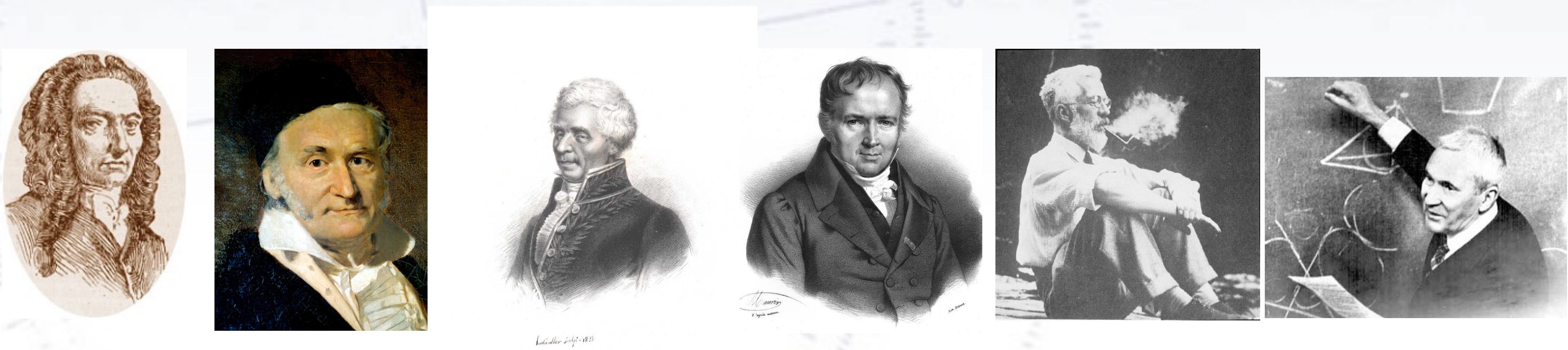
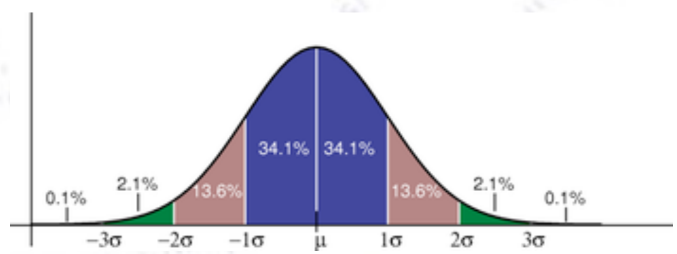
Applied Statistics

Measuring the length of a Table...

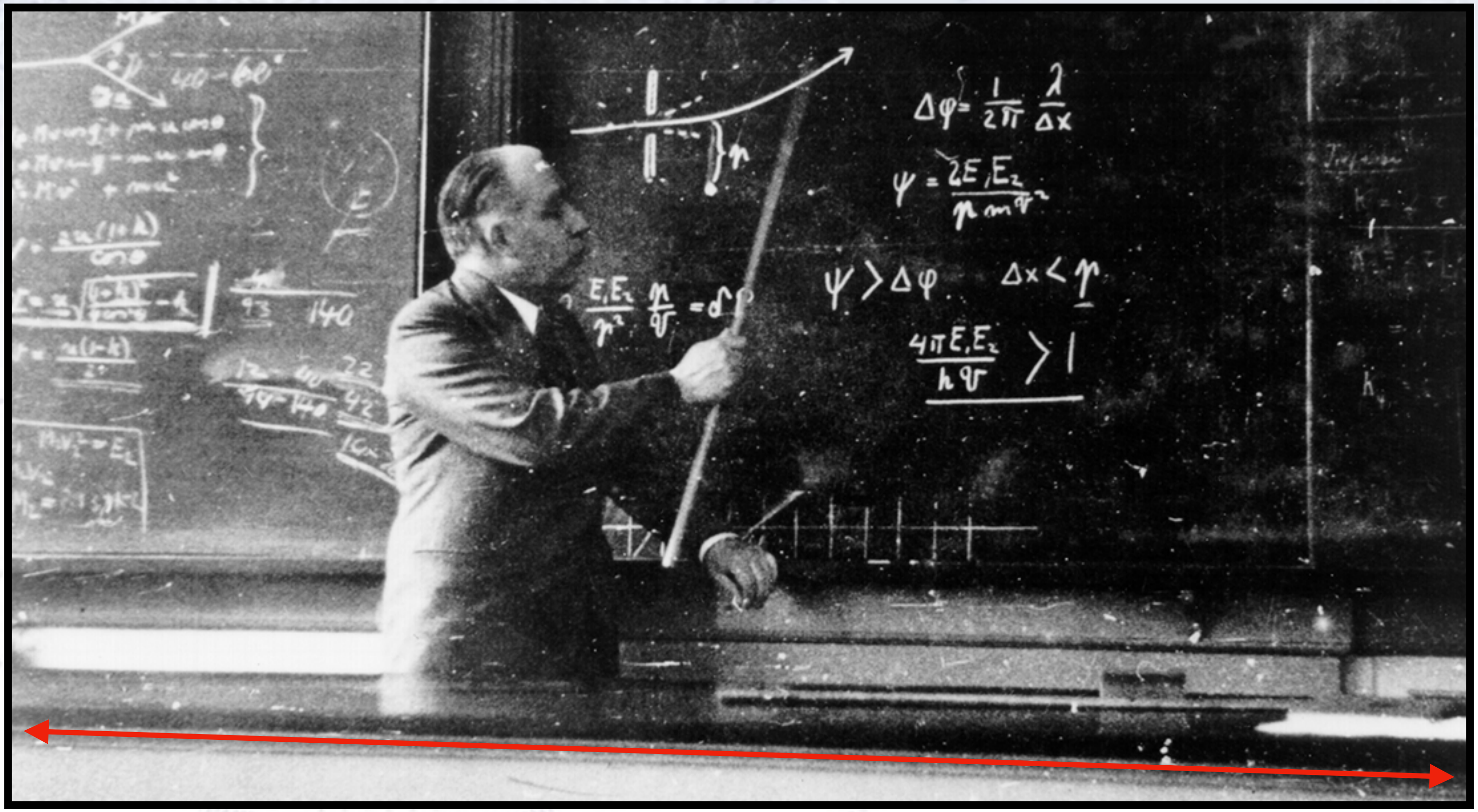


Troels C. Petersen (NBI)



"Statistics is merely a quantisation of common sense"

The table in auditorium A



"Everything is vague to a degree you do not realise till you have tried to make it precise."

[Bertrand Russell, 1872-1970]

My analysis (2009-2025 data)

The table measurement data

The initial dataset contains (valid measurements):

- 30cm measurements: **1007** Range: [0.0, 6.0] m
- 2m measurements: **1005** Range: [0.0, 6.0] m

