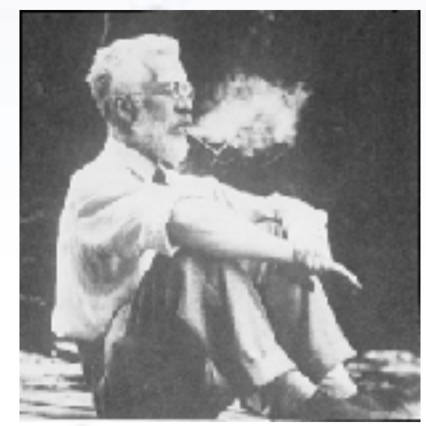
# Applied Statistics

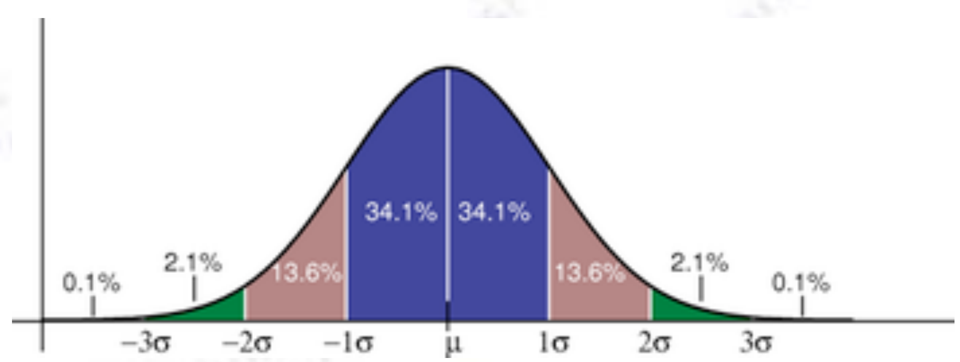
## Simpson's Paradox








Troels C. Petersen (NBI)




*"Statistics is merely a quantisation of common sense"*

# Case: Berkeley admission

In 1973, University of California, Berkeley, were considering which of their applicants got admitted.
As can be seen below, there is seemingly a **bias against women**, as a smaller fraction of women are admitted.
Is that really the case, or is there more to the data than first glance reveals?

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**Sex Bias in Graduate Admissions: Data from Berkeley**

Measuring bias is harder than is usually assumed, and the evidence is sometimes contrary to expectation.

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Table 1. Decisions on applications to Graduate Division for fall 1973, by sex of applicant—naive aggregation. Expected frequencies are calculated from the marginal totals of the observed frequencies under the assumptions (1 and 2) given in the text. $N = 12{,}763$, $\chi^2 = 110.8$, d.f. $= 1$, $P = 0$ (*18*).

| Applicants | Outcome | | | | Difference | |
|---|---|---|---|---|---|---|
| | Observed | | Expected | | | |
| | Admit | Deny | Admit | Deny | Admit | Deny |
| Men | 3738 | 4704 | 3460.7 | 4981.3 | 277.3 | – 277.3 |
| Women | 1494 | 2827 | 1771.3 | 2549.7 | – 277.3 | 277.3 |

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As already noted, we are aware of the pitfalls ahead in this naive approach, but we intend to stumble into every one of them for didactic reasons.

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Bickel et al. goes on to analyse the data further with several interesting findings:

> sex. Our computations, therefore, except where otherwise noted, will be based on the remaining 85. For a start let us identify those of the 85 with bias sufficiently large to occur by chance less than five times in a hundred. There prove to be four such departments. The deficit in the number of women admitted to these four (under the assumptions for calculating expected frequencies as given above) is 26. Looking further, we find six departments biased in the opposite direction, at the same probability levels; these account for a deficit of 64 men.

Out of 85 departments with relevant data, a few seem to show a bias… in both directions, and mostly agains men!!! What!

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Out of 85 departments with relevant data*, a few seem to show a bias… in both directions, and mostly agains men!!! What!

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*Here you should ALWAYS ask, what this involves!
In this case, 16 departments either had no women applying, or did not deny any students admission.

