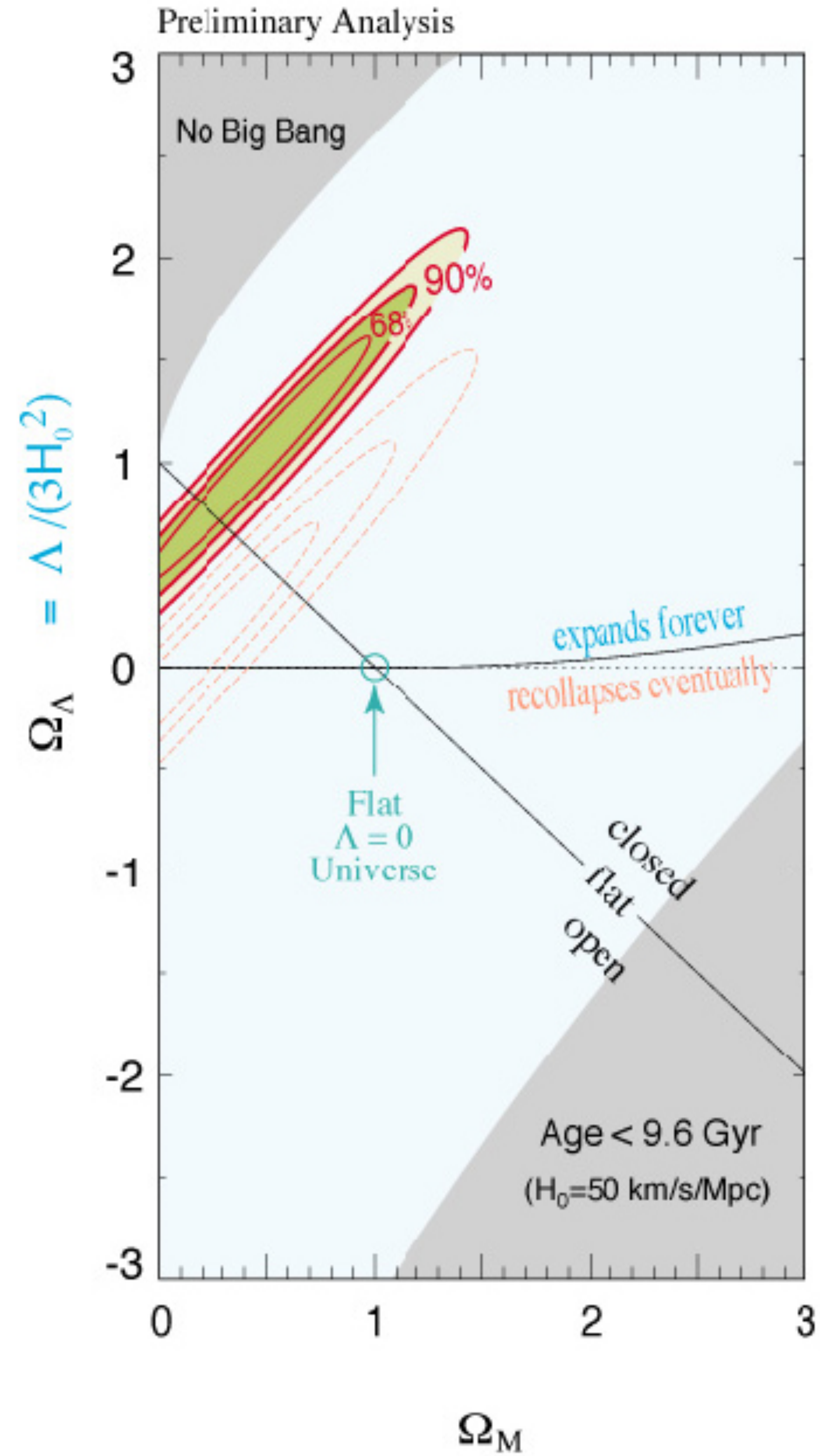
# Example

from cosmology

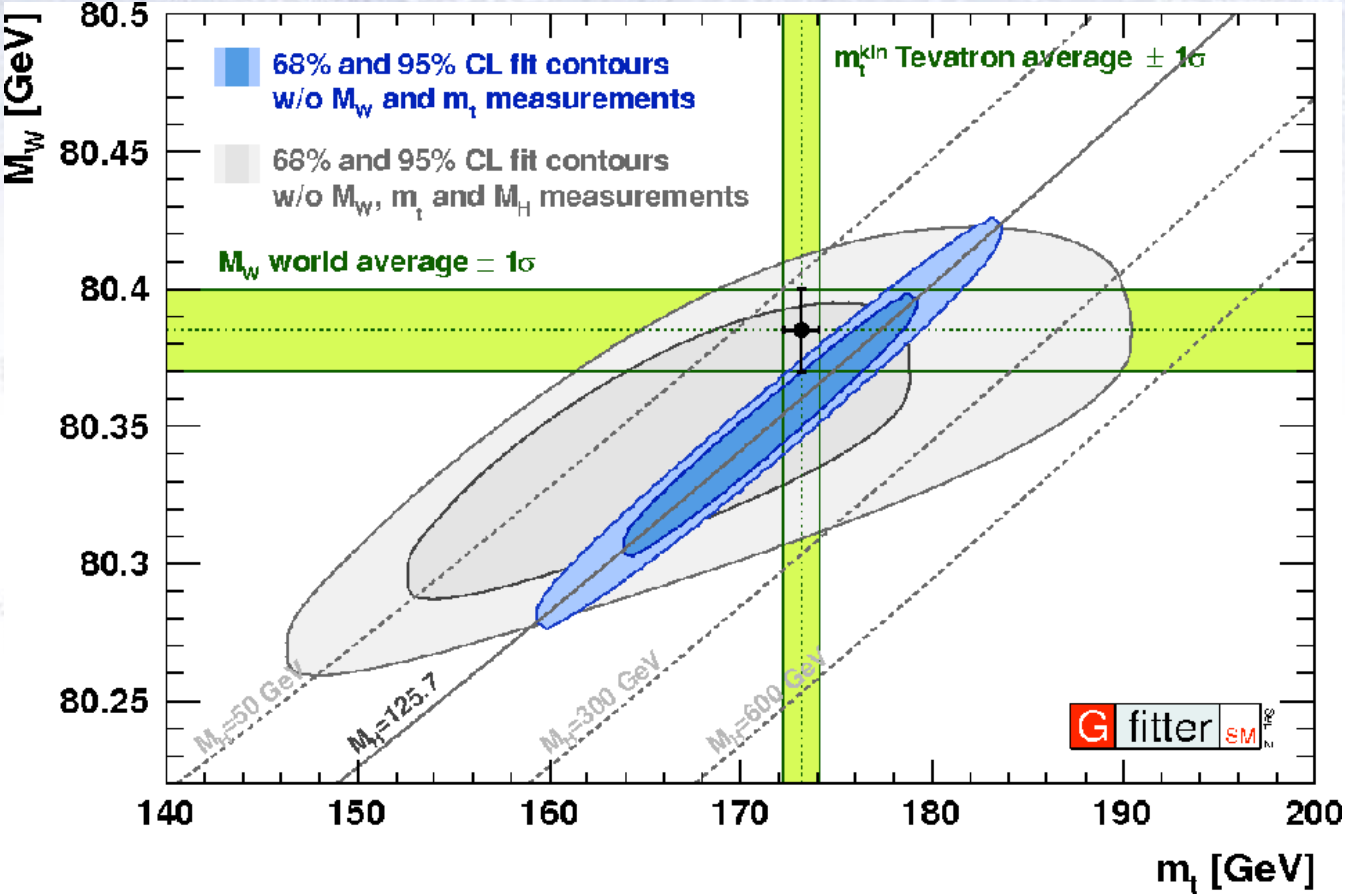
Results: Ω vs Λ from 6 supernovae




Results: Ω vs Λ from 40 supernovae



# Example from particle physics



# Confidence limits

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Imagine that you do an experiment to search for an unknown but predicted phenomenon (aether, planet Vulcan, dark matter, medical effect, etc.), and that find

…**nothing**!