Note: Both 30cm and 2m measurements were **blinded** with an unknown offset chosen from a 10cm wide Gaussian.

```
if blinded:
    blinding30cm = r.normal(0, 0.1)      # I add a constant (Gaussian with +-10cm) to remain "blind"
    blinding2m   = r.normal(0, 0.1)      # I add a constant (Gaussian with +-10cm) to remain "blind"
else:
    blinding30cm = 0
    blinding2m = 0
```

```
# Loop over files and open them
for infile in infiles:

    tmp_L30cm, tmp_eL30cm, tmp_L2m, tmp_eL2m = np.loadtxt(infile, skiprows=2, unpack=True)

    # Note that blinding is applied before storing the values read:
    L30cm = np.append(L30cm, tmp_L30cm + blinding30cm)
    eL30cm = np.append(eL30cm, tmp_eL30cm)
    L2m = np.append(L2m, tmp_L2m + blinding2m)
    eL2m = np.append(eL2m, tmp_eL2m)
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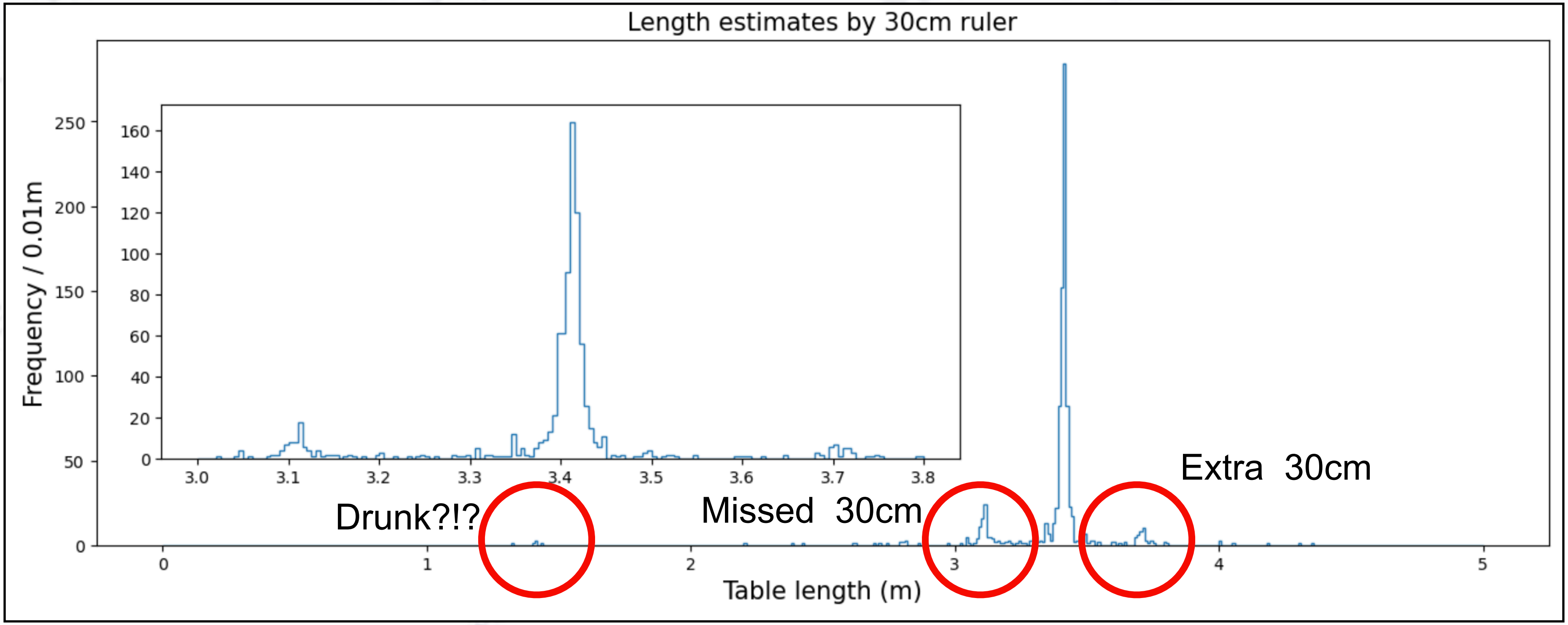

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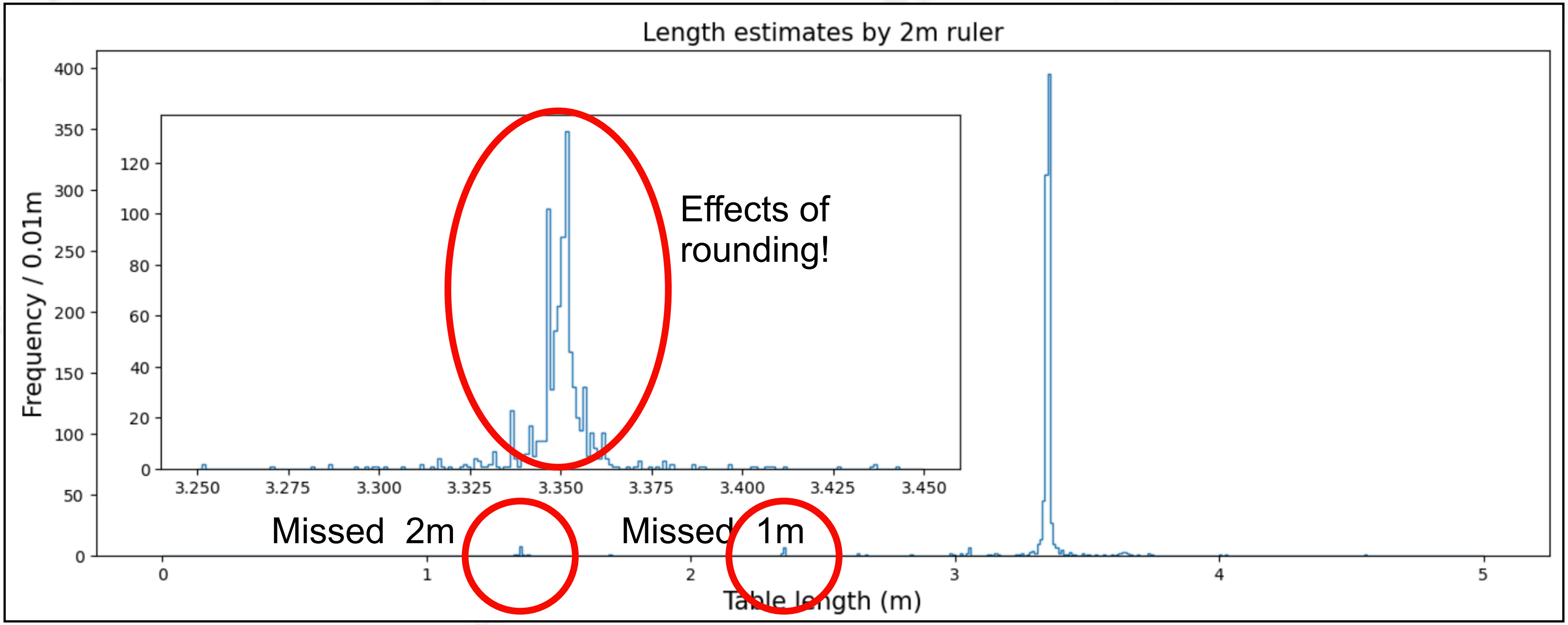

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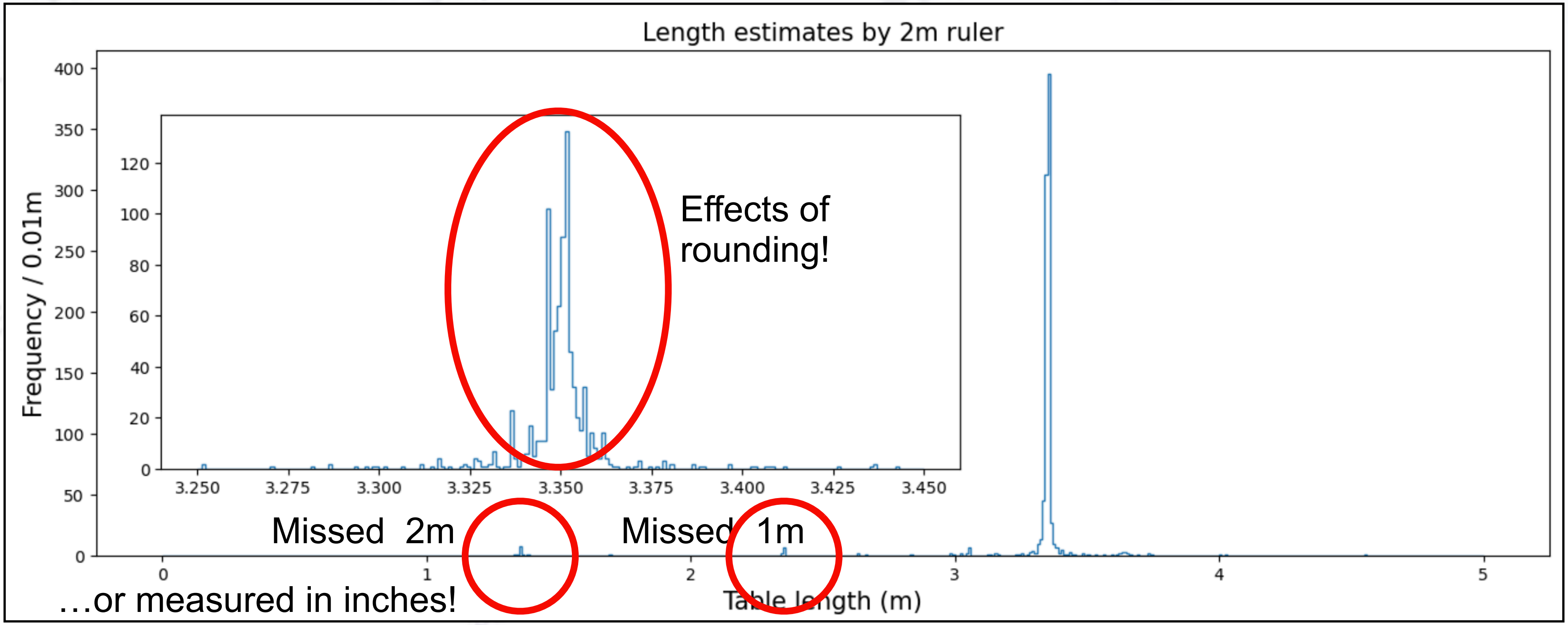
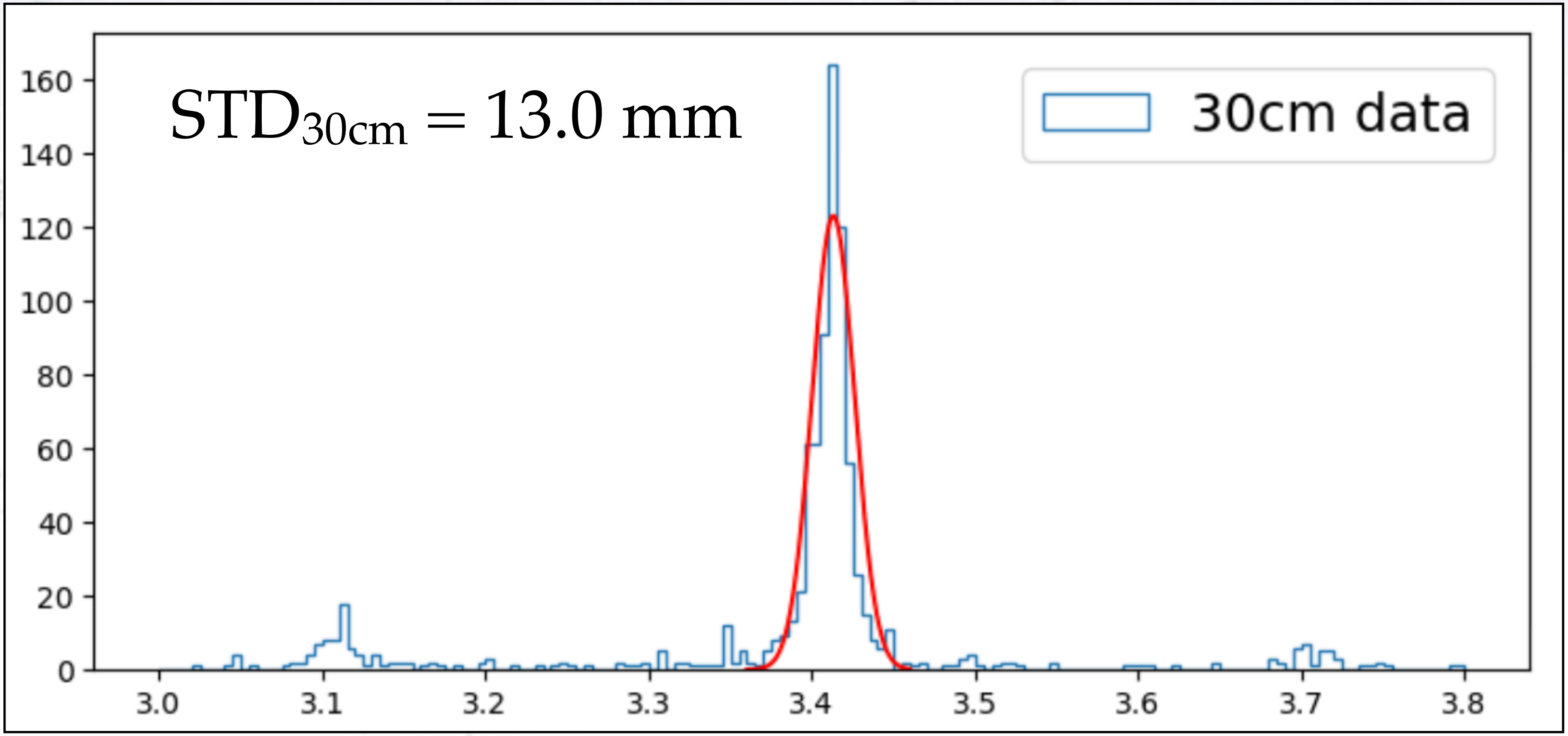


…or measured in inches!

3.36 / 2.54 = 1.32

Inspecting the data

The 30cm peak seems somewhat Gaussian (p=2.4%) with binning 0.005m (smaller binning shows discontinuities, i.e. gives peaks).
The 2m peak does not seem Gaussian with any binning (here 0.005), yet “collected”.



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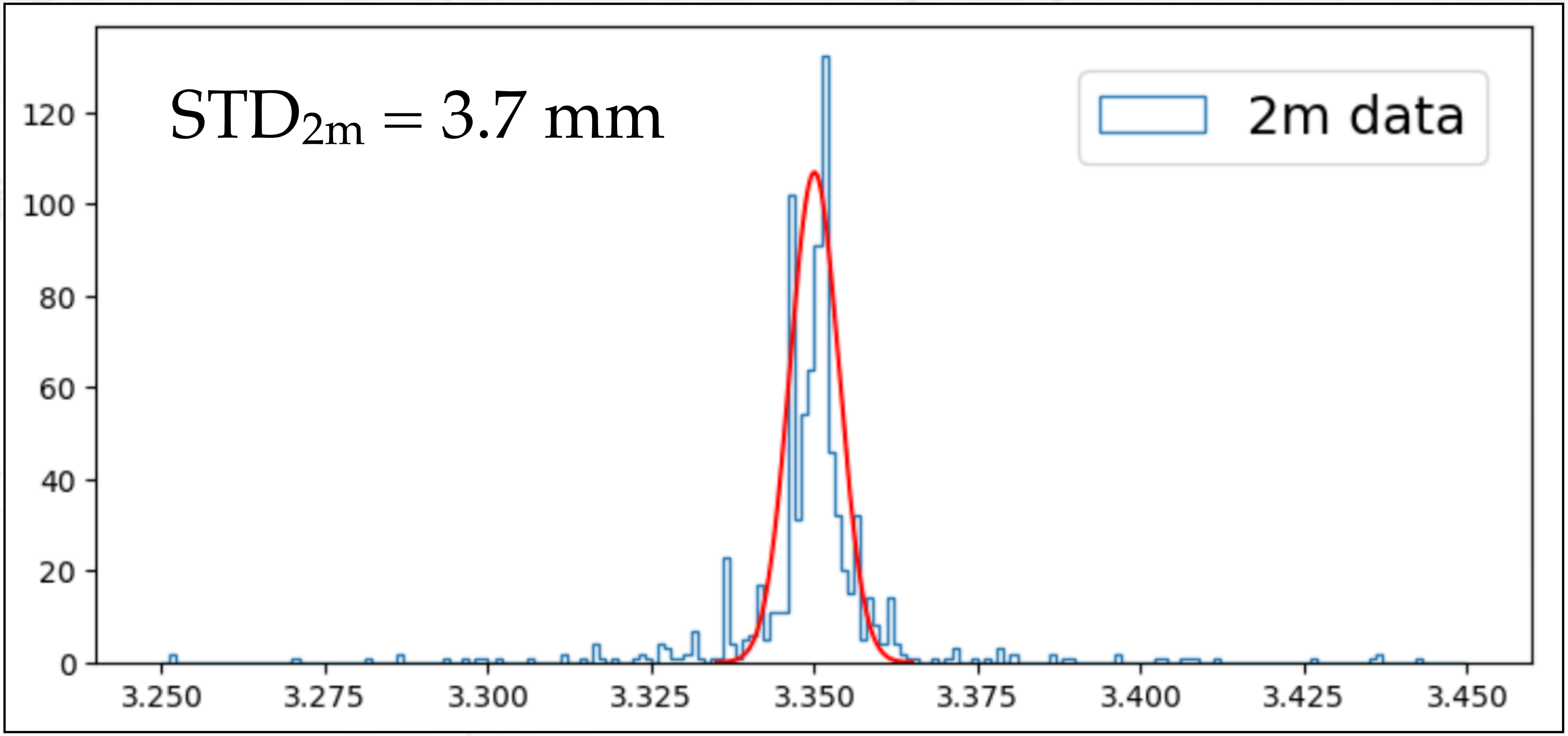
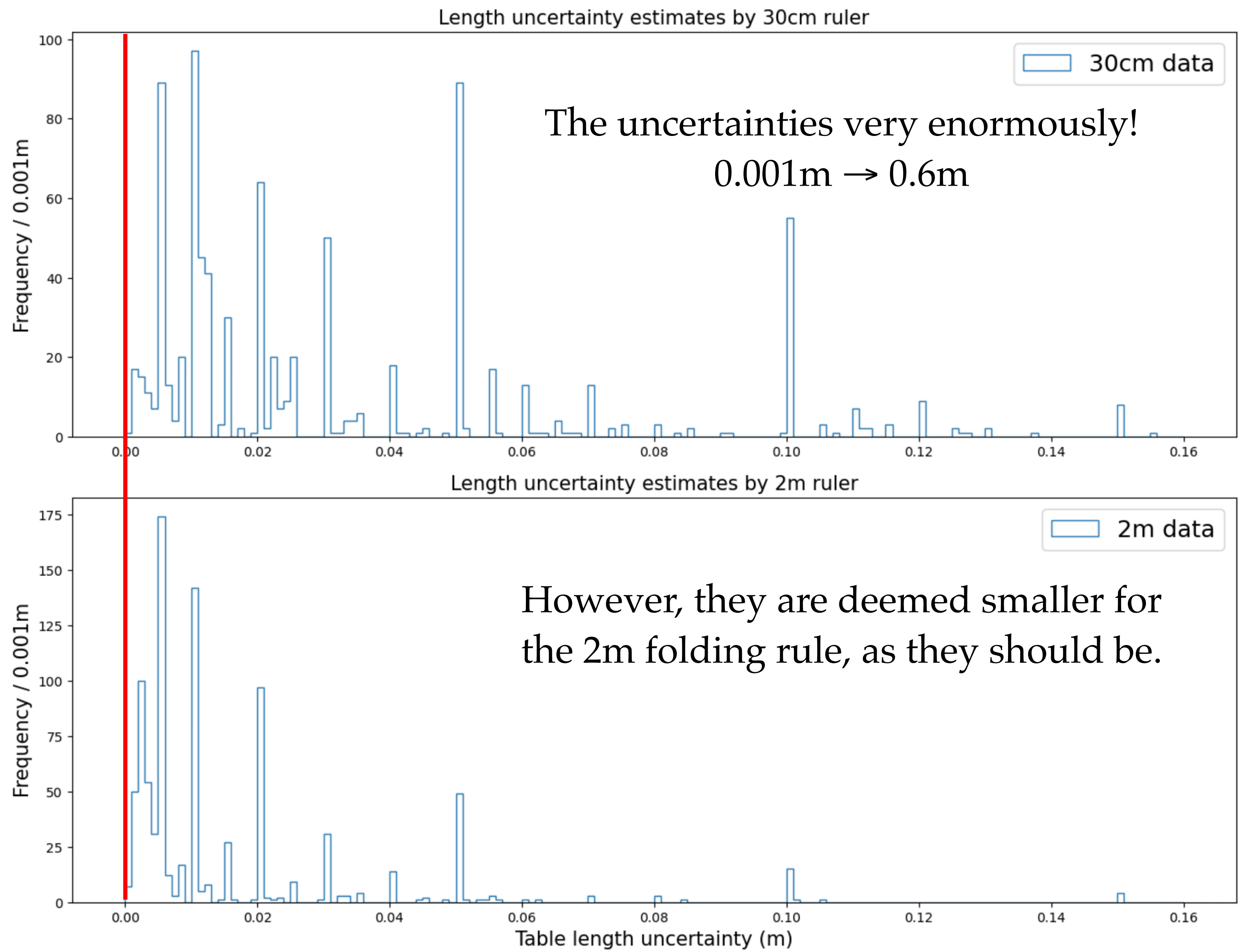


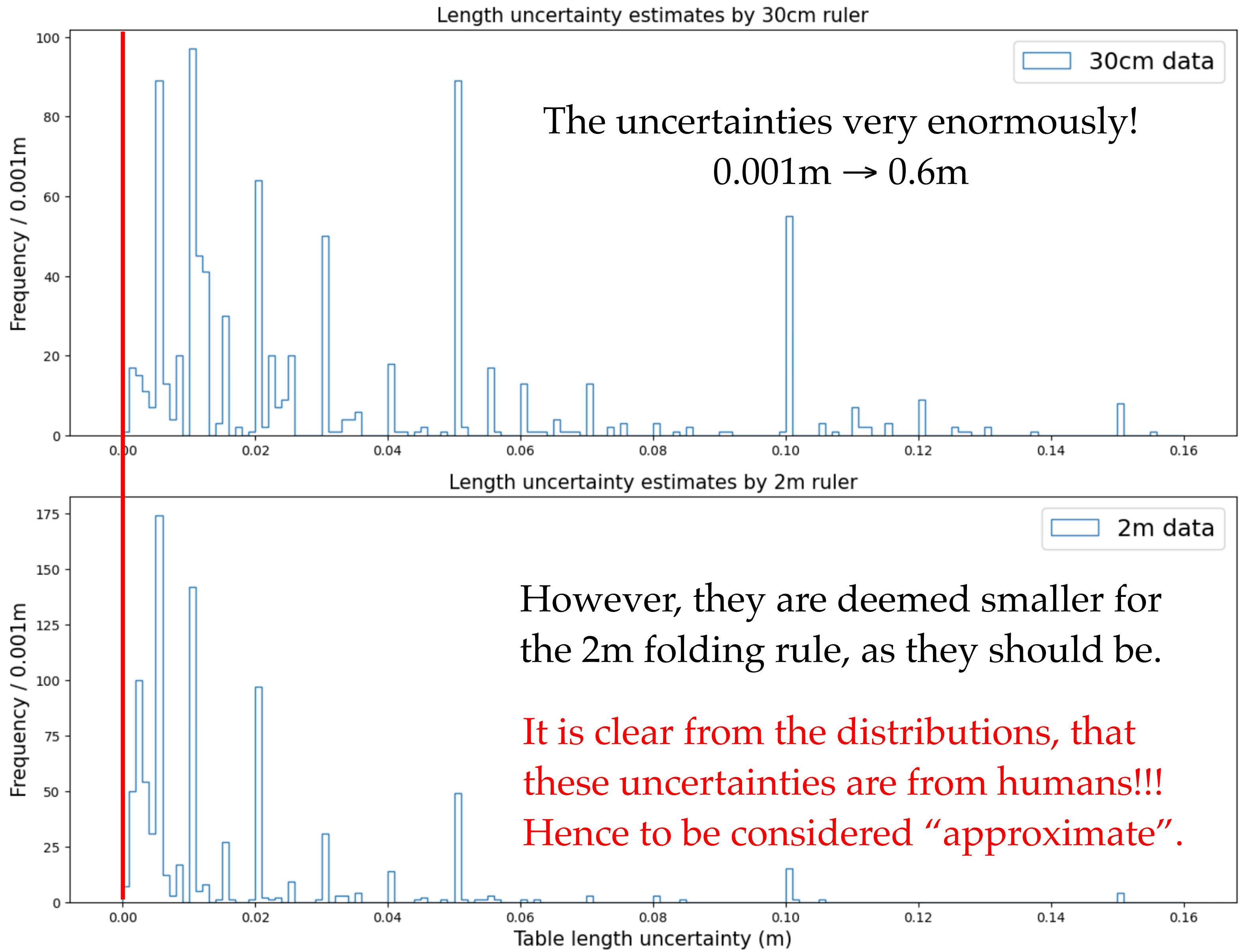
Uncertainties



The uncertainties very enormously!
0.001m → 0.6m

However, they are deemed smaller for the 2m folding rule, as they should be.

Uncertainties



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It is clear from the distributions, that these uncertainties are from humans!!! Hence to be considered "approximate".

Raw ("Naive") results

<u>30cm:</u>

Mean = 3.3728 ± 0.0077 m

Std. = 0.24 m (N = 1007)

<u>2m:</u>

Mean = 3.3088 ± 0.0092 m

Std. = 0.29 m (N = 1005)

From the Std. values alone, it is clear that something is terribly wrong, which is also why the uncertainties on the mean are almost a centimeter!

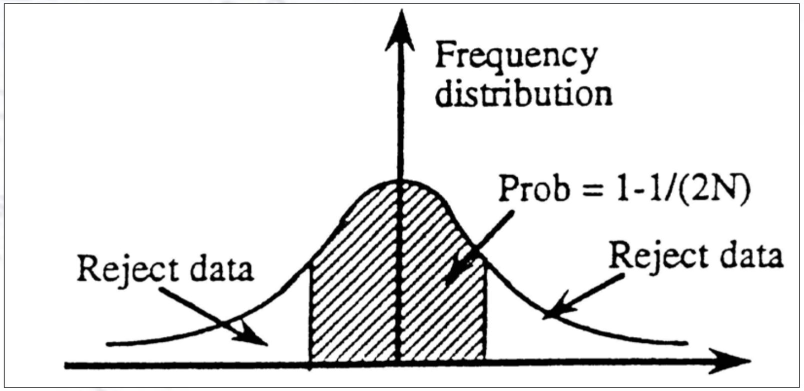
A bit of measurement philosophy

Removing data - General

Some (very "purist") scientists would never allow for the reject of data points! They would argue, that data reflects reality, and that one should simply model this, including imperfections.

Less "purist" scientists accept exclusion of some data points. However, **one should always be very careful about removing data points**, and only be willing to do so, if very good arguments can be found:

- It is clearly an errornous measurement.
- Measurement is highly improbable.



Preferably, one would like to understand why data points seem faulty.

The procedures for removing points are:

- Without errors: **Chauvenet's Criterion,** removing furthest point if unlikely.
- With errors: Simply reject based on the z-value = $(x - \mu)/\sigma$ of the point

However, **ALWAYS keep a record of your original data**, as it may contain more effects than you originally thought. And **all points have the right to a fair hearing!**

Removing data - without errors

Removing improbable data points when no error is given is formalised in **Chauvenet's Criterion,** though many other methods exists (Pierce, Grubbs, etc.)

The overall idea is to assume that the distribution is Gaussian.
One calculates the mean (μ) and standard deviation (σ), and then:
1. Ask what the probability of the farthest point is (given the number of points)
2. Remove point, if it is below some value (e.g. 0.05, preferably decided in advance)
3. If the furthest point was removed, then recalculate μ and σ, and go to 1.

How to calculate the probability of the furthest point with value x (given μ and σ)?

1. Calculate z:

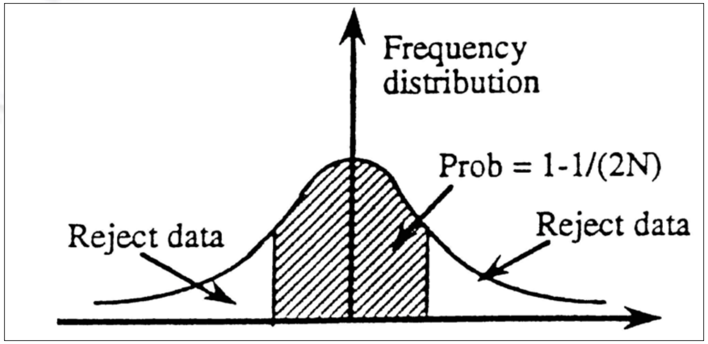
$$z = (x - \mu)/\sigma$$

2. Find the probability of this z, p_{local}:

$$p_{\text{local}} = \int_{-\infty}^{-z} G(z)\,dz + \int_{z}^{\infty} G(z)\,dz$$

3. Take number of point into account, to get p_{global}:

$$p_{\text{global}} = 1 - (1 - p_{\text{local}})^{N_{\text{points}}}$$



Key question:
Is $p_{global} < 0.05$?

Removing data - with errors

Removing improbable data points when each point has an associated uncertainty is much simpler.

The overall idea is that all points should be consistent with a mean value.
One calculates the weighted mean (μ), and then removes all points that are more than z_{cut} sigma away. Done!

No iterative procedure is needed. One can can calculate the value of z_{cut} ahead of applying it as:

$$p_{\text{local}} = 1 - (1 - p_{\text{global}})^{1/N_{\text{points}}}$$

Example:
You have 1000 measurements, all with uncertainties, and decide to discard all points which are less likely than $p_{global} = 0.05$. This yields a cut at $p_{local} = 0.000051$ or 4.05σ. Thus, one would reject all data, which are more than 4.05σ away from the mean.