# Case: Berkeley admission

In order to illustrate the point, Bickel et al. gives a hypothetical (and fun!) case:

Table 2. Admissions data by sex of applicant for two hypothetical departments. For total, $\chi^2 = 5.71$, d.f. = 1, $P = 0.19$ (one-tailed).

| Applicants | Outcome: Observed Admit | Observed Deny | Expected Admit | Expected Deny | Difference Admit | Difference Deny |
|---|---|---|---|---|---|---|
| | | | *Department of machismatics* | | | |
| Men | 200 | 200 | 200 | 200 | 0 | 0 |
| Women | 100 | 100 | 100 | 100 | 0 | 0 |
| | | | *Department of social warfare* | | | |
| Men | 50 | 100 | 50 | 100 | 0 | 0 |
| Women | 150 | 300 | 150 | 300 | 0 | 0 |
| | | | *Totals* | | | |
| Men | 250 | 300 | 229.2 | 320.8 | 20.8 | − 20.8 |
| Women | 250 | 400 | 270.8 | 379.2 | − 20.8 | 20.8 |

The two (very hypothetical) departments are clearly very fair regarding gender, but still a difference appears between the overall resulting observation and expectation.

# Case: Berkeley admission

The “apparent conclusion” (Berkeley discriminates against applications from women) is a result of Simpson’s Paradox (my text):

**“Effect for group, which disappears or reverses, when considering subgroups”.**

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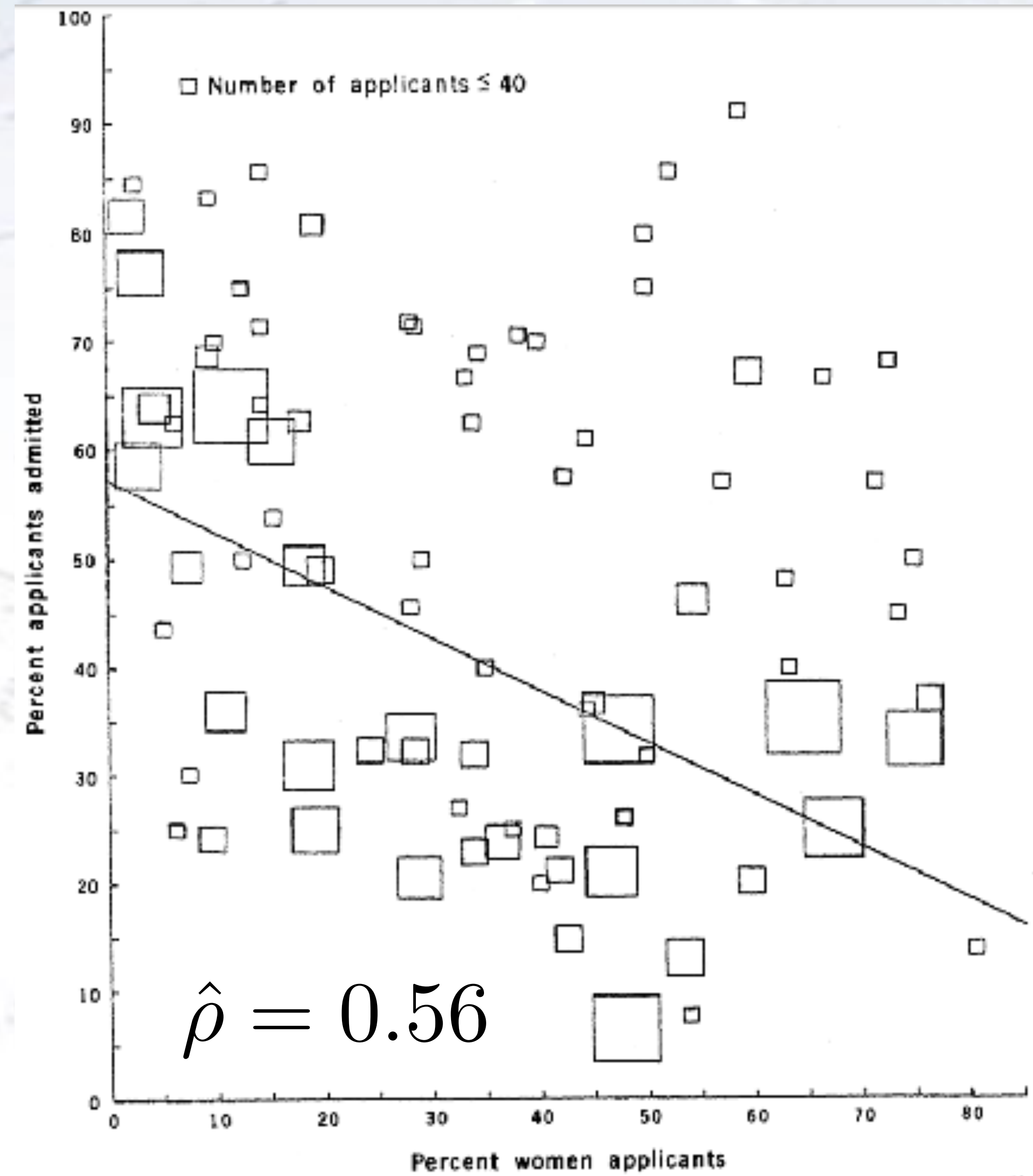