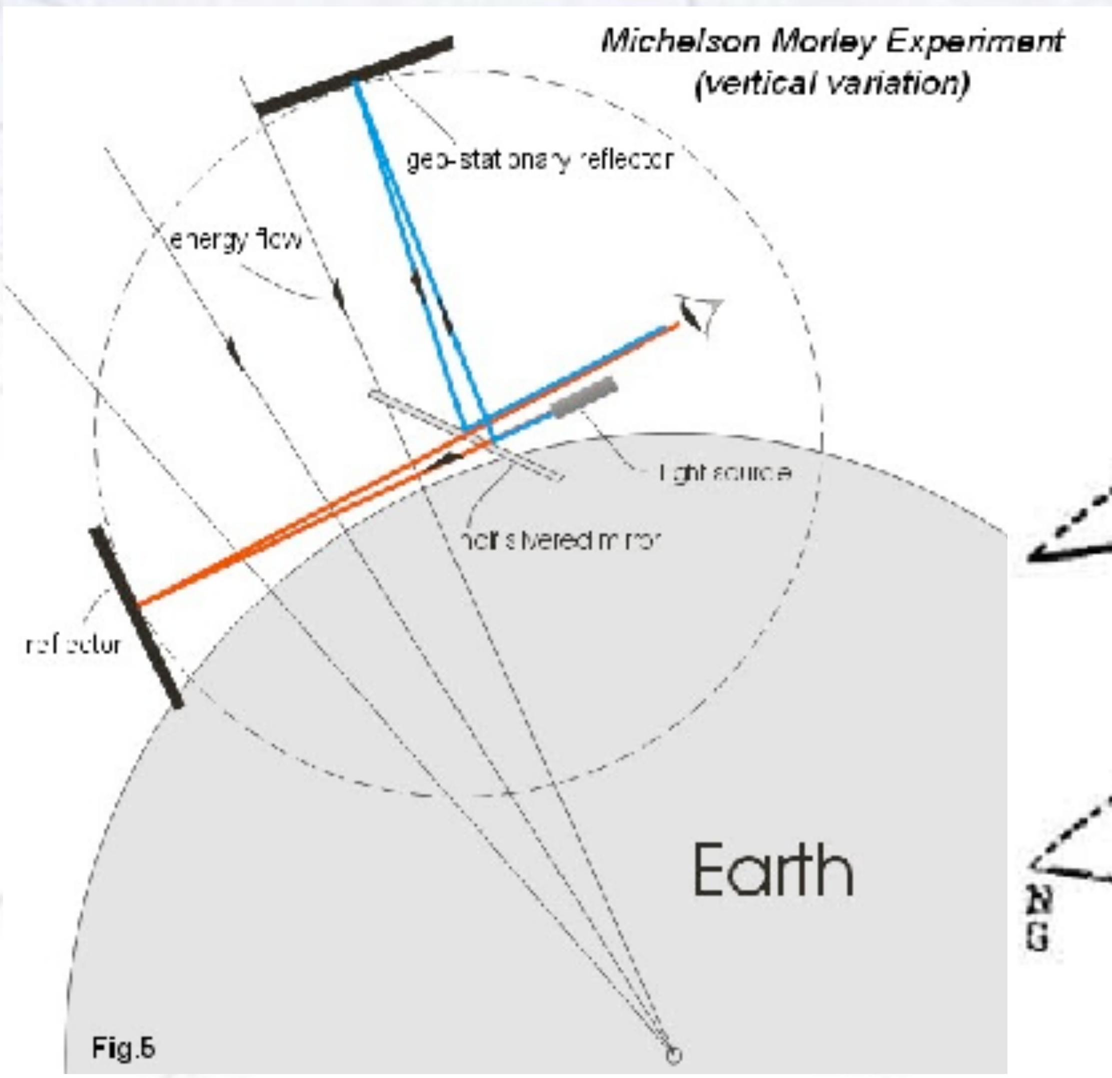
Reporting this result, you wish to state ***what you would have discovered, if it had been there***, i.e. something along the lines:

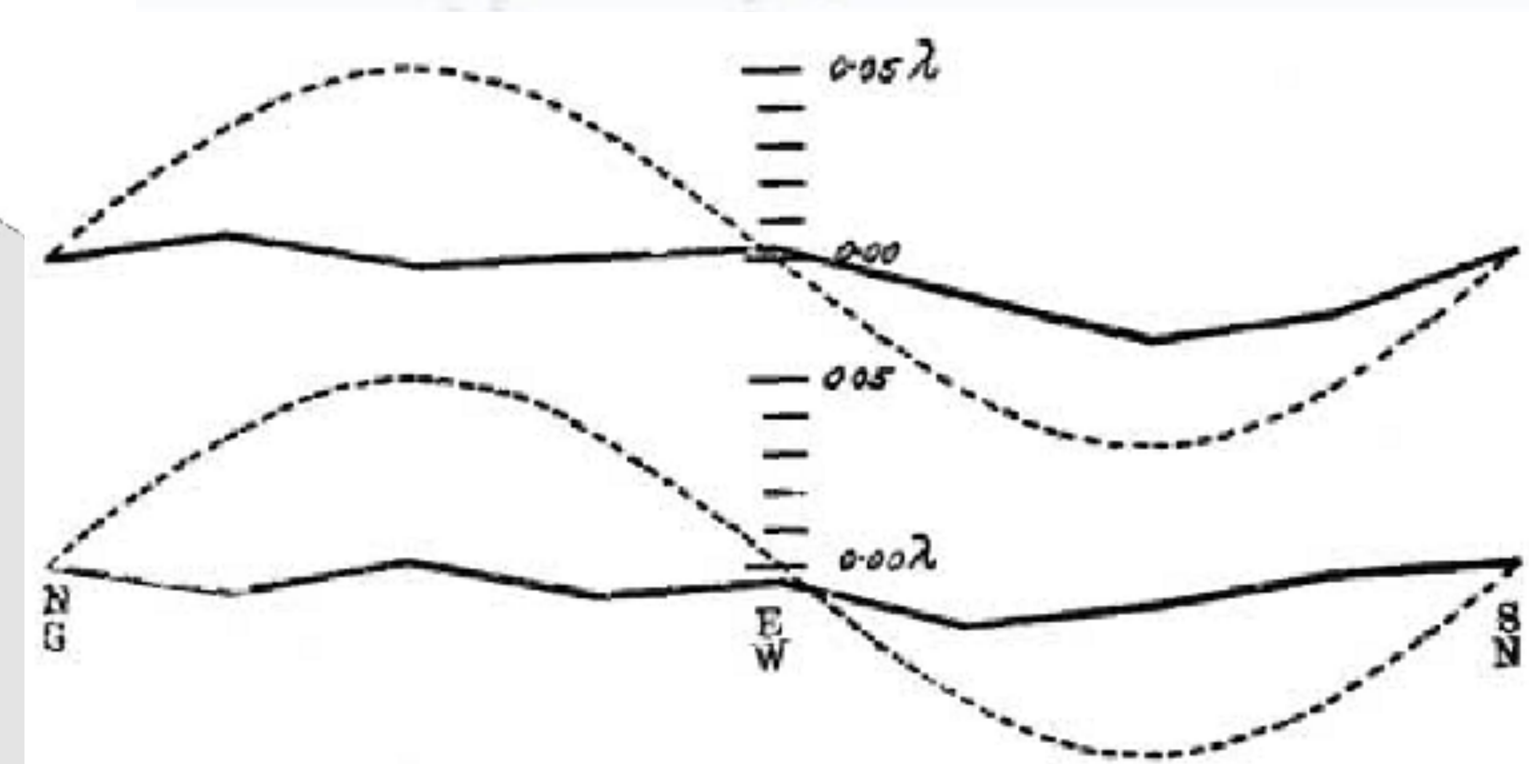
***"If the aether had affected the speed of light by X%, we would have seen the effect with 95% confidence".***

This is a **confidence limit** (much like a one-sided confidence interval).

# Confidence limits example

In the case of Michelson-Morley, a limit could be set on the “degree of dragging” of the aether (though they didn’t do this, as statistics was still in its infancy!).



# Confidence limits - Poisson

Poisson statistics is a neat special case, perhaps best explained by numbers:

<u>Example:</u>
If you in a day observe 0 red cars on Blegdamsvej, you can with 95% confidence say that there are less than 3.00 pr. day, and with 90% confidence say that there are less than 2.30 pr. day.
If you in a day observe 2 red cars, you can say at 95% CL that there are more than 0.355 and less than 6.30 red cars.

| | $1-\alpha = 90\%$ | | $1-\alpha = 95\%$ | |
|---|---|---|---|---|
| $n$ | $\mu_{lo}$ | $\mu_{up}$ | $\mu_{lo}$ | $\mu_{up}$ |
| 0 | – | $\boxed{2.30}$ | – | $\boxed{3.00}$ |
| 1 | 0.105 | 3.89 | 0.051 | 4.74 |
| 2 | 0.532 | 5.32 | $\boxed{0.355}$ | $\boxed{6.30}$ |
| 3 | 1.10 | 6.68 | 0.818 | 7.75 |
| 4 | 1.74 | 7.99 | 1.37 | 9.15 |
| 5 | 2.43 | 9.27 | 1.97 | 10.51 |
| 6 | 3.15 | 10.53 | 2.61 | 11.84 |
| 7 | 3.89 | 11.77 | 3.29 | 13.15 |
| 8 | 4.66 | 12.99 | 3.98 | 14.43 |
| 9 | 5.43 | 14.21 | 4.70 | 15.71 |
| 10 | 6.22 | 15.41 | 5.43 | 16.96 |