```
p_local = 1.0 - (1.0 - p_global)**(1.0/Ndata)
Nsigma = np.abs(stats.norm.ppf(p_local/2.0))
```

Unweighted analysis

...a fair hearing?

```
 -----------------   Chauvenet's Criterion (30cm)  -----------------
 14: L=1.271  dL=2.102  Nsig= 8.63 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1007 mean=3.3728  std=0.2436  -> Rejected
693: L=1.325  dL=2.050  Nsig= 8.74 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1006 mean=3.3749  std=0.2346  -> Rejected
154: L=1.405  dL=1.972  Nsig= 8.74 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1005 mean=3.3769  std=0.2256  -> Rejected
190: L=1.413  dL=1.966  Nsig= 9.06 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1004 mean=3.3789  std=0.2169  -> Rejected
645: L=1.415  dL=1.966  Nsig= 9.45 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1003 mean=3.3808  std=0.2080  -> Rejected
922: L=1.420  dL=1.963  Nsig= 9.89 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1002 mean=3.3828  std=0.1986  -> Rejected
 69: L=1.424  dL=1.961  Nsig=10.39 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1001 mean=3.3848  std=0.1887  -> Rejected
281: L=1.434  dL=1.953  Nsig=10.95 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1000 mean=3.3867  std=0.1783  -> Rejected
254: L=2.206  dL=1.183  Nsig= 7.07 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=999 mean=3.3887  std=0.1674  -> Rejected
353: L=2.383  dL=1.007  Nsig= 6.17 p_loc=0.00000000  p_glob=0.00000034 >? pmin=0.100   N=998 mean=3.3899  std=0.1632  -> Rejected
121: L=2.422  dL=0.969  Nsig= 6.05 p_loc=0.00000000  p_glob=0.00000071 >? pmin=0.100   N=997 mean=3.3909  std=0.1601  -> Rejected
202: L=4.355  dL=0.963  Nsig= 6.12 p_loc=0.00000000  p_glob=0.00000046 >? pmin=0.100   N=996 mean=3.3918  std=0.1572  -> Rejected
918: L=4.310  dL=0.919  Nsig= 5.95 p_loc=0.00000000  p_glob=0.00000131 >? pmin=0.100   N=995 mean=3.3909  std=0.1543  -> Rejected
878: L=4.190  dL=0.800  Nsig= 5.27 p_loc=0.00000007  p_glob=0.00006631 >? pmin=0.100   N=994 mean=3.3900  std=0.1516  -> Rejected
865: L=2.613  dL=0.776  Nsig= 5.19 p_loc=0.00000010  p_glob=0.00010359 >? pmin=0.100   N=993 mean=3.3891  std=0.1496  -> Rejected
211: L=2.625  dL=0.765  Nsig= 5.18 p_loc=0.00000011  p_glob=0.00010725 >? pmin=0.100   N=992 mean=3.3899  std=0.1476  -> Rejected
304: L=2.700  dL=0.691  Nsig= 4.74 p_loc=0.00000105  p_glob=0.00103608 >? pmin=0.100   N=991 mean=3.3907  std=0.1457  -> Rejected
 38: L=2.704  dL=0.688  Nsig= 4.77 p_loc=0.00000090  p_glob=0.00089425 >? pmin=0.100   N=990 mean=3.3914  std=0.1441  -> Rejected
859: L=2.720  dL=0.672  Nsig= 4.72 p_loc=0.00000118  p_glob=0.00116584 >? pmin=0.100   N=989 mean=3.3921  std=0.1425  -> Rejected
690: L=4.054  dL=0.661  Nsig= 4.69 p_loc=0.00000137  p_glob=0.00134971 >? pmin=0.100   N=988 mean=3.3928  std=0.1409  -> Rejected
587: L=2.750  dL=0.642  Nsig= 4.61 p_loc=0.00000203  p_glob=0.00200255 >? pmin=0.100   N=987 mean=3.3921  std=0.1394  -> Rejected
 19: L=2.758  dL=0.635  Nsig= 4.60 p_loc=0.00000208  p_glob=0.00205091 >? pmin=0.100   N=986 mean=3.3928  std=0.1380  -> Rejected
384: L=4.010  dL=0.616  Nsig= 4.51 p_loc=0.00000319  p_glob=0.00314082 >? pmin=0.100   N=985 mean=3.3934  std=0.1365  -> Rejected
269: L=4.009  dL=0.616  Nsig= 4.56 p_loc=0.00000261  p_glob=0.00256778 >? pmin=0.100   N=984 mean=3.3928  std=0.1352  -> Rejected
 71: L=4.005  dL=0.613  Nsig= 4.58 p_loc=0.00000236  p_glob=0.00231757 >? pmin=0.100   N=983 mean=3.3922  std=0.1338  -> Rejected
690: L=4.005  dL=0.613  Nsig= 4.63 p_loc=0.00000184  p_glob=0.00180428 >? pmin=0.100   N=982 mean=3.3915  std=0.1325  -> Rejected
534: L=2.794  dL=0.597  Nsig= 4.56 p_loc=0.00000260  p_glob=0.00254932 >? pmin=0.100   N=981 mean=3.3909  std=0.1311  -> Rejected
439: L=2.800  dL=0.592  Nsig= 4.56 p_loc=0.00000254  p_glob=0.00248514 >? pmin=0.100   N=980 mean=3.3915  std=0.1297  -> Rejected
330: L=2.808  dL=0.584  Nsig= 4.55 p_loc=0.00000267  p_glob=0.00261183 >? pmin=0.100   N=979 mean=3.3921  std=0.1284  -> Rejected
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104: L=2.815  dL=0.579  Nsig= 4.60 p_loc=0.00000212  p_glob=0.00206727 >? pmin=0.100   N=977 mean=3.3933  std=0.1258  -> Rejected
743: L=2.818  dL=0.576  Nsig= 4.63 p_loc=0.00000184  p_glob=0.00179668 >? pmin=0.100   N=976 mean=3.3939  std=0.1245  -> Rejected
646: L=2.819  dL=0.576  Nsig= 4.67 p_loc=0.00000147  p_glob=0.00143633 >? pmin=0.100   N=975 mean=3.3945  std=0.1232  -> Rejected
258: L=2.864  dL=0.531  Nsig= 4.36 p_loc=0.00000648  p_glob=0.00628935 >? pmin=0.100   N=974 mean=3.3951  std=0.1219  -> Rejected
294: L=3.891  dL=0.495  Nsig= 4.10 p_loc=0.00002061  p_glob=0.01985353 >? pmin=0.100   N=973 mean=3.3956  std=0.1207  -> Rejected
  0: L=3.827  dL=0.432  Nsig= 3.60 p_loc=0.00015662  p_glob=0.14122203 >? pmin=0.100   N=972 mean=3.3951  std=0.1197  -> Accepted
```

...a fair hearing?

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 258: L=2.864  dL=0.531  Nsig= 4.36 p_loc=0.00000648  p_glob=0.00628935 >? pmin=0.100   N=974 mean=3.3951  std=0.1219  -> Rejected
 294: L=3.891  dL=0.495  Nsig= 4.10 p_loc=0.00002061  p_glob=0.01985353 >? pmin=0.100   N=973 mean=3.3956  std=0.1207  -> Rejected
   0: L=3.827  dL=0.432  Nsig= 3.60 p_loc=0.00015662  p_glob=0.14122203 >? pmin=0.100   N=972 mean=3.3951  std=0.1197  -> Accepted
```

...a fair hearing?

```
 -----------------   Chauvenet's Criterion (30cm)  -----------------
  14: L=1.271  dL=2.102  Nsig= 8.63 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1007 mean=3.3728  std=0.2436  -> Rejected
 693: L=1.325  dL=2.050  Nsig= 8.74 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1006 mean=3.3749  std=0.2346  -> Rejected
 154: L=1.405  dL=1.972  Nsig= 8.74 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1005 mean=3.3769  std=0.2256  -> Rejected
 190: L=1.413  dL=1.966  Nsig= 9.06 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1004 mean=3.3789  std=0.2169  -> Rejected
 645: L=1.415  dL=1.966  Nsig= 9.45 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1003 mean=3.3808  std=0.2080  -> Rejected
 922: L=1.420  dL=1.963  Nsig= 9.89 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1002 mean=3.3828  std=0.1986  -> Rejected
  69: L=1.424  dL=1.961  Nsig=10.39 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1001 mean=3.3848  std=0.1887  -> Rejected
 281: L=1.434  dL=1.953  Nsig=10.95 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=1000 mean=3.3867  std=0.1783  -> Rejected
 254: L=2.206  dL=1.183  Nsig= 7.07 p_loc=0.00000000  p_glob=0.00000000 >? pmin=0.100   N=999 mean=3.3887  std=0.1674  -> Rejected
 353: L=2.383  dL=1.007  Nsig= 6.17 p_loc=0.00000000  p_glob=0.00000034 >? pmin=0.100   N=998 mean=3.3899  std=0.1632  -> Rejected
 121: L=2.422  dL=0.969  Nsig= 6.05 p_loc=0.00000000  p_glob=0.00000071 >? pmin=0.100   N=997 mean=3.3909  std=0.1601  -> Rejected
 202: L=4.355  dL=0.963  Nsig= 6.12 p_loc=0.00000000  p_glob=0.00000046 >? pmin=0.100   N=996 mean=3.3918  std=0.1572  -> Rejected
 918: L=4.310  dL=0.919  ...  d=0.1543  -> Rejected
 878: L=4.190  dL=0.800  ...  d=0.1516  -> Rejected
 865: L=2.613  dL=0.776  ...  d=0.1496  -> Rejected
 211: L=2.625  dL=0.765  ...  d=0.1476  -> Rejected
 304: L=2.700  dL=0.691  ...  d=0.1457  -> Rejected
  38: L=2.704  dL=0.688  ...  d=0.1441  -> Rejected
 859: L=2.720  dL=0.672  ...  d=0.1425  -> Rejected
 690: L=4.054  dL=0.661  ...  d=0.1409  -> Rejected
 587: L=2.750  dL=0.642  ...  d=0.1394  -> Rejected
  19: L=2.758  dL=0.635  ...  d=0.1380  -> Rejected
 384: L=4.010  dL=0.616  ...  d=0.1365  -> Rejected
 269: L=4.009  dL=0.616  ...  d=0.1352  -> Rejected
  71: L=4.005  dL=0.613  ...  d=0.1338  -> Rejected
 690: L=4.005  dL=0.613  Nsig= 4.63 p_loc=0.00000184  p_glob=0.00180428 >? pmin=0.100   N=982 mean=3.3915  std=0.1325  -> Rejected
 534: L=2.794  dL=0.597  Nsig= 4.56 p_loc=0.00000260  p_glob=0.00254932 >? pmin=0.100   N=981 mean=3.3909  std=0.1311  -> Rejected
 439: L=2.800  dL=0.592  Nsig= 4.56 p_loc=0.00000254  p_glob=0.00248514 >? pmin=0.100   N=980 mean=3.3915  std=0.1297  -> Rejected
 330: L=2.808  dL=0.584  Nsig= 4.55 p_loc=0.00000267  p_glob=0.00261183 >? pmin=0.100   N=979 mean=3.3921  std=0.1284  -> Rejected
 512: L=2.809  dL=0.584  Nsig= 4.59 p_loc=0.00000217  p_glob=0.00212091 >? pmin=0.100   N=978 mean=3.3927  std=0.1271  -> Rejected
 104: L=2.815  dL=0.579  Nsig= 4.60 p_loc=0.00000212  p_glob=0.00206727 >? pmin=0.100   N=977 mean=3.3933  std=0.1258  -> Rejected
 743: L=2.818  dL=0.576  Nsig= 4.63 p_loc=0.00000184  p_glob=0.00179668 >? pmin=0.100   N=976 mean=3.3939  std=0.1245  -> Rejected
 646: L=2.819  dL=0.576  Nsig= 4.67 p_loc=0.00000147  p_glob=0.00143633 >? pmin=0.100   N=975 mean=3.3945  std=0.1232  -> Rejected
 258: L=2.864  dL=0.531  Nsig= 4.36 p_loc=0.00000648  p_glob=0.00628935 >? pmin=0.100   N=974 mean=3.3951  std=0.1219  -> Rejected
 294: L=3.891  dL=0.495  Nsig= 4.10 p_loc=0.00002061  p_glob=0.01985353 >? pmin=0.100   N=973 mean=3.3956  std=0.1207  -> Rejected
   0: L=3.827  dL=0.432  Nsig= 3.60 p_loc=0.00015662  p_glob=0.14122203 >? pmin=0.100   N=972 mean=3.3951  std=0.1197  -> Accepted
```

I rejected:
35 data points from the 30cm sample (*),
145 data points from the 2m sample.
And I inspected each and every one!

Unweighted results

<u>30cm:</u>

Mean = 3.39512 ± 0.00384 m

Std. = 0.120 m (N = 972)

<u>2m:</u>

Mean = 3.34950 ± 0.00018 m

Std. = 0.0054 m (N = 860)

Without corrections and Chavenet's (p=0.10)

While the 2m result starts looking realistic (Std. of 5.5mm) and precise (1/5th of a millimeter), **the 30cm result is still terrible**.

This is because the two ±30cm peaks are not removed by Chavenet's Criterion.

30cm: Reject or correct?

The poor 30cm result is due to the (many) mis-measurements of ± 30cm, which are not rejected by the Chauvenet's Criterion with p_global = 0.10.