Fig. 1. Proportion of applicants that are women plotted against proportion of applicants admitted, in 85 departments. Size of box indicates relative number of applicants to the department.

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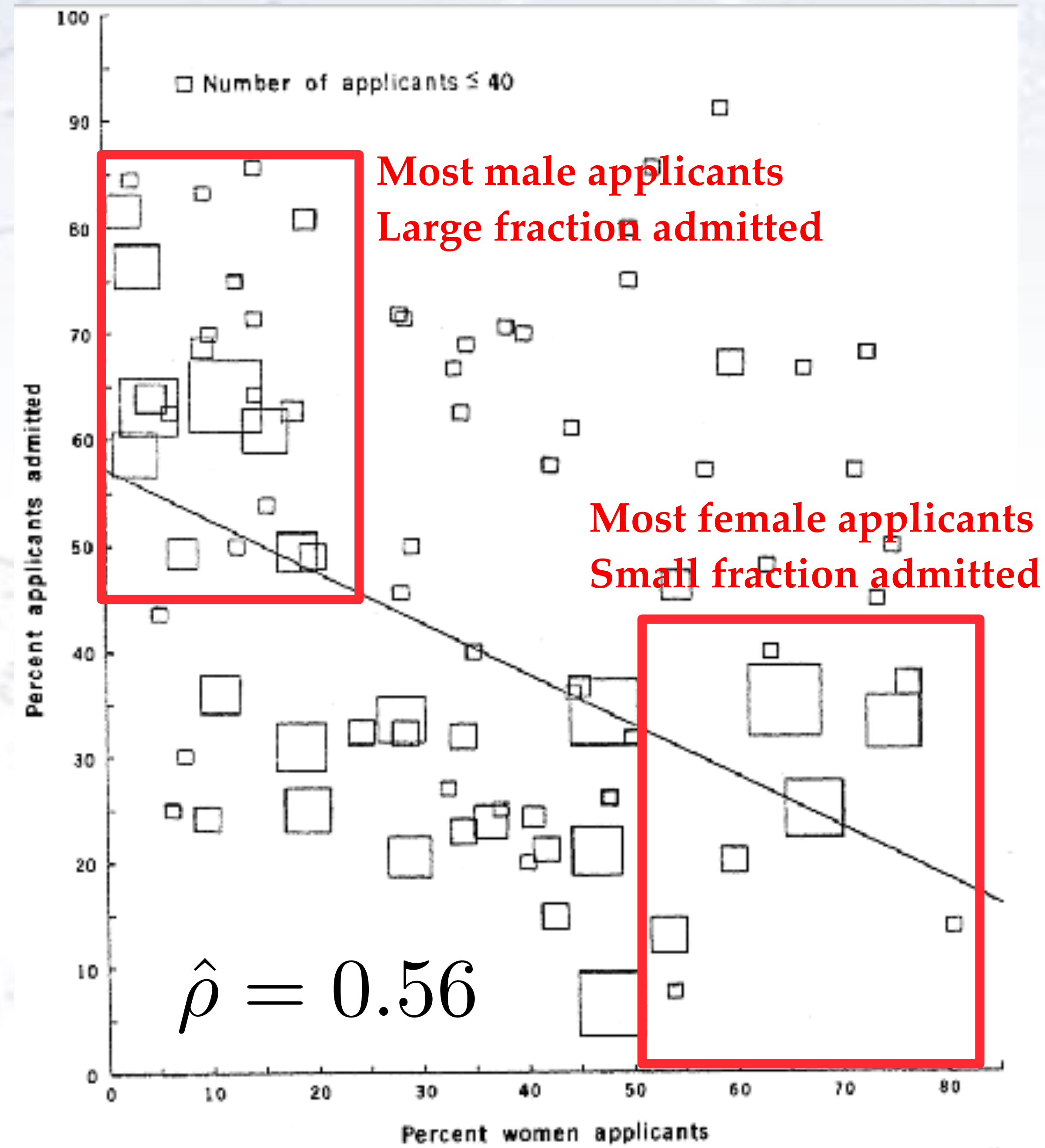