At least two solutions exist:

1. Decide to **reject** all measurements more than 15cm away from the mean and run Chauvenet's Criterion again.
2. Decide to **correct** measurements 15-45cm away from the mean, and run Chauvenet's Criterion again.

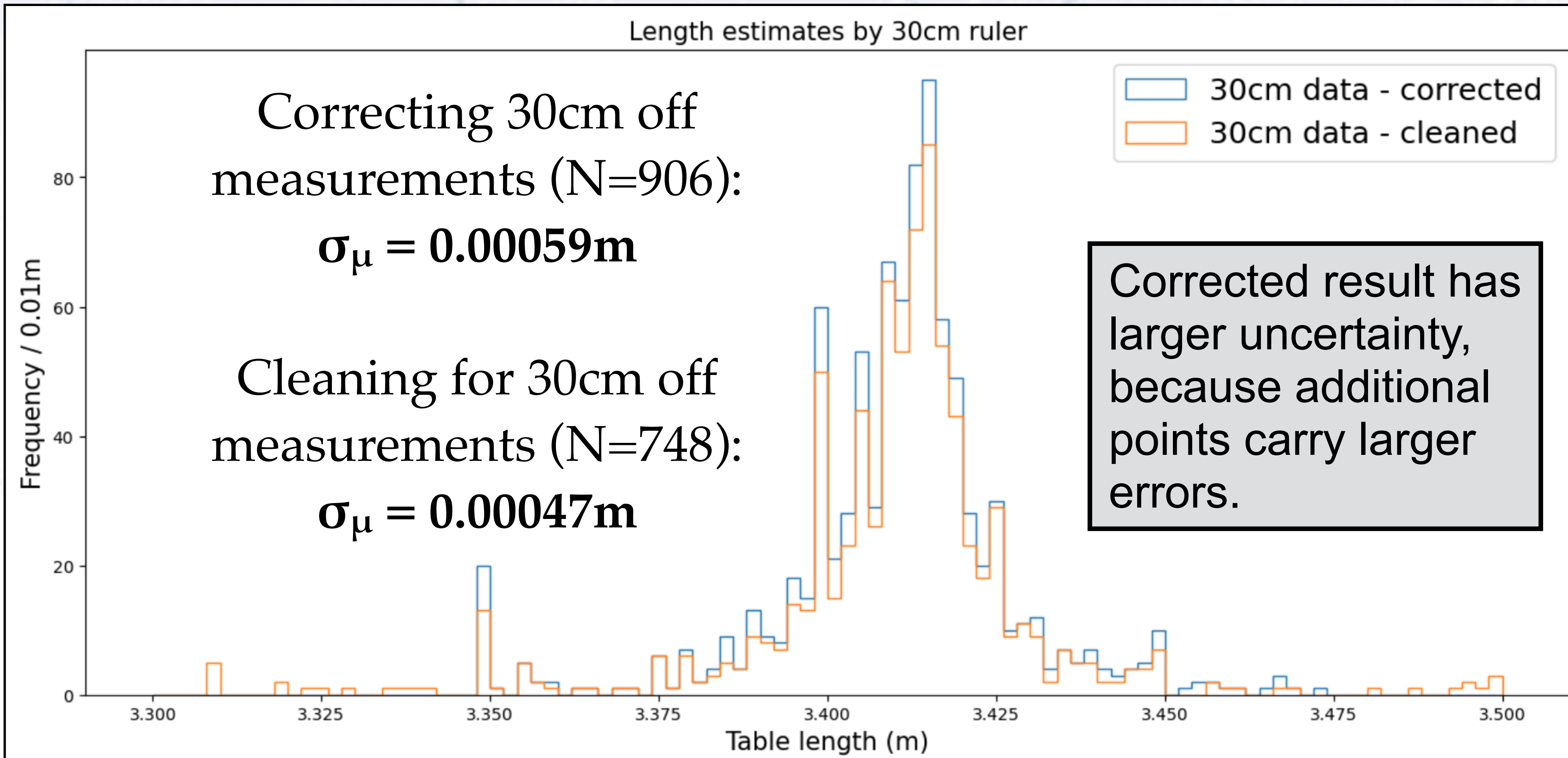
Rejecting the events removes a total of 189 (± 15cm) + 70 (CC) measurements.
Correcting the events first removes a total of 91 measurements.

It would generally be nice not to have to correct any results!

But let us focus on the good measurements, if this can be argued.

30cm: Reject or correct?

There are some clear and understandable mis-measurements.
Should one correct and include these?
Depends on resulting improvement, but decide without seeing the final result.



The correct way to combine

If one needs to combine results of different quality, the correct way is to:

- First, produce results for each group of measurements.
- Then combine these in a weighted average, provided they are consistent.

In this way, the poor quality results do not "degrade" those of high quality.

In our case, one would get the following results (cleaning around each peak):

Low peak result:	3.41540 ± 0.00370 m	(Std = 0.036 m, N = 97)
Central peak result:	3.41198 ± 0.00047 m	(Std = 0.013 m, N = 748)
High peak result:	3.39919 ± 0.00750 m	(Std = 0.055 m, N = 54)

The combination in a weighted mean gives:

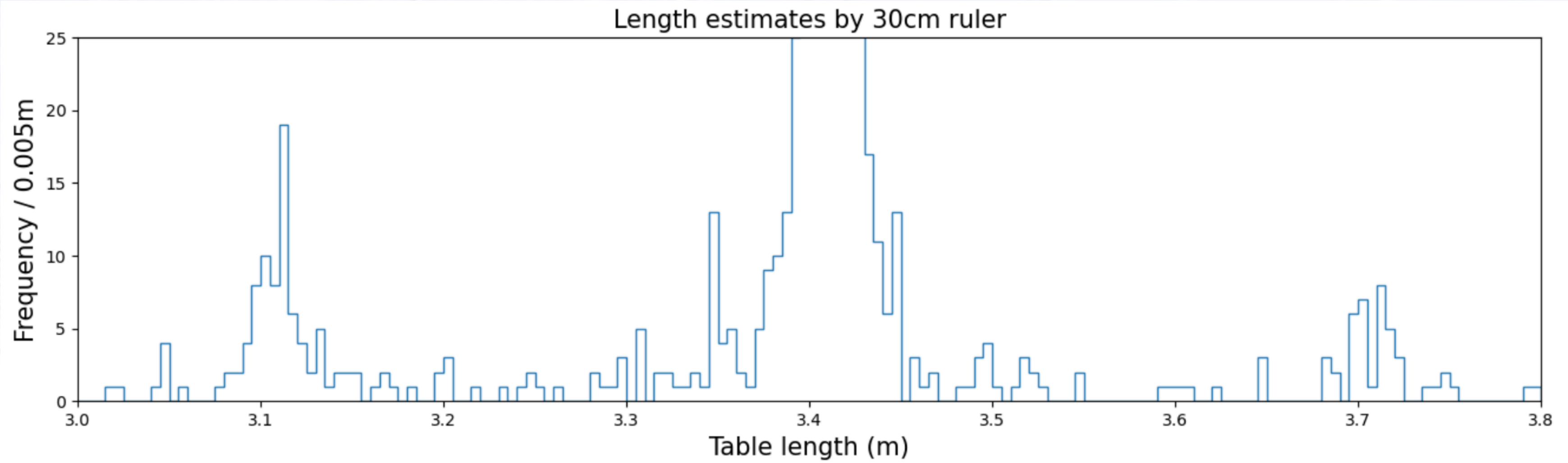
Combined result: 3.41198 ± 0.00046 m Prob(Chi2 = 3.8, N_{dof} = 2) = 0.152

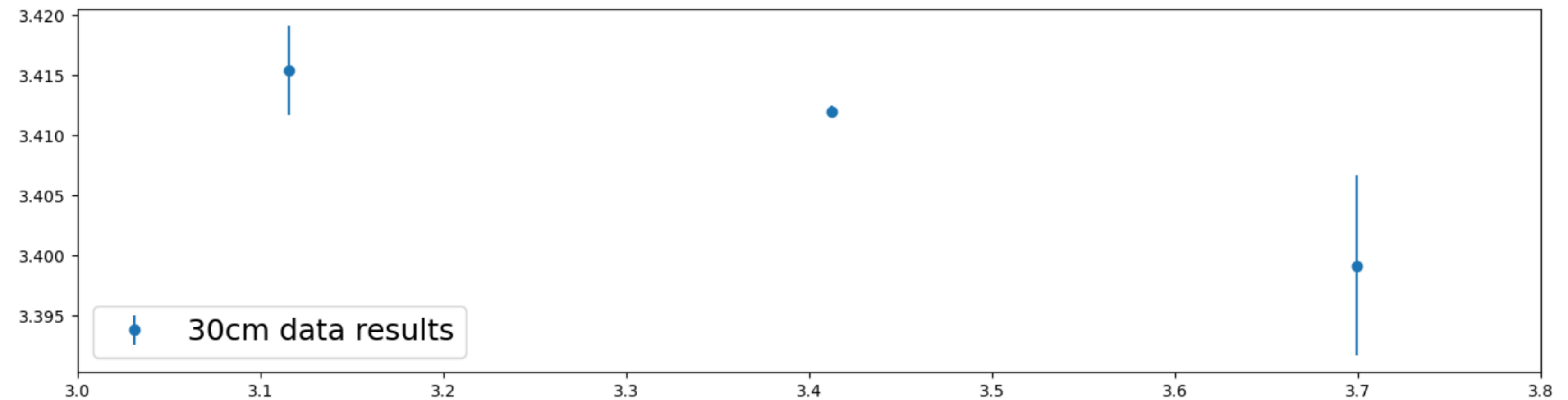
Conclusion:

Thus it all combines well, but the improvement is minuscule compared to the complications and questions it raises.

The correct way to combine

Visual inspection of the data and numbers:





Unweighted results

30cm:

Mean = 3.41198 ± 0.00047 m

Std. = 0.0128 m (N = 748)

2m:

Mean = 3.34950 ± 0.00018 m

Std. = 0.0054 m (N = 860)

With cleaning and Chavenet's (p=0.10)

Now the results are precise, and the 2m result is about a factor 2.5 more so, as would also be expected from the initial Std. observed for the peaks.

The improvement over the naive 30cm / 2m results are factores of 15 / 40

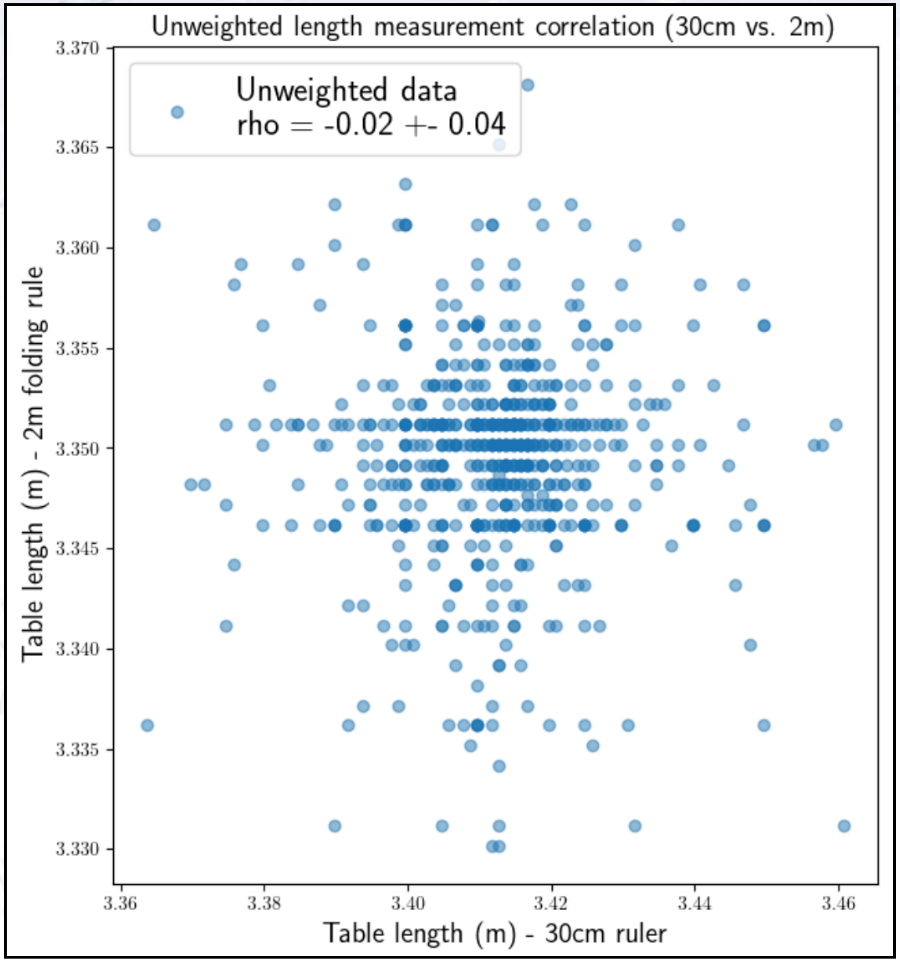
Cross Check

One of the purposes of measuring with both the 30cm ruler and 2m folding rule was to have a **cross check**!

However, it might be that the measurements are **correlated**, and could thus be **commonly shifted and still be in agreement**.

In order to check this, we want to plot the two measurements against each other and calculate the correlation coefficient.

We exclude the outliers, as these tend to be random, but potentially have a large impact.



Result: No significant correlation. Uncertainty is from formula in Barlow.

Cross Check

Now we have gotten two precision results that are uncorrelated. How to cross check if there is any realism in the values and uncertainties?

We compare the 30cm and 2m results.

So far, the results have been blinded, and so the difference is very large:
L30cm - L2m (fully blinded) = 0.06248 ± 0.00050 m (124.2σ → prob=0.0000)

Subtracting the *difference in blinding value* yields the real difference:
L30cm - L2m (partially blinded) = **-0.00102 ± 0.00050** m (**2.0σ** → prob=0.0434)

This means that the two results are reasonably within each other, and the results and their uncertainty are (more) trustworthy.

Weighted analysis

Checking for valid errors

In order to do a weighted analyst, the measurements of course have to have valid uncertainties.

You may wonder why there are negative uncertainties!

The reason is, that this is a (good?) way of putting measurements without uncertainties, without putting NaNs into the table.

```
The 30cm entry L =  3.413 +- -1.000 was not considered valid!
The 30cm entry L =  3.412 +- -1.000 was not considered valid!
The 30cm entry L =  3.388 +- -1.000 was not considered valid!
The 30cm entry L =  3.416 +- -1.000 was not considered valid!
The 30cm entry L =  3.421 +- -1.000 was not considered valid!
The 30cm entry L =  3.415 +- -1.000 was not considered valid!
The 30cm entry L =  3.443 +- -1.000 was not considered valid!
The 30cm entry L =  3.422 +- -1.000 was not considered valid!
The 30cm entry L =  3.410 +- -1.000 was not considered valid!
The 30cm entry L =  3.416 +- -1.000 was not considered valid!
The 30cm entry L =  3.416 +- -1.000 was not considered valid!
The 30cm entry L =  3.405 +- -1.000 was not considered valid!
The 30cm entry L =  3.417 +- -1.000 was not considered valid!
The 30cm entry L =  3.414 +- -1.000 was not considered valid!
The 30cm entry L =  3.430 +- -1.000 was not considered valid!
The 30cm entry L =  3.720 +- -1.000 was not considered valid!
The number of accepted / rejected 30cm points is 991 / 16

The 2m entry L =  3.350 +- -1.000 was not considered valid!
The 2m entry L =  3.361 +- -1.000 was not considered valid!
The 2m entry L =  3.351 +-  0.000 was not considered valid!
The 2m entry L =  3.350 +- -1.000 was not considered valid!
The 2m entry L =  1.361 +-  0.000 was not considered valid!
The 2m entry L =  3.342 +- -1.000 was not considered valid!
The 2m entry L =  3.350 +- -1.000 was not considered valid!
The 2m entry L = -1.014 +- -1.000 was not considered valid!
The 2m entry L =  3.355 +- -1.000 was not considered valid!
The 2m entry L =  3.353 +- -1.000 was not considered valid!
The 2m entry L =  3.349 +- -1.000 was not considered valid!
The 2m entry L =  3.356 +- -1.000 was not considered valid!
The 2m entry L =  3.348 +- -1.000 was not considered valid!
The 2m entry L =  3.350 +- -1.000 was not considered valid!
The 2m entry L = -1.014 +- -0.010 was not considered valid!
The 2m entry L =  3.336 +-  0.000 was not considered valid!
The 2m entry L =  3.341 +- -0.010 was not considered valid!
The 2m entry L =  3.346 +- -0.010 was not considered valid!
The 2m entry L =  3.351 +- -0.010 was not considered valid!
The 2m entry L =  3.426 +- -0.010 was not considered valid!
The 2m entry L =  3.626 +- -0.010 was not considered valid!
The number of accepted / rejected 2m points is 986 / 21
```

"Naive" weighted results

<u>30cm:</u>

Mean = 3.38748 ± 0.00017 m

RMS = undefined!   (N = 991)

<u>2m:</u>

Mean = 3.31618 ± 0.00009 m

RMS = undefined! (N = 986)

Now, this looks very good… BUT!