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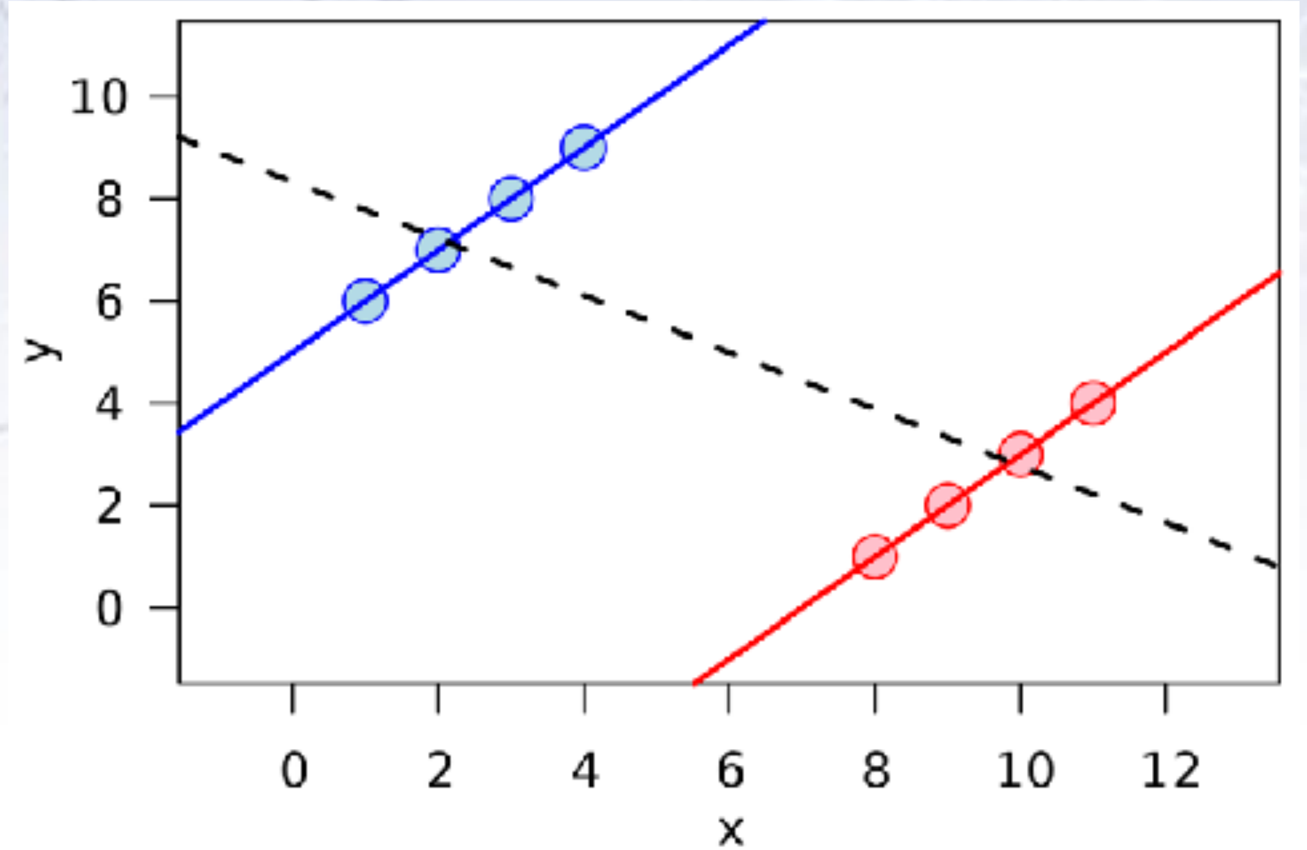
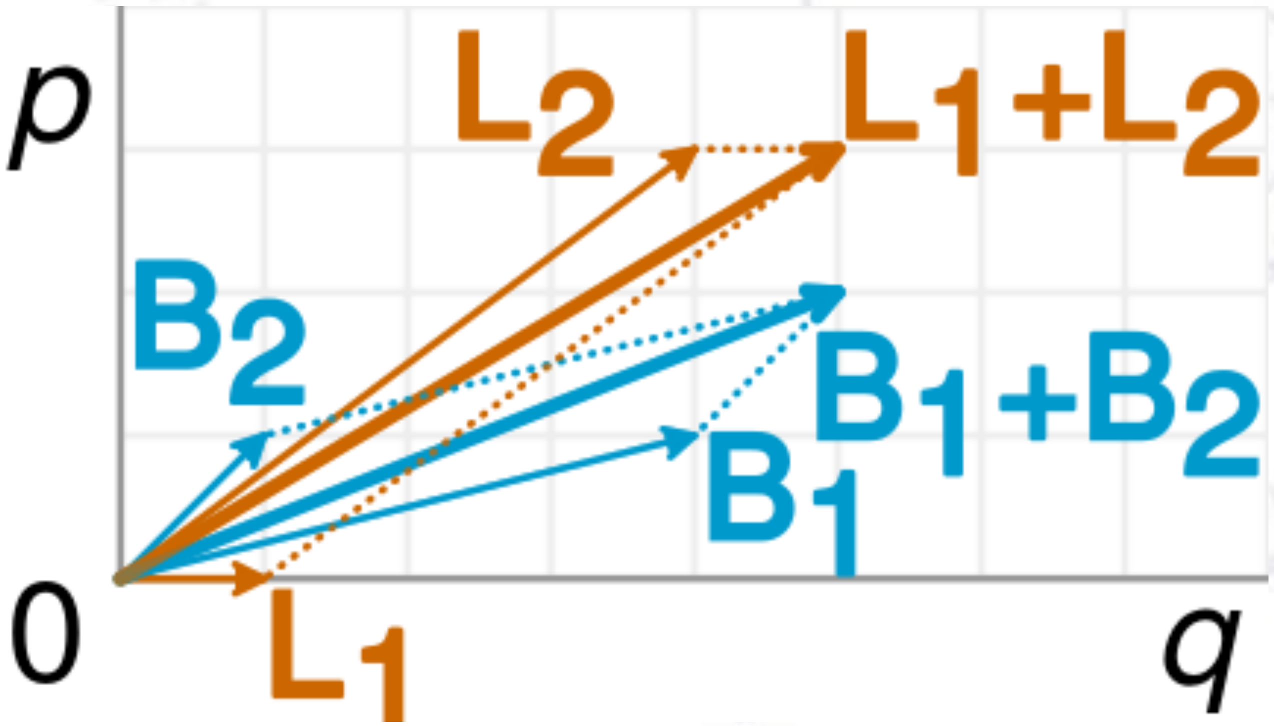
# Simpson's Paradox explained

The reason for the **apparent** paradox arise when frequency data is unduly given causal interpretations.

The figure on the right illustrates the "paradox" nicely.




The situation can be illustrated with 2D vectors, as shown below.




A succes rate p/q (successes / attempts) can be represented by vectors with a slope. Higher slope = higher succes rate.

But though B1 is steeper than L1, and B2 is steeper than L2, then B1+B1 is not as steep as L1+L2.