“Naive” weighted results

30cm:

Mean = 3.38748 ± 0.00017 m

Chi2 = 1787951.4, Ndof = 990, Prob = 0.0!

2m:

Mean = 3.31618 ± 0.00009 m

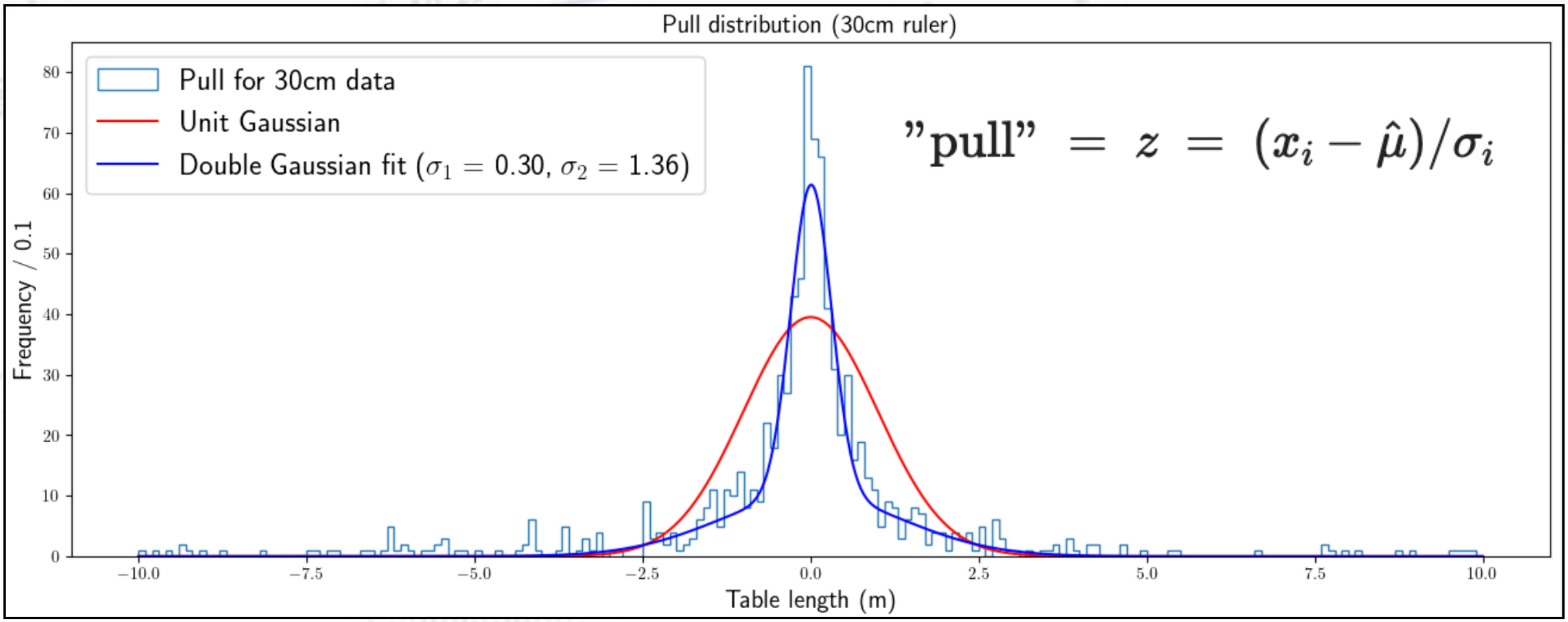
Chi2 = 7824156.4, Ndof = 985, Prob = 0.0!

Now, this looks very good… BUT… While the values of the results may look "alluring" and the uncertainties amazingly small, the ChiSquare reveals that this is not the case. **The measurements disagree enormously when uncertainties are taken into account**. Clearly, the naive approach is way off.

The pull distribution

Considering the quoted uncertainties, we first need to evaluate their quality. The plot to consider is a **PULL** plot, i.e. the distribution of z-values.

The pulls should be unit Gaussian. However, it is far from. In fact, **most pull values are small, which is caused by an overestimation of the uncertainty**. We are too conservative and don't trust, that we can do things fairly accurately.

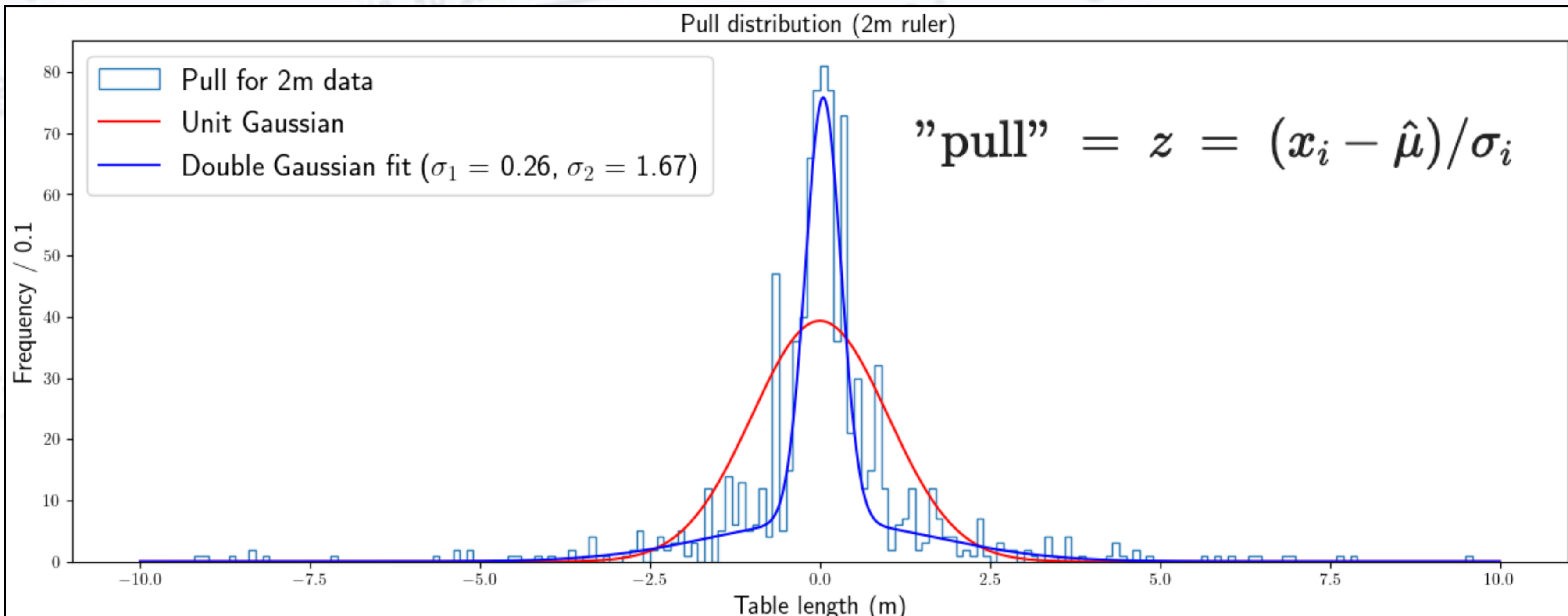


In the case at hand, we take the mean to be the unweighted best result.

The pull distribution

Considering the quoted uncertainties, we first need to evaluate their quality. The plot to consider is a **PULL** plot, i.e. the distribution of z-values.

The pulls should be unit Gaussian. However, it is far from. In fact, **most pull values are small, which is caused by an overestimation of the uncertainty**. We are too conservative and don't trust, that we can do things fairly accurately.



In the case at hand, we take the mean to be the unweighted best result.

Where to select?

Assuming unit Gaussian distributions (not the case, but still) we calculate at what level it is reasonable to discard individual measurement based on their z-value, i.e. how many sigmas they are away from the mean.

This only depends on the number of measurements and $p_{global} = 0.05$, and the result is 4.0σ for both 30cm and 2m data.

```
p_local = 1.0 - (1.0 - p_global)**(1.0/Ndata)
Nsigma = np.abs(stats.norm.ppf(p_local/2.0))
```

Given that the assumption is not really fulfilled, the real level should be set below this value. I chose 80% and 90% of the 4.0σ, i.e. 3.2σ and 3.6σ.

Once the selection level is fitting, we then discard unlikely events (i.e. beyond a certain number of sigmas) and then proceed to calculate the weighted mean (with error, Chi2, and ProbChi2 of course!).

Excluded data due to bad pull

```
Warning! Large pull: L = 3.750 +- 0.013 m     z =   25.98
Warning! Large pull: L = 3.697 +- 0.020 m     z =   14.24
Warning! Large pull: L = 3.323 +- 0.009 m     z =   -9.91
Warning! Large pull: L = 2.815 +- 0.091 m     z =   -6.56
Warning! Large pull: L = 3.250 +- 0.002 m     z =  -81.11
Warning! Large pull: L = 3.395 +- 0.005 m     z =   -3.45
Warning! Large pull: L = 2.422 +- 0.005 m     z = -198.05
Warning! Large pull: L = 3.753 +- 0.010 m     z =   34.08
Warning! Large pull: L = 3.425 +- 0.003 m     z =    4.26
Warning! Large pull: L = 3.085 +- 0.070 m     z =   -4.67
Warning! Large pull: L = 3.115 +- 0.023 m     z =  -12.92
Warning! Large pull: L = 3.595 +- 0.005 m     z =   36.55
Warning! Large pull: L = 3.701 +- 0.030 m     z =    9.63
Warning! Large pull: L = 1.405 +- 0.030 m     z =  -66.91
Warning! Large pull: L = 3.356 +- 0.009 m     z =   -6.25
Warning! Large pull: L = 3.089 +- 0.035 m     z =   -9.24
Warning! Large pull: L = 3.112 +- 0.021 m     z =  -14.30
Warning! Large pull: L = 3.417 +- 0.001 m     z =    4.77
Warning! Large pull: L = 3.405 +- 0.002 m     z =   -3.61
Warning! Large pull: L = 3.409 +- 0.001 m     z =   -3.23
Warning! Large pull: L = 3.743 +- 0.004 m     z =   82.69
Warning! Large pull: L = 3.721 +- 0.005 m     z =   61.75
Warning! Large pull: L = 3.134 +- 0.050 m     z =   -5.56
Warning! Large pull: L = 1.413 +- 0.013 m     z = -153.79
Warning! Large pull: L = 3.142 +- 0.005 m     z =  -54.05
Warning! Large pull: L = 3.107 +- 0.005 m     z =  -61.05
Warning! Large pull: L = 4.355 +- 0.260 m     z =    3.63
Warning! Large pull: L = 3.795 +- 0.100 m     z =    3.83
Warning! Large pull: L = 3.091 +- 0.015 m     z =  -21.42
Warning! Large pull: L = 2.625 +- 0.090 m     z =   -8.75
Warning! Large pull: L = 3.106 +- 0.073 m     z =   -4.19
Warning! Large pull: L = 3.132 +- 0.022 m     z =  -12.74
Warning! Large pull: L = 3.168 +- 0.045 m     z =   -5.43
Warning! Large pull: L = 2.206 +- 0.002 m     z = -603.11
Warning! Large pull: L = 3.094 +- 0.002 m     z = -159.11
Warning! Large pull: L = 3.085 +- 0.005 m     z =  -65.45
Warning! Large pull: L = 3.320 +- 0.005 m     z =  -18.45
Warning! Large pull: L = 3.106 +- 0.050 m     z =   -6.12
Warning! Large pull: L = 3.110 +- 0.010 m     z =  -30.22
Warning! Large pull: L = 3.717 +- 0.061 m     z =    5.00
```

```
Warning! Large pull: L = 3.696 +- 0.050 m     z =    6.93
Warning! Large pull: L = 3.371 +- 0.005 m     z =    4.35
Warning! Large pull: L = 2.350 +- 0.015 m     z =  -66.62
Warning! Large pull: L = 3.746 +- 0.020 m     z =   19.84
Warning! Large pull: L = 3.051 +- 0.002 m     z = -149.13
Warning! Large pull: L = 2.350 +- 0.001 m     z = -999.25
Warning! Large pull: L = 1.351 +- 0.020 m     z =  -99.91
Warning! Large pull: L = 3.122 +- 0.025 m     z =   -9.09
Warning! Large pull: L = 3.631 +- 0.006 m     z =   46.96
Warning! Large pull: L = 3.535 +- 0.005 m     z =   37.15
Warning! Large pull: L = 1.350 +- 0.010 m     z = -199.93
Warning! Large pull: L = 4.550 +- 0.023 m     z =   52.21
Warning! Large pull: L = 3.322 +- 0.001 m     z =  -27.25
Warning! Large pull: L = 3.166 +- 0.020 m     z =   -9.16
Warning! Large pull: L = 3.344 +- 0.001 m     z =   -5.25
Warning! Large pull: L = 3.653 +- 0.025 m     z =   12.15
Warning! Large pull: L = 3.386 +- 0.010 m     z =    3.67
Warning! Large pull: L = 3.660 +- 0.084 m     z =    3.70
Warning! Large pull: L = 3.353 +- 0.001 m     z =    3.75
Warning! Large pull: L = 3.324 +- 0.001 m     z =  -25.25
Warning! Large pull: L = 2.351 +- 0.015 m     z =  -66.55
Warning! Large pull: L = 1.337 +- 0.003 m     z = -670.75
Warning! Large pull: L = 3.641 +- 0.014 m     z =   20.84
Warning! Large pull: L = 1.351 +- 0.005 m     z = -399.65
Warning! Large pull: L = 3.357 +- 0.002 m     z =    3.87
Warning! Large pull: L = 3.299 +- 0.002 m     z =  -25.13
Warning! Large pull: L = 3.286 +- 0.005 m     z =  -12.65
Warning! Large pull: L = 2.636 +- 0.050 m     z =  -14.27
Warning! Large pull: L = 3.361 +- 0.002 m     z =    5.87
Warning! Large pull: L = 2.631 +- 0.010 m     z =  -71.83
Warning! Large pull: L = 2.665 +- 0.150 m     z =   -4.56
Warning! Large pull: L = 3.358 +- 0.002 m     z =    4.37
Warning! Large pull: L = 3.341 +- 0.001 m     z =   -8.25
Warning! Large pull: L = 3.344 +- 0.001 m     z =   -5.25
Warning! Large pull: L = 2.341 +- 0.025 m     z =  -40.33
Warning! Large pull: L = 3.618 +- 0.013 m     z =   20.67
Warning! Large pull: L = 3.341 +- 0.002 m     z =   -4.13
Warning! Large pull: L = 3.378 +- 0.002 m     z =   14.37
Warning! Large pull: L = 3.335 +- 0.002 m     z =   -7.13
Warning! Large pull: L = 3.354 +- 0.001 m     z =    4.75
```

Excluded data due to bad pull

```
Warning! Large pull: L = 3.750 +- 0.013 m      z =   25.98
Warning! Large pull: L = 3
Warning! Large pull: L = 3
Warning! Large pull: L = 2
Warning! Large pull: L = 3
Warning! Large pull: L = 3
Warning! Large pull: L = 2
Warning! Large pull: L = 3
Warning! Large pull: L = 3
Warning! Large pull: L = 3
Warning! Large pull: L = 3.115 +- 0.023 m      z =  -12.92
Warning! Large pull: L = 3.595 +- 0.005 m      z =   36.55
Warning! Large pull: L = 3.701 +- 0.030 m      z =    9.63
Warning! Large pull: L = 1.405 +- 0.030 m      z =  -66.91
Warning! Large pull: L = 3.356 +- 0.009 m      z =   -6.25
Warning! Large pull: L = 3.089 +- 0.035 m      z =   -9.24
Warning! Large pull: L = 3.112 +- 0.021 m      z =  -14.30
Warning! Large pull: L = 3.417 +- 0.001 m      z =    4.77
Warning! Large pull: L = 3.405 +- 0.002 m      z =   -3.61
Warning! Large pull: L = 3.409 +- 0.001 m      z =   -3.23
Warning! Large pull: L = 3.743 +- 0.004 m      z =   82.69
Warning! Large pull: L = 3.721 +- 0.005 m      z =   61.75
Warning! Large pull: L = 3.134 +- 0.050 m      z =   -5.56
Warning! Large pull: L = 1.413 +- 0.013 m      z = -153.79
Warning! Large pull: L = 3.142 +- 0.005 m      z =  -54.05
Warning! Large pull: L = 3.107 +- 0.005 m      z =  -61.05
Warning! Large pull: L = 4.355 +- 0.260 m      z =    3.63
Warning! Large pull: L = 3.795 +- 0.100 m      z =    3.83
Warning! Large pull: L = 3.091 +- 0.015 m      z =  -21.42
Warning! Large pull: L = 2.625 +- 0.090 m      z =   -8.75
Warning! Large pull: L = 3.106 +- 0.073 m      z =   -4.19
Warning! Large pull: L = 3.132 +- 0.022 m      z =  -12.74
Warning! Large pull: L = 3.168 +- 0.045 m      z =   -5.43
Warning! Large pull: L = 2.206 +- 0.002 m      z = -603.11
Warning! Large pull: L = 3.094 +- 0.002 m      z = -159.11
Warning! Large pull: L = 3.085 +- 0.005 m      z =  -65.45
Warning! Large pull: L = 3.320 +- 0.005 m      z =  -18.45
Warning! Large pull: L = 3.106 +- 0.050 m      z =   -6.12
Warning! Large pull: L = 3.110 +- 0.010 m      z =  -30.22
Warning! Large pull: L = 3.717 +- 0.061 m      z =    5.00
```

Largest pull is about 1000, the result of a fine measurement with tiny uncertainty…

…but 1 meter wrong!

```
Warning! Large pull: L = 3.696 +- 0.050 m      z =    6.93
Warning! Large pull: L = 3.371 +- 0.005 m      z =    4.35
Warning! Large pull: L = 2.350 +- 0.015 m      z =  -66.62
Warning! Large pull: L = 3.746 +- 0.020 m      z =   19.84
Warning! Large pull: L = 3.051 +- 0.002 m      z = -149.13
Warning! Large pull: L = 2.350 +- 0.001 m      z = -999.25
Warning! Large pull: L = 1.351 +- 0.020 m      z =  -99.91
Warning! Large pull: L = 3.122 +- 0.025 m      z =   -9.09
Warning! Large pull: L = 3.631 +- 0.006 m      z =   46.96
Warning! Large pull: L = 3.535 +- 0.005 m      z =   37.15
Warning! Large pull: L = 1.350 +- 0.010 m      z = -199.93
Warning! Large pull: L = 4.550 +- 0.023 m      z =   52.21
Warning! Large pull: L = 3.322 +- 0.001 m      z =  -27.25
Warning! Large pull: L = 3.166 +- 0.020 m      z =   -9.16
Warning! Large pull: L = 3.344 +- 0.001 m      z =   -5.25
Warning! Large pull: L = 3.653 +- 0.025 m      z =   12.15
Warning! Large pull: L = 3.386 +- 0.010 m      z =    3.67
Warning! Large pull: L = 3.660 +- 0.084 m      z =    3.70
Warning! Large pull: L = 3.353 +- 0.001 m      z =    3.75
Warning! Large pull: L = 3.324 +- 0.001 m      z =  -25.25
Warning! Large pull: L = 2.351 +- 0.015 m      z =  -66.55
Warning! Large pull: L = 1.337 +- 0.003 m      z = -670.75
Warning! Large pull: L = 3.641 +- 0.014 m      z =   20.84
Warning! Large pull: L = 1.351 +- 0.005 m      z = -399.65
Warning! Large pull: L = 3.357 +- 0.002 m      z =    3.87
Warning! Large pull: L = 3.299 +- 0.002 m      z =  -25.13
Warning! Large pull: L = 3.286 +- 0.005 m      z =  -12.65
Warning! Large pull: L = 2.636 +- 0.050 m      z =  -14.27
Warning! Large pull: L = 3.361 +- 0.002 m      z =    5.87
Warning! Large pull: L = 2.631 +- 0.010 m      z =  -71.83
Warning! Large pull: L = 2.665 +- 0.150 m      z =   -4.56
Warning! Large pull: L = 3.358 +- 0.002 m      z =    4.37
Warning! Large pull: L = 3.341 +- 0.001 m      z =   -8.25
Warning! Large pull: L = 3.344 +- 0.001 m      z =   -5.25
Warning! Large pull: L = 2.341 +- 0.025 m      z =  -40.33
Warning! Large pull: L = 3.618 +- 0.013 m      z =   20.67
Warning! Large pull: L = 3.341 +- 0.002 m      z =   -4.13
Warning! Large pull: L = 3.378 +- 0.002 m      z =   14.37
Warning! Large pull: L = 3.335 +- 0.002 m      z =   -7.13
Warning! Large pull: L = 3.354 +- 0.001 m      z =    4.75
```

Weighted results

30cm:

Mean = 3.41279 ± 0.00026 m

RMS = undefined!   (N = 796)

2m:

Mean = 3.34981 ± 0.00011 m

RMS = undefined! (N = 872)

Now the results are really precise, and the 2m result is about a factor 2.5 more so.

Improvement over the unweighted 30cm / 2m results are factores of 1.8 / 1.7

So the uncertainties carry information about the measurement quality.

Weighted results

<u>30cm:</u>

Mean = 3.41279 ± 0.00026 m

Chi2 = 720.9, Ndof = 795, Prob = 0.97

<u>2m:</u>

Mean = 3.34981 ± 0.00011 m

Chi2 = 770.5, Ndof = 871, Prob = 0.99

Now the results are really precise, and the 2m result is about a factor 2.5 more so.

Improvement over the unweighted 30cm / 2m results are factores of 1.8 / 1.7

So the uncertainties carry information about the measurement quality.

Cross Checks

Once again, we compare the 30cm and 2m weighted results.

Subtracting the *difference in blinding value* yields the real difference:
L30cm - L2m (partially blinded) = **-0.00052 ± 0.00028** m (1**.9σ** → prob=0.06)

This means that the two results are reasonably within each other, and the results and their uncertainty are (more) trustworthy.

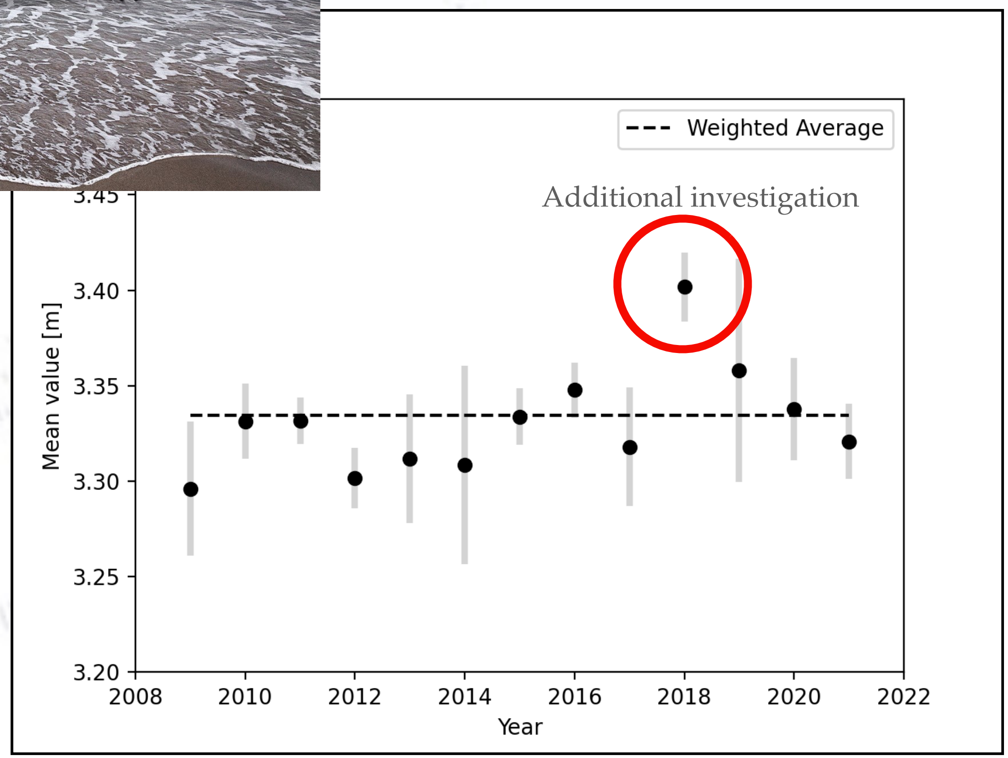
We can also check the unweighted against the weighted results. Here, there is not even a partial unblinding, as they have the same offsets.

30cm: Unweighted-Weighted = -0.00081 ± 0.00053 m (1.5σ → prob = 0.13)
2m: Unweighted-Weighted = -0.00031 ± 0.00021 m (1.5σ → prob = 0.14)

Thus, now the four results (30cm, 2m) x (unweighted, weighted) seem to be in agreement.

A problem?

Things may look very good, yet it remains to investigate the data further.
A question is the homogeneity of the data. And here we find problems!



Somehow, the 2018 data seems different (read: biased) compared to the other years.

This would require further investigations to understand!

Fitting analysis

Fitting for a result

A completely different approach is to fit the RAW data, hence describing **all** data points instead of excluding some.

This approach is philosophically more clean, but certainly not easy!

Challenges:

- Measurements has many different resolutions.
- There are several peaks in the data (30cm case).
- Some measurements are clearly rounded.

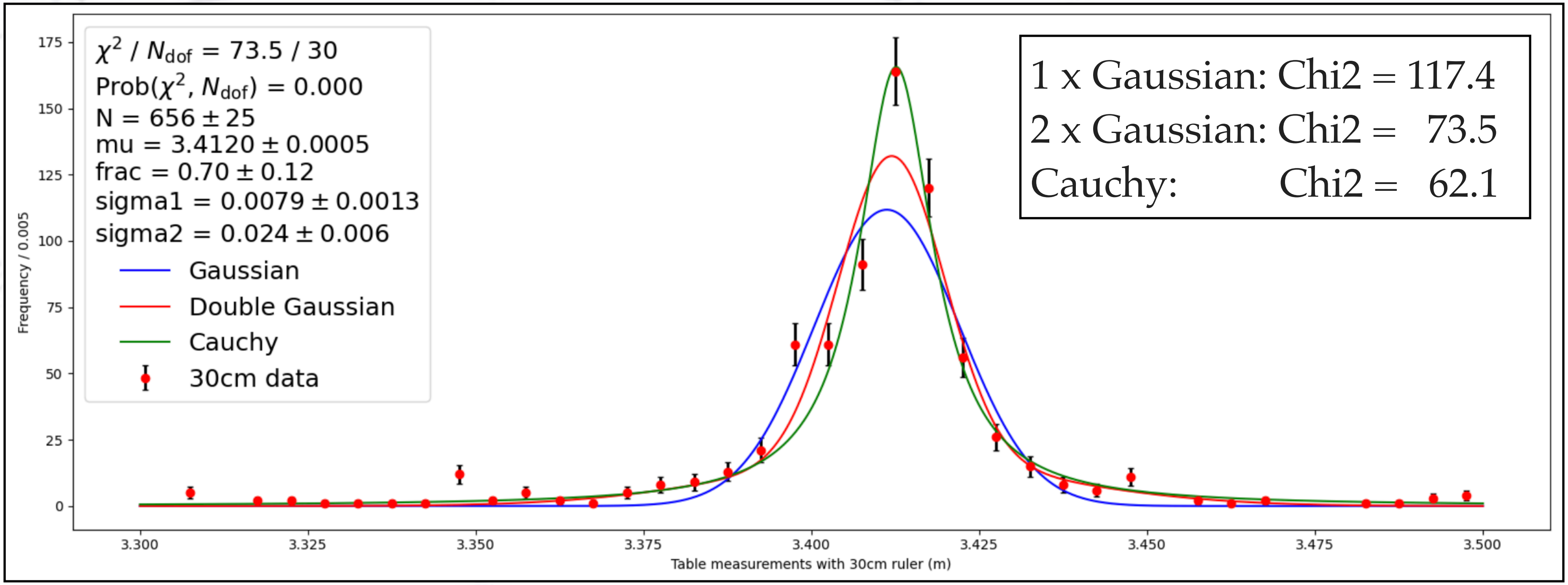
While all of these can be accommodated, it is still a challenge, at the following “fitting around” took me several hours!

Fitting for a result

First step is to establish what PDF the measurements follow.

I have tried the following three:

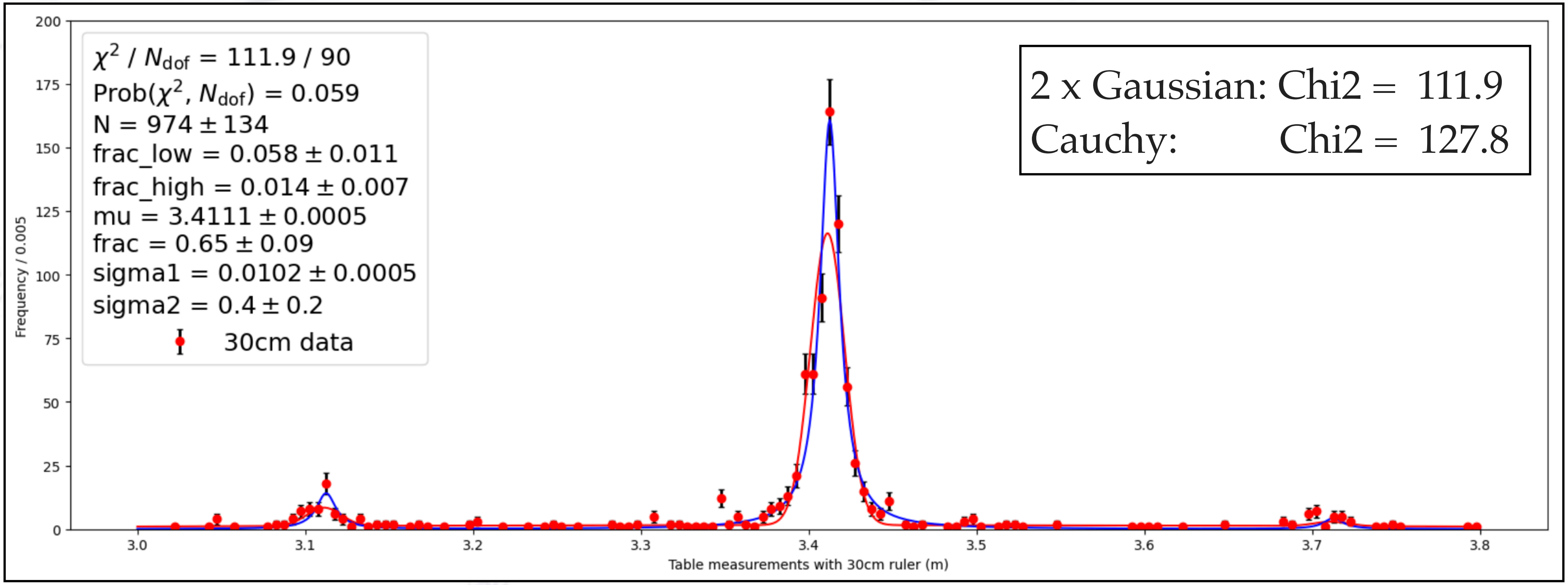
- Single Gaussian: Simplest and mandatory first step.
- Double Gaussian: To accommodate different resolutions.
- Cauchy: Alternative to Gaussian with long tails as expected.



Fitting for a result

The fits converge and gives OK values.However, both models have a problem modelling the far outliers. The second Gaussian starts being used for this, thus not matching the peak.

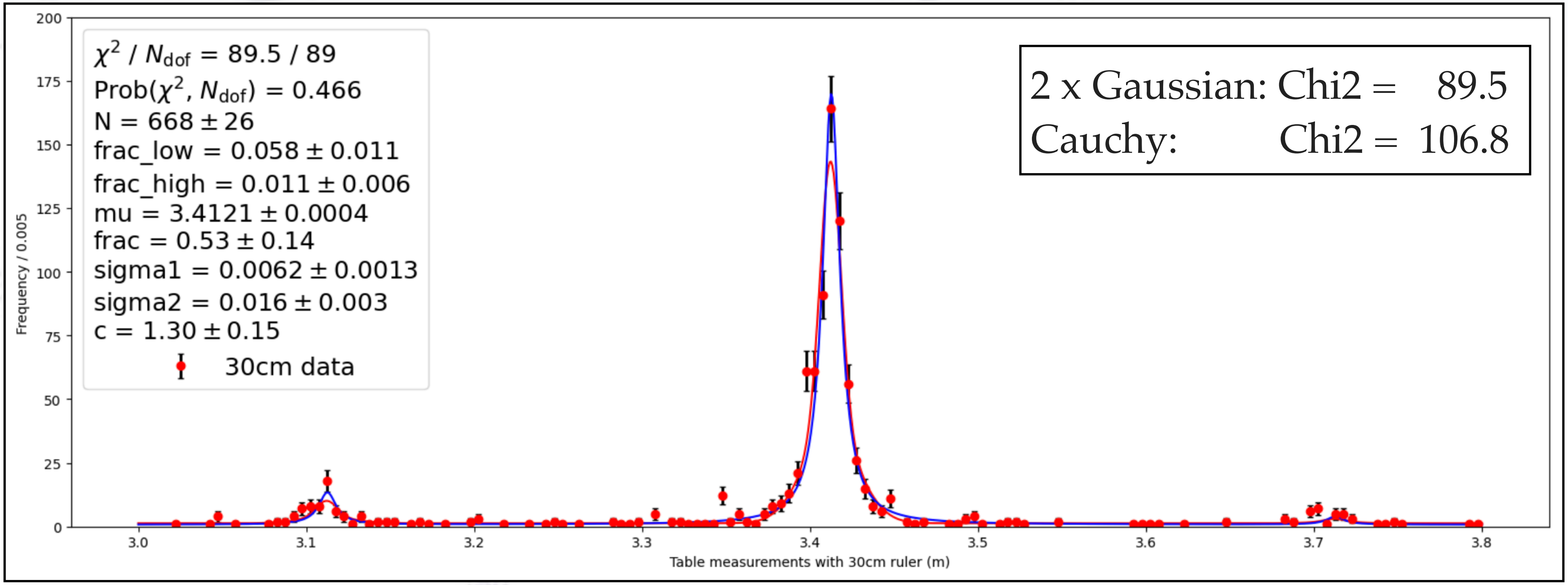
A better model, which avoids this problem should have a separate PDF for the far outliers.



Fitting for a result

The fits converge and gives OK values.However, both models have a problem modelling the far outliers. The second Gaussian starts being used for this, thus not matching the peak.

A better model, which avoids this problem should have a separate PDF for the far outliers. Adding a constant improves the fits, especially the double Gaussian.



Fitting results

Summarising all the fitting results (below), it is clear that the quality of the fit slowly improves.

```
Chi2 - Single Gaussian:              Prob(chi2 = 117.4, Ndof = 94) = 0.052    Mu = 3.411227 +- 0.000455
Chi2 - Double Gaussian:              Prob(chi2 =  73.5, Ndof = 92) = 0.922    Mu = 3.411950 +- 0.000547
Chi2 - 3 x Double Gaussian:          Prob(chi2 = 111.9, Ndof = 90) = 0.059    Mu = 3.411138 +- 0.000452
Chi2 - 3 x Double Gaussian + c:      Prob(chi2 =  89.5, Ndof = 89) = 0.466    Mu = 3.412124 +- 0.000448
ULLH - 3 x Double Gaussian + c:      Likelihood value = -6327.9               Mu = 3.413071 +- 0.000321
ULLH - 3 x Cauchy + c:               Likelihood value = -6241.9               Mu = 3.413009 +- 0.000318
```

The double Gaussian triple peak fit has a good ChiSquare, but the statistics is often low, and hence a likelihood fit is used.

Even with a good PDF, this was not easy to get running, and amendments were needed. However, the result is significantly more precise, and in the end we **reach an uncertainty of 0.32mm**.

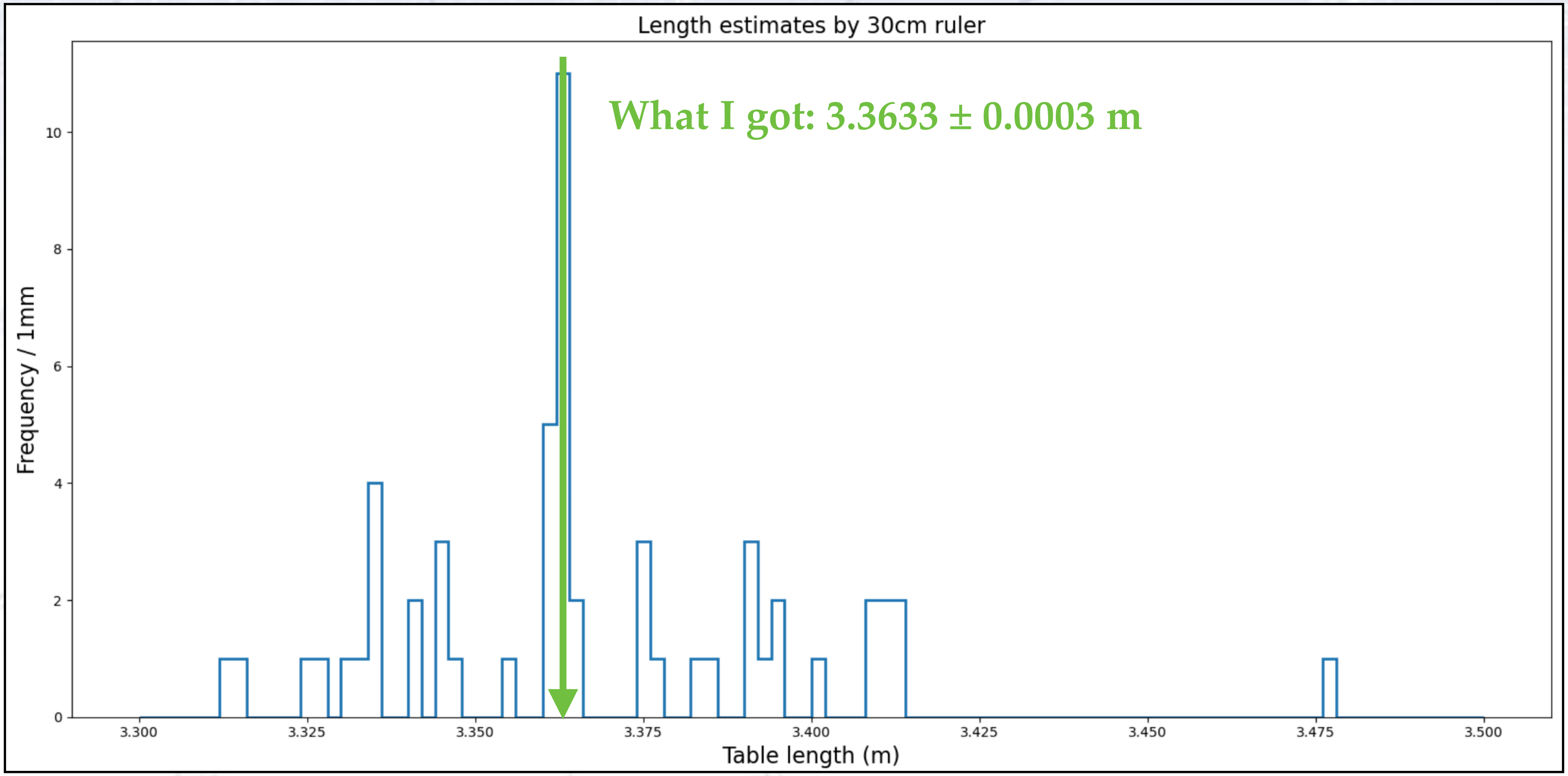
Both value and uncertainty are remarkably comparable to the weighted mean result: Weighted mean = 3.41279 +- 0.00026m Fit: 3.41307 +- 0.00032m

The fitting PDF and method starts being a significant systematic uncertainty!

Student analyses comparison

Your measurement value

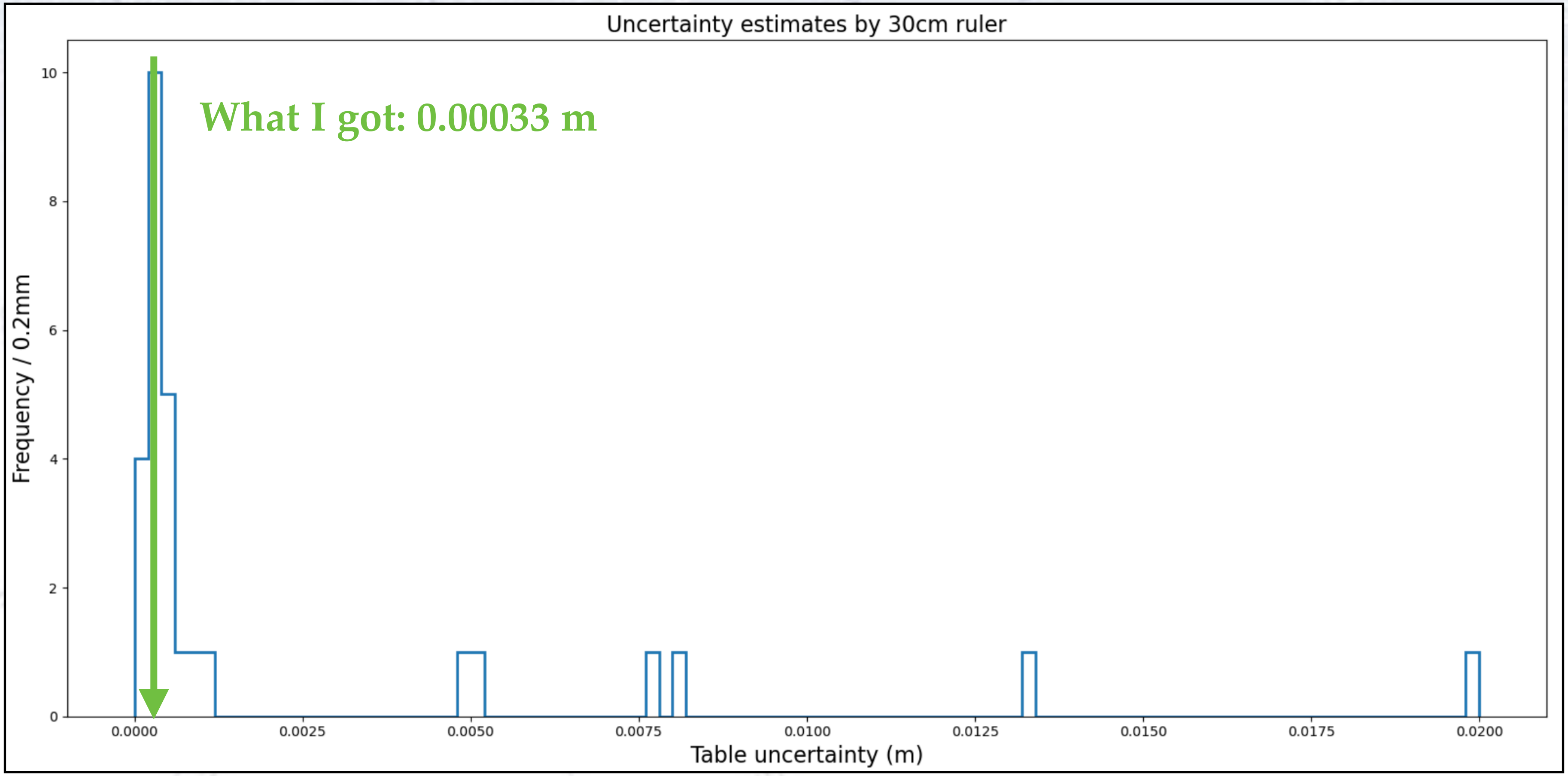
The uncertainties varied quite a bit…. from 3.3 to beyond 3.4.



Estimating uncertainties is (still) hard.

Your measurement uncertainty

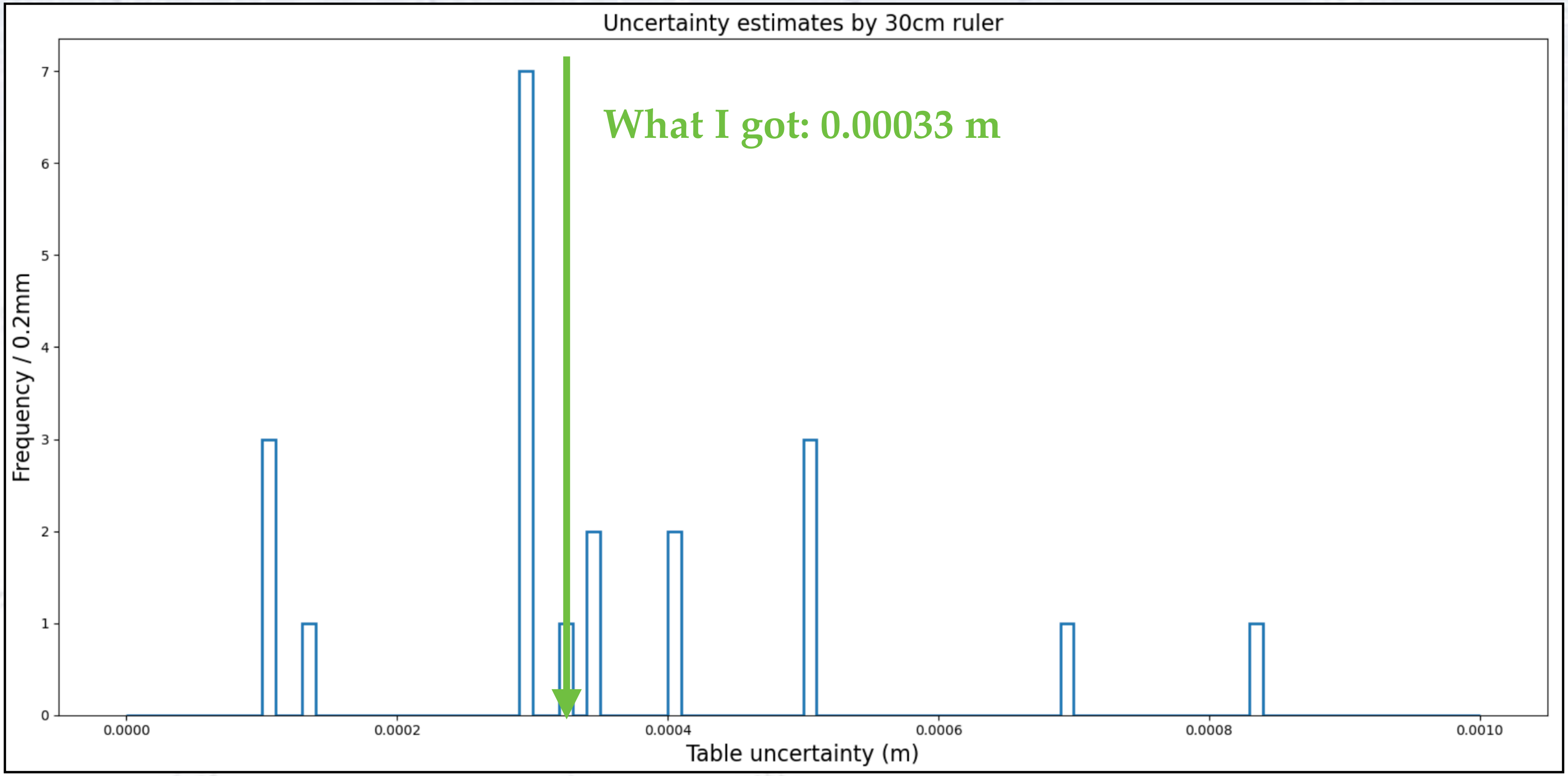
The measurement uncertainties varied even more wildly!!!



The lowest was 0.0001, while the highest was 0.02 (two orders of magnitude).

Your measurement uncertainty

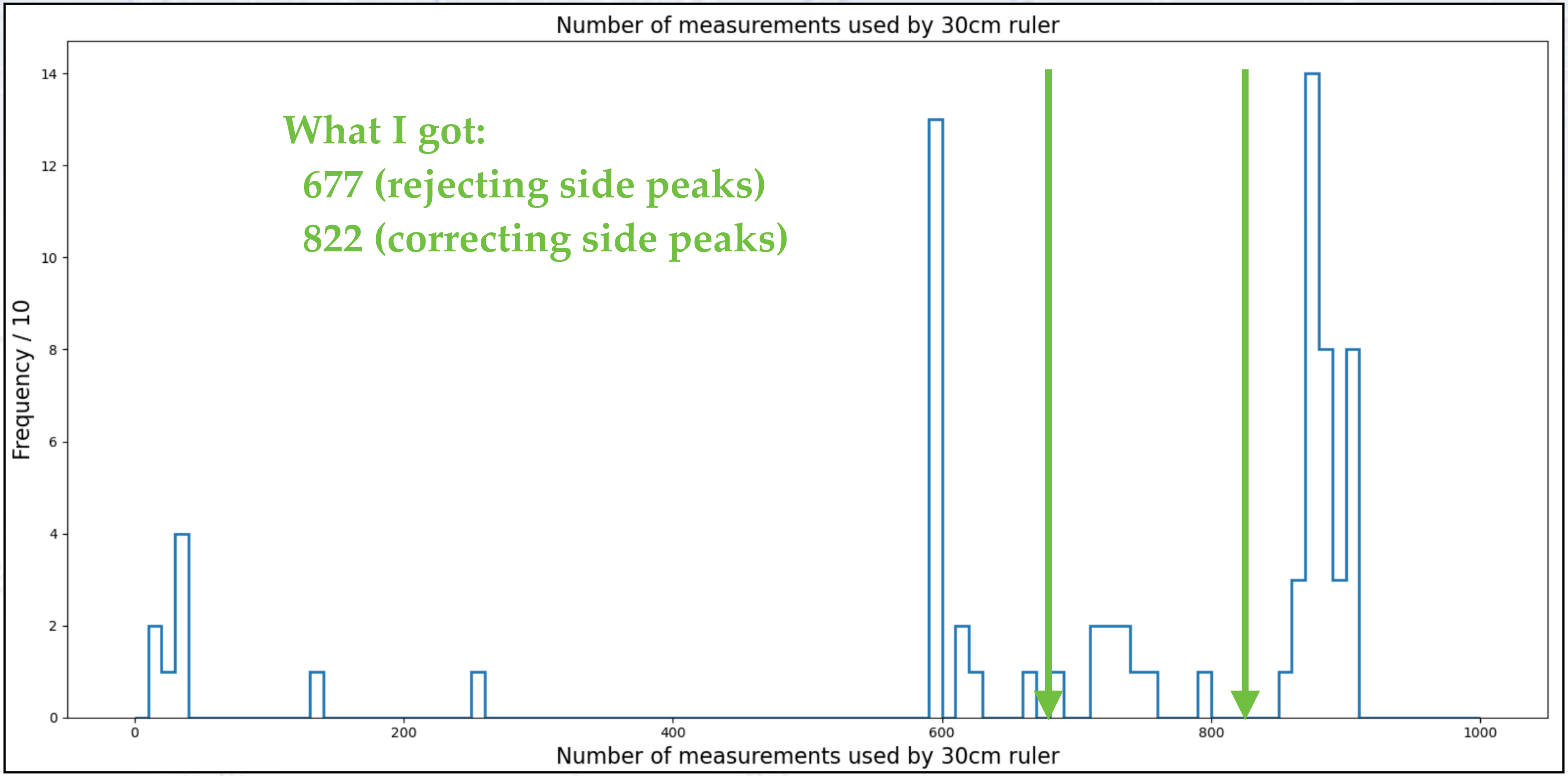
The measurement uncertainties varied even more wildly!!!



The lowest was 0.0001, while the highest was 0.02 (two orders of magnitude).

Your number of measurements

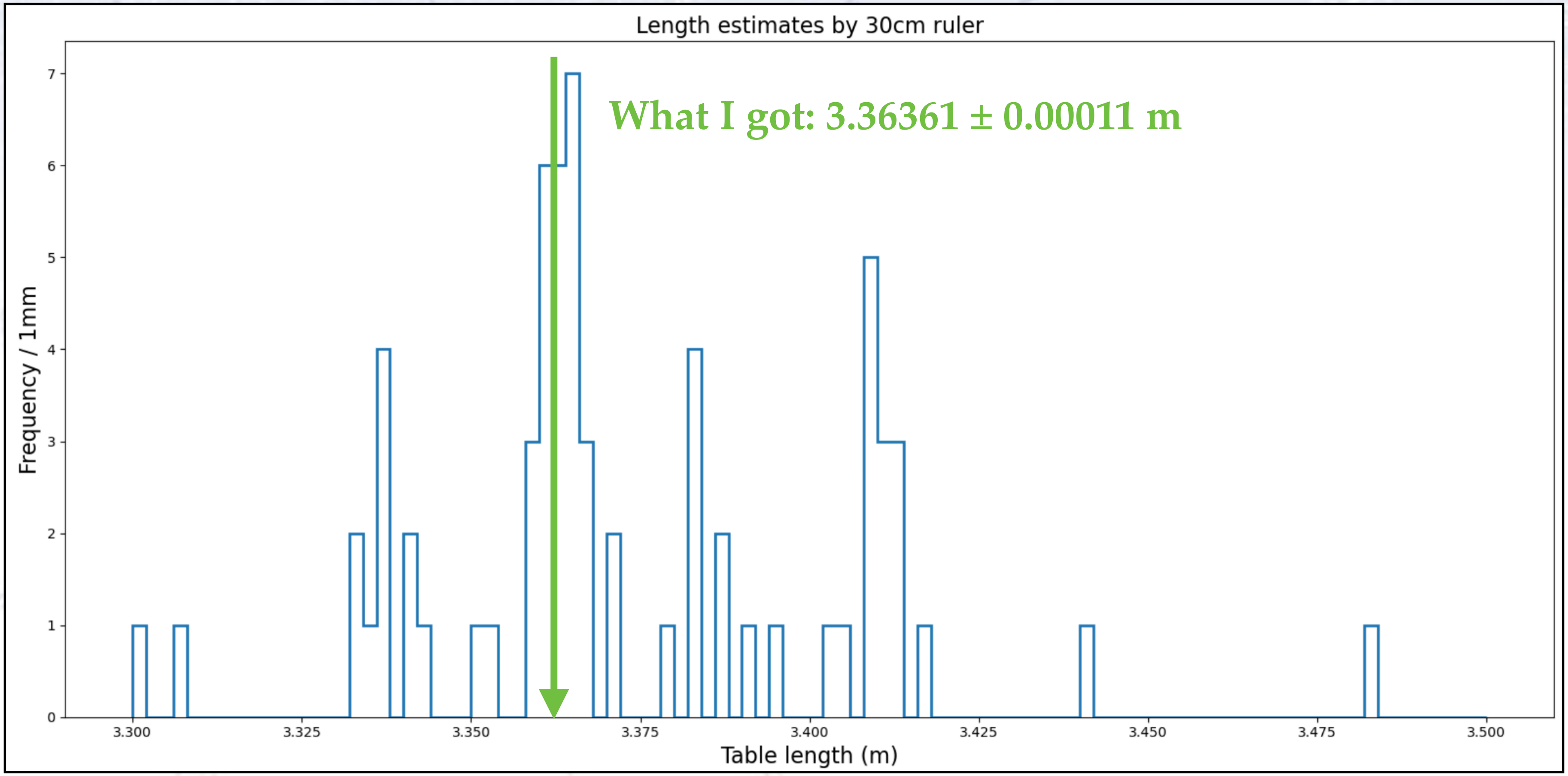
The number of measurements also varied, but some were in the right ballpark.



Remember that the impact is only sqrt(N), and thus not overly important!

Your measurement uncertainty

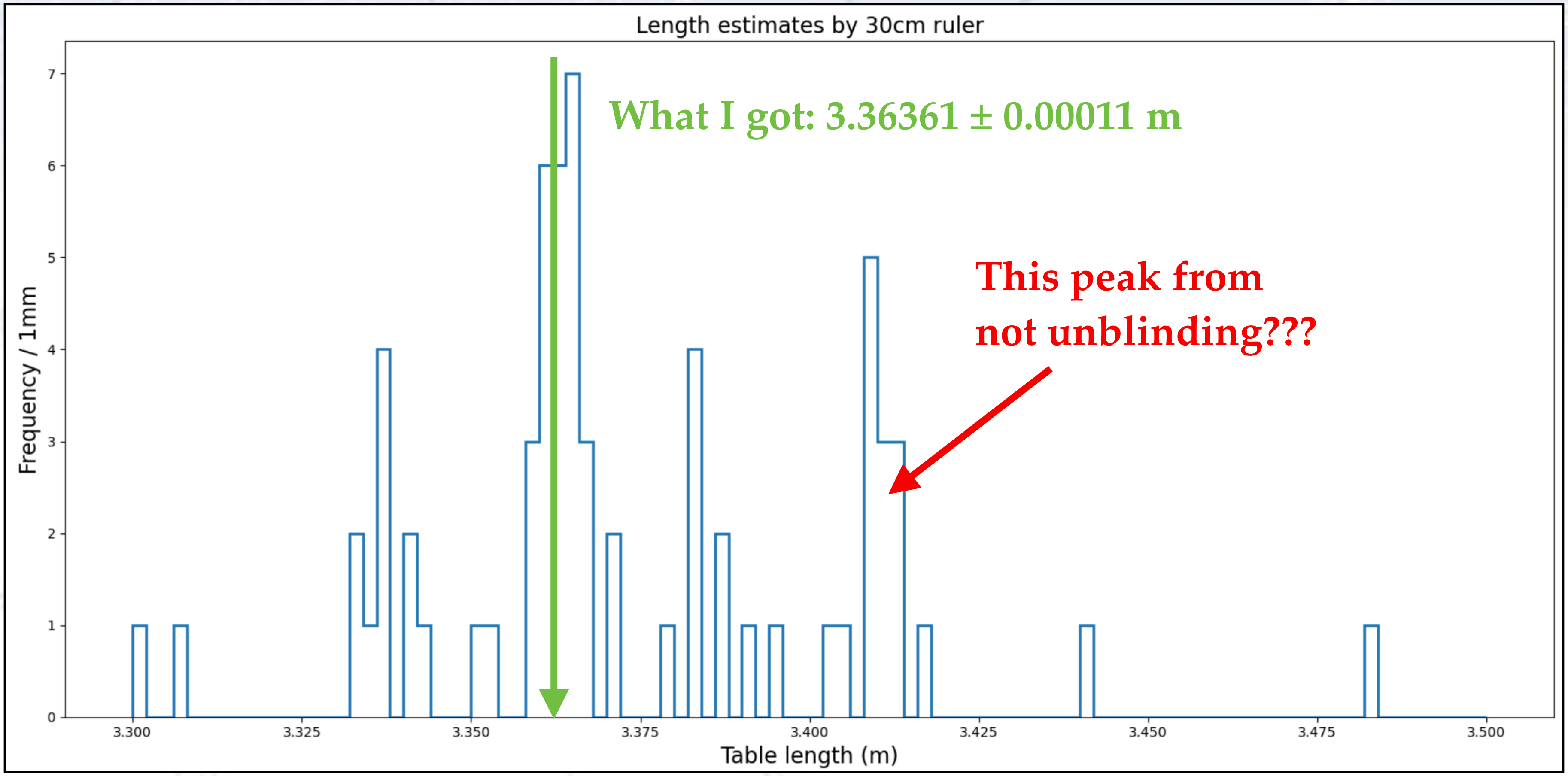
The uncertainties varied quite a bit…. from 3.3 to beyond 3.4.



Estimating uncertainties is (still) hard.

Your measurement uncertainty

The uncertainties varied quite a bit…. from 3.3 to beyond 3.4.

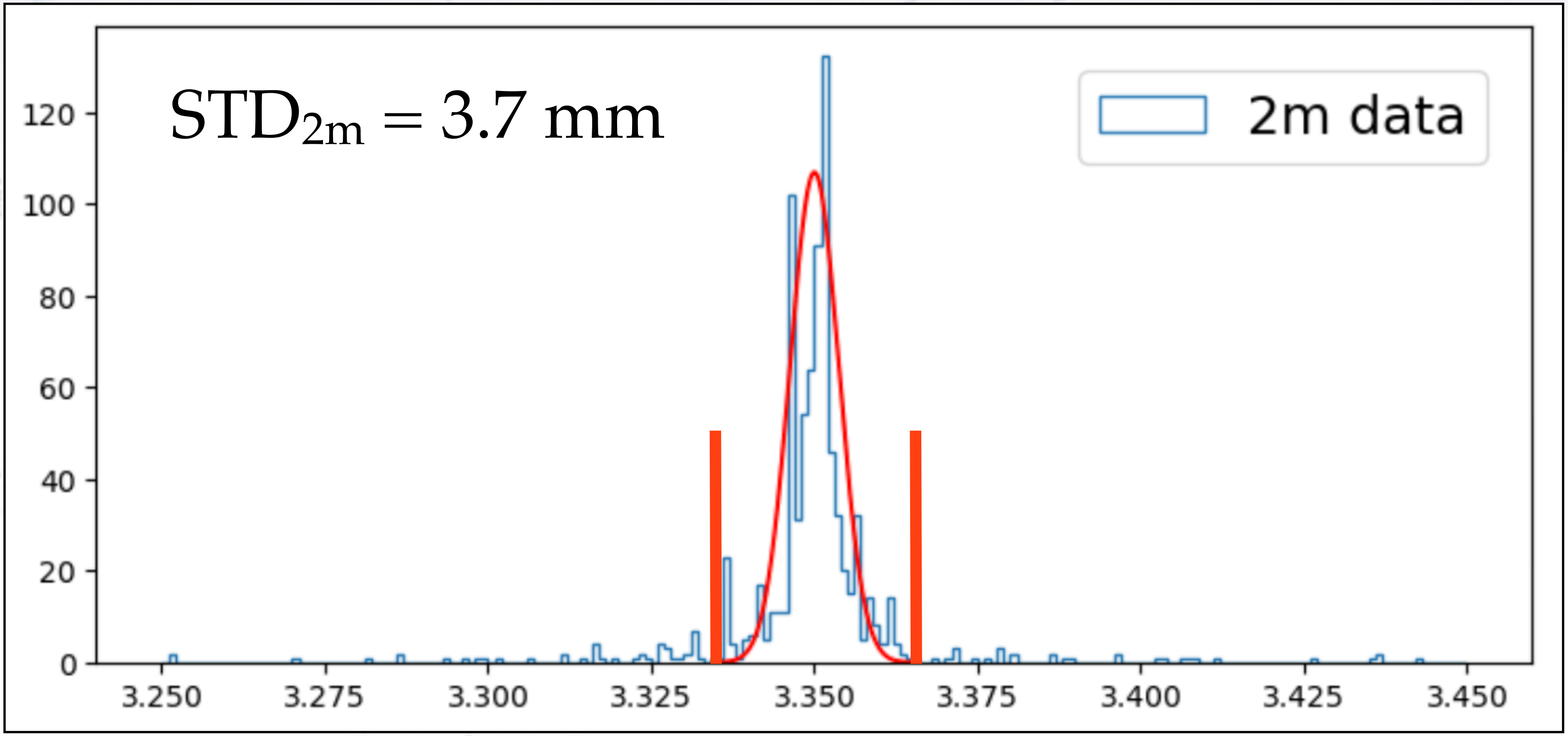


Estimating uncertainties is (still) hard.

The Quick & Dirty

Inspecting the data

There are clearly some **mis-measurements**, which we would like to **exclude**. Using the fitted width, and accepting that this only includes the best measurements, I could **decide** to include measurements within **4 × STD**:



The quick and dirty solution(s):

The above analysis is some work, but once you get the hang of it, and have previous produced (or copied understood) code for the task, it is less cumbersome. Once you see a plot of the data and understand what is happening, **the essense of this data analysis is to only consider the reasonable measurements** and extract a value from these.

The below code does that in a quick and dirty manner, which fails to do all the checks that are needed, if the data is important and the situation calls for it.

```
# Looking at the initial (30cm in particular) plots, it is clear that there is a central peak of valid measurements.
# By eye, the 30cm / 2m peak is at 3.415m / 3.350m (blinded!) and the width about 2.5cm / 1cm, so discarting all measurements
# outside +- 7.5cm / 3cm is a crude but fast way forward.
m30cm = np.abs(L30cm - 3.415) < 0.075
mu30cm = np.mean(np.array(L30cm)[m30cm])
sig30cm = np.std(np.array(L30cm)[m30cm])
print(f"  The crude (unweighted) mean = {mu30cm:7.5f} +- {sig30cm/np.sqrt(len(L30cm)):7.5f} m" +
      f"  (brutally raw (gu)estimate) from {len(np.array(L30cm)[m30cm]):d} measurements")

m2m = np.abs(L2m - 3.350) < 0.030
mu2m = np.mean(np.array(L2m)[m2m])
sig2m = np.std(np.array(L2m)[m2m])
print(f"  The crude (unweighted) mean = {mu2m:7.5f} +- {sig2m/np.sqrt(len(L2m)):7.5f} m" +
      f"  (brutally raw (gu)estimate) from {len(np.array(L2m)[m2m]):d} measurements")
```

```
  The crude (unweighted) mean = 3.41055 +- 0.00057 m  (brutally raw (gu)estimate) from 705 measurements
  The crude (unweighted) mean = 3.34932 +- 0.00022 m  (brutally raw (gu)estimate) from 801 measurements
```

However, this is approach is not advicable. Give it a little more consideration, and **promise yourself that you'll at the very minimum always do the following three key things**:

1. Blind the data
2. Plot the data
3. Make reassuring cross check and info print statements throughout your code.

By doing these things, you might only have to debug your way out of the unforeseen (e.g. negative uncertainties) to get to a decent result, that you can convince yourself and others is in the right ballpark. Good luck.

Notice that even though there has been a lot of analysis, comparison, and discussion of the result, the actual value of the table length has not yet been unblinded!

Conclusions

<u>Specifically on the analysis:</u>

- Greatest improvement came from simply removing mis-measurements!
- Weighted result was a further improvement, but required good uncertainties.
- The uncertainties are accepted as “reasonable”, as they have good pull distributions, and improve the result.
- The 30cm and 2m results match, giving credibility to the stated precision.

<u>More generally:</u>

- What appears to be a trivial task, turns out to require some thought anyhow.
  (Ask yourself how many fellow students would have been able to get a good result and error?)
- There were several choices to be made in the analysis:
  1. Which measurements to accept.
  2. Which uncertainties to accept.
  3. To correct or discard understood mis-measurements.
- All this can be solved with simple Python code.